RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2613

Statistics 1

Friday **11 JUNE 2004** Morning 1 hour 20 minutes

Additional materials:
- Answer paper
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 A random sample of 25 aeroplane arrivals at an airport was taken on a particular day. The numbers of minutes early (–) or late (+) were as follows.

+8	+1	+8	–12	+10	+20	+21	+5	+4	+2
–20	–12	+15	–6	–19	–11	+21	+29	–3	+3
+36	+34	+4	+26	+21					

(i) Copy and complete the following sorted stem and leaf diagram. [The items already shown in the diagram represent the numbers $-20, -19, -12$ and -6, taken from the above list.] [3]

–2	0
–1	9 2
–0	6
0	
1	
2	
3	

(ii) Find the mode and median of the data. Which one of these is more appropriate as a measure of central tendency and why? [3]

(iii) Calculate the inter-quartile range and so determine whether there are any outliers. [5]

(iv) Use medians and ranges to compare the distributions for aeroplanes that landed late and aeroplanes that landed early. [4]

2 Alec and Bob play each other at snooker. Each snooker match consists of a sequence of "frames". On average, Alec wins 60% and Bob wins 40% of the frames. The winner of a match is the first to win n frames, where n is a positive integer.

For example, for the case $n = 3$, the following sequences are three of the possible ways in which Alec could win the match. [**A** represents a frame won by Alec, **B** represents a frame won by Bob.]

AAA **ABAA** **BAABA**

(i) For the case $n = 2$, write down the three sequences in which Alec wins the match. [1]

(ii) For the case $n = 2$, find the probability that

(A) Alec wins the match without Bob winning a frame, [1]

(B) Alec wins the match. [3]

(iii) For the case $n = 3$,

(A) find the probability that Bob wins the match with Alec winning exactly one frame, [3]

(B) show that the probability that Bob wins the match is 0.317 (correct to 3 significant figures). [4]

(iv) Given that Bob wins a match when $n = 3$, what is the probability that the match contains just 3 frames? [3]

3 A market research firm is to conduct a survey of consumer spending at a local supermarket. The firm wants various age groups to be represented.

(i) Describe how a quota sample of size 100 could be taken from the population of customers. [3]

(ii) Explain why the method of stratified random sampling would not be suitable. [1]

The amount spent, £x, by each of the 100 customers sampled is summarised in the following table.

Amount spent, £x	Number of customers
$0 \leqslant x < 10$	6
$10 \leqslant x < 20$	16
$20 \leqslant x < 30$	35
$30 \leqslant x < 50$	18
$50 \leqslant x < 75$	15
$75 \leqslant x < 100$	10

(iii) Illustrate the spending patterns by a histogram, drawn on graph paper. [4]

(iv) Calculate estimates of the mean and standard deviation of the amount spent. [4]

Five of the 100 customers are chosen at random.

(v) Find the probability that just the first two of these five customers spend less than £50. [3]

4 *Joggers* produce packets of crisps. On average, 1 in every 5 packets, chosen randomly, contains a prize voucher.

A box contains 30 packets of *Joggers* crisps.

(i) State the expected number of packets containing a prize voucher and find the probability of exactly this number occurring. [4]

(ii) Show that it is almost certain that at least one packet will contain a voucher. [2]

Sprinters also produce packets of crisps, some of which contain a prize voucher. Jean wishes to test whether the proportion of packets of *Sprinters* crisps with prize vouchers is also $\frac{1}{5}$.

(iii) State suitable null and alternative hypotheses for the test. [2]

Jean buys 12 packets of *Sprinters* crisps and finds no vouchers at all.

(iv) Carry out the hypothesis test at the 5% significance level, giving the critical region for the test and stating your conclusions carefully. [5]

(v) How many packets of crisps would Jean have to buy for the critical region to have a non-empty lower tail? [2]

Mark Scheme

GENERAL INSTRUCTIONS

Marks in the mark scheme are explicitly designated as **M**, **A**, **B**, **E** or **G**.

M marks ("method") are for an attempt to use a correct method (not merely for stating the method).

A marks ("accuracy") are for accurate answers and can only be earned if corresponding **M** mark(s) have been earned. Candidates are expected to give answers to a sensible level of accuracy in the context of the problem in hand. The level of accuracy quoted in the mark scheme will sometimes deliberately be greater than is required, when this facilitates marking.

B marks are independent of all others. They are usually awarded for a single correct answer. Typically they are available for correct quotation of points such as 1.96 from tables.

E marks ("explanation") are for explanation and/or interpretation. These will frequently be sub divisible depending on the thoroughness of the candidate's answer.

G marks ("graph") are for completing a graph or diagram correctly.

- Insert part marks in **right-hand** margin in line with the mark scheme. For fully correct parts tick the answer. For partially complete parts indicate clearly in the body of the script where the marks have been gained or lost, in line with the mark scheme.

- Please indicate incorrect working by ringing or underlining as appropriate.

- Insert total in **right-hand** margin, ringed, at end of question, in line with the mark scheme.

- Numerical answers which are not exact should be given to at least the accuracy shown. Approximate answers to a greater accuracy *may* be condoned.

- Probabilities should be given as fractions, decimals or percentages.

- FOLLOW-THROUGH MARKING SHOULD NORMALLY BE USED WHEREVER POSSIBLE. There will, however, be an occasional designation of '**c.a.o.**' for "correct answer only".

- Full credit MUST be given when correct alternative methods of solution are used. If errors occur in such methods, the marks awarded should correspond as nearly as possible to equivalent work using the method in the mark scheme.

- The following notation should be used where applicable:

FT	Follow-through marking
BOD	Benefit of doubt
W	Work worthy of credit but of no value

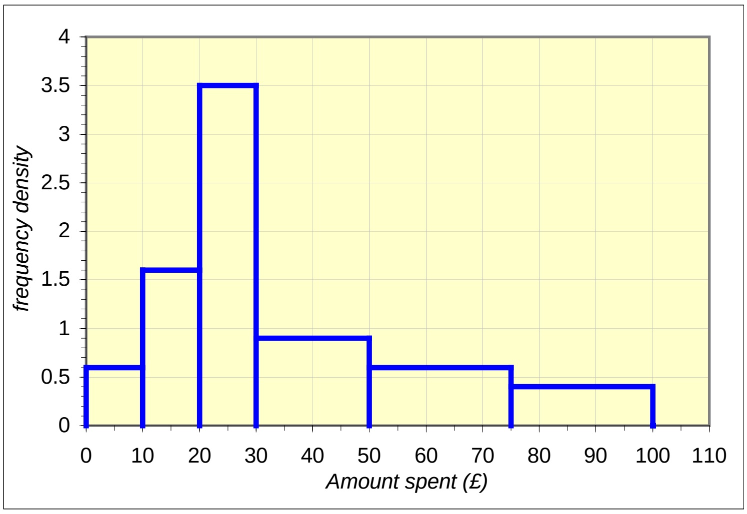
Question 1

(i)	-2 | 0 -1 | 9 2 2 1 -0 | 6 3 0 | 1 2 3 4 4 5 8 8 1 | 0 5 2 | 0 1 1 1 6 9 3 | 4 6 1 | 5 means 15	G1 for negative leaves *sorted or unsorted* [*max. 1 error / omission*] G1 for positive leaves *sorted or unsorted* [*max. 1 error / omission*] G1 for sorted leaves in vertical alignment	**3**
(ii)	Mode = 21 Median = 5 Median is more appropriate ✓ since it may be considered as a representative value ✓ because of its central location ✓ *or similar comment*	B1 **cao** B1 **cao** E1 for any one answer	**3**
(iii)	$Q_1 = [-4.5, -3]$ $Q_3 = 21$ Inter-quartile range = $21 - [-4.5, -3] = [24, 25.5]$ $Q_1 - 1.5 \times IQR = [-4.5, -3] - 1.5 \times [24, 25.5]$ $= [-42.75, -39]$ $Q_3 + 1.5 \times IQR = 21 + 1.5 \times [24, 25.5] = [57, 59.25]$ Hence there are no outliers (according to this definition)	M1 for their $(Q_3 - Q_1)$ A1 **cao** M1 for attempt at finding either boundary A1 for both boundaries E1 for conclusion **cao** *dependent on M1*	**5**
(iv)	Distribution of times for aeroplanes landing late: Median = 12.5 Range = 35 Distribution of times for aeroplanes landing early: Median = –12 Range = 17 Median time late $\approx$ median time early Range of times late > range of times early *Allow spread, variability, dispersion, etc.*	B1 **cao** for 2 or 3 median / range values B1 **cao** for 4^{th} median / range value E1 **cao** comparing medians E1 **cao** comparing ranges	**4**
			15

Question 2

(i)	Three sequences in which Alec wins the match: AA, ABA, BAA	B1 **cao**	**1**
(ii)	(*A*) P(Alec wins wins without Bob winning a frame) $= 0.6^2 = 0.36$ *or* $\frac{9}{25}$	B1 **cao**	**1**
	(*B*) P(Alec wins the match) $= 0.6^2 + 2 \times 0.6^2 \times 0.4$ $= 0.36 + 0.288$ $= 0.648$ *or* 0.65 (to 2 s.f.)	M1 for "$2 \times 0.6^2 \times 0.4$" M1 for sum of 2nd and 3rd order terms A1 **cao**	**3**
(iii)	(*A*) P(Bob wins match with Alec taking just one frame) $= 3 \times 0.4^3 \times 0.6$ $= 0.115$ (to 3 s.f.) *or* 0.12 (to 2 s.f.)	M1 for "$0.4^3 \times 0.6$" M1 for "$3 \times \ldots$ " A1 **cao**	**3**
	(*B*) P(Bob wins the match) = P(Bob wins in 3 frames *or* Bob wins in 4 frames *or* Bob wins in 5 frames) $= 0.4^3 + 3 \times 0.4^3 \times 0.6 + 6 \times 0.4^3 \times 0.6^2$ [$= 0.064 + 0.1152 + 0.13824 = 0.31744$] $= 0.317$ (to 3 s.f.)	M1 for structure s.o.i. M1 for "$6 \times 0.4^3 \times 0.6^2$" M1 for sum of 3rd, 4th and 5th order terms * A1 for accuracy of * *Beware printed answer*	**4**
(iv)	P(Match contains just 3 frames | Bob won the match) $= \dfrac{\text{P(Match contains 3 frames } and \text{ Bob wins the match)}}{\text{P(Bob wins the match)}}$ $= \dfrac{0.064}{0.31744}$ *or* $\dfrac{0.064}{0.317}$ $= 0.202$ (to 3 s.f.) *or* 0.20 (to 2 s.f.) [*or* 0.2 *with working shown*]	M1 for "0.064" on its own or as numerator of quotient M1 for 0.317... as denominator of quotient A1	**3**
			15

Question 3

(i)	• How the interviewer chooses customers (*not* random) • Asks customers from various age groups (strata) • Continues until quota for each stratum is complete *or* determines number in each age group	E1 E1 E1	**3**
(ii)	Stratified random sampling not suitable because Cannot define the population *or* Difficult to define a sampling frame *or* *equivalent*	E1 for reference to population	**1**
(iii)		G1 for linear scaled axes with attempt at a statistical diagram G1 for heights of first 3 bars [0.6k, 1.6k, 3.5k] G1 for height of at least 1 of last 3 joined bars G1 for heights of remaining joined bars [0.9k, 0.6k, 0.4k]	**4**
(iv)	Mid-interval points: 5, 15, 25, 40, 62.5, 87.5 Mean $= \frac{3677.5}{100} =$ £36.78 (to 2 d.p.) *or* £36.80 *or* £37 s.d. $= \sqrt{\frac{189581.25}{100} - 36.775^2} =$ £23.31 (to 2 d.p.) *or* £23.30 *or* £23	B1 for mid-interval points [*max. 1 error*] s.o.i. B1 **cao** M1 for variance A1 **cao**	**4**

(v)	P(just first two customers spend < £50) $= \frac{75}{100}\times\frac{74}{99}\times\frac{25}{98}\times\frac{24}{97}\times\frac{23}{96}$ $=$ 0.00848 (to 3 s.f.) *or* 0.0085 (to 2 s.f.)	M1 for numerators M1 for denominators A1 *Special case:* Max 2 for $0.75^2 \times 0.25^3$ = 0.0088 (2 s.f.)	**3**
			15

Question 4

(i)	Expected number of packets with vouchers $= 30 \times 0.2 = 6$ P(6 packets contain vouchers) $= \binom{30}{6} \times 0.2^6 \times 0.8^{24} = 0.179$ (to 3 s.f.) $= 0.18$ (to 2 s.f.)	B1 **cao** M1 for "$0.2^6 \times 0.8^{24}$" M1 for "$\binom{30}{6} \times p^6 \times q^{24}$" A1 **cao**	**4**
(ii)	P(at least 1 packet contains a voucher) = 1 – P(0 packets contain a voucher) $= 1 - 0.8^{30}$ = 1 – 0.00124 = 0.99876 (5 s.f.) *or* 0.9988 (4 s.f.) *or* 0.999 (3 s.f.) which is very nearly 1 *or equivalent*	M1 for their attempt at "1 – P(X = 0)" A1 **cao** E1 for comment	**3**
(iii)	H_0: $p = 0.2$ H_1: $p \neq 0.2$	B1 for H_0 B1 for H_1	**2**
(iv)	$P(X \leq 0) = 0.0687$ $P(X \geq 5) = 0.0726$ *or* $P(X \geq 6) = 0.0194$ At least one comparison with 0.025 (*or* 0 not in CR) There is not enough evidence to reject the hypothesis that the proportion of packets of crisps with vouchers is 0.2. The critical region for the test is {6, 7, 8, 9, 10, 11, 12} *or* $\{6 \leq x \leq 12\}$	M1 for probability M1 for probability M1 for comparison A1 for conclusion in words *dep on 1st and 3rd M1* A1 for region *dep 3rd M1*	**5**
(v)	From tables the first value of n for which $P(X \leq 0) < 0.025$ is $n = 17$	B1 for value of n = 17 seen	**1**
			15

Examiner’s Report

2613 Statistics 1

General Comments

The overall performance of candidates was moderate. There were fewer than usual scoring high marks but many scripts gaining single figure marks were seen. Even very good candidates found it difficult to score well on question 1. Question 2 was a high scoring question for those who understood the question. Some candidates were confused about what '*n*' meant. In question 3 the histogram was generally well drawn with the follow up work on the mean and standard deviation often completed well. The work on hypothesis testing in question 4 still poses a challenge for many candidates with much unconvincing work seen.

Comments on Individual Questions

Q.1 Although the arithmetic involved in this question was straightforward, candidates' application of basic ideas of data analysis often proved inadequate. Several parts demanded 'thinking skills' which were often sadly absent. The data analysis question is usually a good bet for a mark in 'double figures', but the majority of candidates failed to achieve this goal.

(i) This was surprisingly badly done for a first part. Many were confused by the negative values. The leaves were often sorted as 'all positive' or 'all negative' or the negatives were sorted and the positives not sorted at all or vice versa. Even when sorted the alignment of the numbers was poorly done in many cases.

(ii) Virtually all candidates stated the value of the mode correctly. The median was also well answered, but not as consistently as the mode. Sometimes the median was given as 4.5 from $\frac{n}{2}$ instead of $\frac{n+1}{2}$. Reasons for taking the median as the more appropriate measure of central tendency often lacked conviction.

(iii) Despite there being a very similar question last year, this part was very badly done. Most candidates were able to find 'something' that was their Q_3 – Q_1 with $\frac{n}{4}, \frac{n+1}{4}, \frac{n+2}{4}, \frac{n+3}{4}$ all used for the lower quartile, yielding answers of –5.25, –4.5, –3.75, and –3 respectively, of which only the first was disallowed as a valid answer. The upper quartile was more often correct. Many who got, for example, 21 and –4.5 often expressed the inter-quartile range as 21 – 4.5 = 16.5, rather than 21 – (–4.5) = 25.5. Sadly, the majority still did not know the 'boundaries' for an outlier as Q_1 – 1.5 IQR and Q_3 + 1.5 IQR. The incorrect 'median $\pm$ 1.5 IQR' was so common throughout many scripts. Many did not appreciate that a calculation was needed prior to their conclusion about outliers. Only candidates gaining a method mark for a valid attempt at the 'boundaries' could get credit for saying that there were no outliers.

(iv) Whilst there were many correct answers, many candidates did not appreciate that they needed to separate out the early and late arrivals to get the medians and ranges. The range for the negative results was often given as –17 instead of 17. Some calculated the IQR instead of the range. Often comments were related to the *number* of aeroplanes rather than the *times* of the aeroplanes thus missing the point of the question.

Q.2 This probability question produced the full spectrum of marks. Well prepared candidates found little trouble in achieving close to full marks, but weaker candidates often erroneously tried to apply the binomial distribution and so lost many method and accuracy marks. Accuracy to 3 significant figures is expected, but candidates writing answers correct to 2 significant figures were given full credit.

(i) Most candidates found the three sequences AA, ABA, and BAA. A few wrote down AAB instead of AA.

(ii) For candidates who knew what was going on this proved an easy part but some had no idea how to proceed. A common mistake was to use 3 (or ${}^3C_1$) as the coefficient of $0.6^2 \times 0.4$. [The examiners occasionally saw one fluked answer involving $0.6^3 + 3 \times 0.6^2 \times 0.4$ which also came to 0.648 – but for no credit.]

(iii) In part (A), $3 \times 0.4^3 \times 0.6$ was often achieved, but $4 \times 0.4^3 \times 0.6$ was seen. In part (B), a large number got 0.31744 before rounding to the given answer. There was some fudging by many candidates to get 0.317. The most common errors by candidates 'on the right track' were multiples of 4 and 10 (going down the binomial route) instead of the correct 3 and 6 – or in some cases no multiples at all. Some lists of sequences were not exhaustive with only 8 or 9 instead of 10 ways. A generous mark scheme allowed some credit for candidates in this section.

(iv) Those candidates who got this far often found the correct answer, spotting the need for a conditional probability. The correct response of $\dfrac{0.064}{0.31744}$ was often seen although a common error was $\dfrac{0.064 \times 0.31744}{0.31744}$.

Q.3 Candidates still do not seem very confident in descriptive responses about sampling techniques. Responses to the first two parts of the question were generally very poor. However, the examiners were pleased with the standard of histograms, and calculation of mean and standard deviation for the grouped frequency distribution was encouraging. The probability part met mixed responses, with most candidates **once again** confusing sampling *with* and sampling *without* replacement.

(i) Very few candidates understood what was required here when three bullet points would have sufficed. The worst error was the use of the word 'random' for the selection of people to be asked – 'opportunistic' sampling by the interviewer would have been a satisfactory response.

(ii) Again there were very few sensible responses. Many did not realise that stratified random sampling could not be used because there the population cannot be defined or it would be difficult to set up a sampling frame.

(iii) Attempts at the histogram were pleasing with many aware of the frequency density concept when drawing the heights of the bars.

(iv) For the mean and standard deviation there were surprisingly few fully correct attempts. The condoned mistakes – not giving answers to 2 d.p. and omitting the '£' sign were very common. Curiously 36.775 was frequently rounded to 36.76 instead of 36.78. To find the standard deviation most used their rounded mean instead of the exact value – seemingly unaware that a rounding error is almost certain to result, as in fact it did. The examiners saw the usual problems of calculating $\sum (fx)^2$ or $\sum f^2x$ or even $(\sum fx)^2$.

Candidates should be reminded that processing of data on calculators, scientific or graphical, should be encouraged, where appropriate. Full marks are given to correct answers.

(v) Most candidates assumed this was a binomial probability and the most common response was ${}^5C_2 \times 0.75^2 \times 0.25^3$. Very few recognised the ‘without replacement’ nature of the question.

Q.4 A full spectrum of marks was seen. This remains a challenging topic for some candidates, where many misconceptions abound.

(i) Almost all candidates found the expected number of 6, but a small minority failed to compute the associated probability. $P(X = 5)$ was sometimes seen instead of $P(X = 6)$.

(ii) A common error was to express $P(X \geq 1)$ as $1 – P(X = 1)$ instead of $1 – P(X = 0)$. A few candidates attempted $1 – [P(X = 0) + P(X = 1)]$. Many responses failed to gain full credit by not explaining why the event was ‘almost certain’, i.e. the probability was ‘very close to 1’.

(iii) Most candidates correctly stated the hypotheses, but the examiners continue to be surprised, despite previous warnings, by sloppy notation such as ‘$H_0 = 0.2$’, which is penalised. The parameter p should always be used in such statements.

(iv) Many candidates were not confident with the two-tail test. Even when a two-tailed test was identified, many still used 5% at both ends (i.e. 10% in total). Many were confused by the empty lower tail but most managed to write down the probability 0.0687.

Many only considered one tail end even though their H_1 was $p \neq 0.2$. Using 0.05 rather than 0.025 meant that the final two accuracy marks could not be scored. Many candidates still do not know the meaning of *critical region*. Comparison with 0.025 was expected to determine the ‘critical value’ of 6 before writing down the values in the critical region.

(v) The majority got either the correct $n = 17$ or $n = 14$ (presumably from a one-tail approach). Those who could not do it guessed various numbers from 0 to 20.