OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Wednesday **16 JUNE 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 3 printed pages and 1 blank page.

1 **(a)** Express $3\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $3\sin\theta - 4\cos\theta = 5$, where $0° \leqslant \theta < 360°$. [5]

(b) Calculate $\int_0^1 x\,\mathrm{e}^{2x}\,\mathrm{d}x$, giving your answer in terms of e. [4]

(c) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xy.$$ [4]

(d) Given that $y = \sin^3 x$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\cos x - 3\cos^3 x$. [3]

[Total 16]

2 Fig. 2 shows a section of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{9-3x}{(1+x)(2-x)}$.

The lines l_1 and l_2 are asymptotes to the curve.

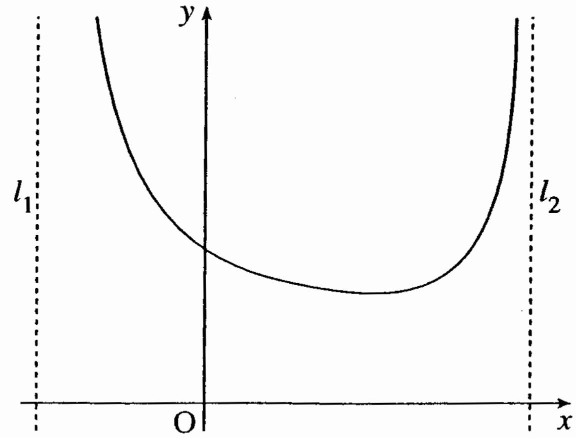


Fig. 2

(i) Write down the equations of the two asymptotes l_1 and l_2. [2]

(ii) Express $\mathrm{f}(x)$ in partial fractions. [3]

(iii) Using suitable binomial expansions, show that, for small values of x, $\mathrm{f}(x)$ may be approximated by the quadratic function $\mathrm{g}(x)$, where

$$\mathrm{g}(x) = 4.5 - 3.75x + 4.125x^2.$$ [5]

(iv) Verify that $\mathrm{f}(0) = \mathrm{g}(0)$ and that $\mathrm{f}'(0) = \mathrm{g}'(0)$. [5]

[Total 15]

3 The lines l_1 and l_2 have equations as follows:

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix},$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

(i) The point A lies on l_1 and has parameter $\lambda = 2$. The point B lies on l_2 and has parameter $\mu = 3$. Find the distance AB. [4]

(ii) Verify that the point $C(1, 1, 2)$ lies on l_1 and l_2.

Calculate angle ACB. [6]

(iii) Verify that the vector $\mathbf{n} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ is normal to the plane containing l_1 and l_2.

Hence or otherwise find the cartesian equation of this plane. [4]

[Total 14]

4 A curve C has equation $x^2 - 8y + 4y^2 = 0$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{x}{4(1-y)}$. [3]

(ii) Hence find the coordinates of the two points on the curve where the gradient is zero, and the two points where the tangent to the curve is parallel to the y-axis. [6]

(iii) Verify that

$$x = 2\cos\theta, \quad y = 1 + \sin\theta$$

are parametric equations for C. [2]

(iv) The curve D has parametric equations

$$x = 2\cos\theta, \quad y = \sin\theta.$$

What type of curve is D? Specify the transformation that maps the curve D onto the curve C.

Sketch the curves D and C on the same set of coordinate axes. [4]

[Total 15]

Mark Scheme

General Instructions

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓
   - For error in follow through work, use ✗

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not be earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

c.a.o.	: correct answer only
b.o.d.	: benefit of doubt (where full work is not shown)
X | X	: work of no mark value between crosses
s.o.i.	: seen or implied
s.c.	: special case (as defined in the mark scheme)
w.w.w	: without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting, send a further sample of about 40 scripts, following the same procedure as in para 2.

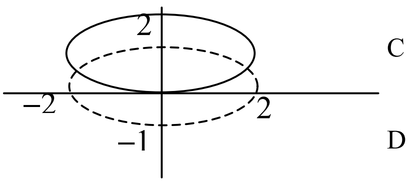
4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

1(a) $3\sin\theta - 4\cos\theta = R\sin(\theta - \alpha)$ $= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 3$, $R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2$, $R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 53.13°$ $5\sin(\theta - 53.13°) = 5$ $\Rightarrow \theta - 53.13° = 90°$ $\Rightarrow \theta = 143.13°$	M1 B1 B1 M1 A1cao [5]	Identity s.o.i by correct answers or two correct equations. Accept 53.1° or 0.93 radians Or sin(θ–53.13°)=1 but not if followed by sin θ – sin 53.13°=1 Accept 143.1° or 2.50radians
(b) $\int_0^1 xe^{2x}dx$ let $u = x$, $dv/dx = e^{2x}$ $\Rightarrow v = ½ e^{2x}$ $= \left[x.\frac{1}{2}e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}.1dx$ $= \frac{1}{2}e^2 - \left[\frac{1}{4}e^{2x}\right]_0^1$ $= e^2/2 - e^2/4 + ¼$ $= \frac{e^2+1}{4}$	M1 A1 A1ft A1c.a.o. [4]	Integration by parts. The correct choice of u and $\frac{dv}{dx}$ plus an attempt to integrate e^{2x} $x.\frac{1}{2}e^{2x}$ $-(\frac{1}{4}e^{2x})$ or f.t their v. Ignore subsequent errors in simplification
(c) $\frac{dy}{dx} = xy$ $\Rightarrow \int \frac{1}{y}dy = \int xdx$ $\Rightarrow \ln y = \frac{1}{2}x^2 + c$	M1 A1 A1 A1 [4]	Separating variables. Condone the omission of the integral signs <u>or</u> dx,dy $\ln y$ $\frac{1}{2}x^2$ $+ c$ (no need to solve for y)
(d) $y = \sin^3 x$ $\Rightarrow dy/dx = 3\sin^2 x\cos x$ $= 3(1 - \cos^2 x)\cos x$ $= 3\cos x - 3\cos^3 x$ *	M1 A1 E1 [3] Total [16]	Chain rule Or using a product and double angle formulae: M1 for the differentiation of a product (correct form) <u>and</u> the chain rule as appropriate. A1 any correct result E1 the required result

2 (i) $x = -1, x = 2$	B1B1 [2]	Accept $x = -1, 2$ SC $-1, 2$ seen B1
(ii) $\dfrac{9-3x}{(1+x)(2-x)} = \dfrac{A}{1+x} + \dfrac{B}{2-x}$ $\Rightarrow 9 - 3x = A(2 - x) + B(1 + x)$ $x = 2 \Rightarrow 3 = 3B \Rightarrow B = 1$ $x = -1 \Rightarrow 12 = 3A \Rightarrow A = 4$	M1 B1 B1 [3]	Correct identity s.o.i. by A and B correct <u>or</u> two correct equations.
(iii) $f(x) = \dfrac{4}{1+x} + \dfrac{1}{2-x}$ $= 4(1 + x)^{-1} + ½ (1 - ½ x)^{-1}$ $= 4(1 - x + x^2 - \ldots) + ½ (1 + ½ x + ¼ x^2 + \ldots)$ $= 4.5 - 3.75 x + 4.125 x^2 + \ldots$*	M1 M1 A1 A1 E1 [5]	$½(1- \dfrac{1}{2} x)^{-1}$ or binomial series for $(2-x)^{-1}$ binomial series with $n = -1$ $1 - x + x^2 - \ldots$ $1 + ½ x + ¼ x^2 + \ldots$
(iv) $f(0) = 4.5 = g(0)$ $f'(x) = -\dfrac{4}{(1+x)^2} + \dfrac{1}{(2-x)^2}$ **or** $f'(x) = \dfrac{(2+x-x^2)(-3)-(9-3x)(1-2x)}{(2+x-x^2)^2}$ $f'(0) = -\dfrac{4}{1} + \dfrac{1}{4} = -3.75$ $g'(x) = -3.75 + 8.25 x$ $\Rightarrow g'(0) = -3.75$ $\Rightarrow f'(0) = g'(0)$	E1 M1 A1 M1 B1 [5] Total [15]	 Must see $\dfrac{-1}{(1+x)^2}$ <u>or</u> $\dfrac{\pm 1}{(2-x)^2}$ M1 correct form of the quotient rule Without wrong working No 4.5 <u>and</u> <u>either</u> -3.75 <u>or</u> $8.25x$ Without wrong working

3 (i) A: $\begin{pmatrix}0\\1\\0\end{pmatrix}+2\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}2\\1\\4\end{pmatrix}$ so A is (2, 1, 4) B: $\begin{pmatrix}0\\2\\2\end{pmatrix}+3\begin{pmatrix}-1\\1\\0\end{pmatrix}=\begin{pmatrix}-3\\5\\2\end{pmatrix}$ so B is (–3, 5, 2) $AB^2 = (2+3)^2 + (1-5)^2 + (4-2)^2$ $= 45$ $\Rightarrow AB = \sqrt{45} = 6.708\ldots$	B1 B1 M1 A1 cao [4]	Accept column vectors Distance formula. Ft their A and B. Condone one slip. Implied by 45 or the correct result √45 or 6.708…Accept 6.71
(ii) On l_1: $\begin{pmatrix}\lambda\\1\\2\lambda\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ when $\lambda = 1$.(by inspection) On l_2: $\begin{pmatrix}-\mu\\2+\mu\\2\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ when $\mu = -1$. Angle ACB = angle between $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ $\cos\theta = \dfrac{1\times(-1)+0\times1+2\times0}{\sqrt{5}\times\sqrt{2}} = -\dfrac{1}{\sqrt{10}}$ $\Rightarrow \theta = 108.4°$	B1 B1 M1 M1 A1 A1 cao [6]	**Or** from $\begin{pmatrix}0\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}0\\2\\2\end{pmatrix}+\mu\begin{pmatrix}-1\\1\\0\end{pmatrix}$ $\Rightarrow \lambda = -\mu;\ 1 = 2 + \mu;\ 2\lambda = 2$ or use of **CA** = $\begin{pmatrix}1\\0\\2\end{pmatrix}$ and **CB** = $\begin{pmatrix}-4\\4\\0\end{pmatrix}$ ft their A and B Use of scalar product ft their vectors $-1/\sqrt{10}$ allow + for this A1but final answer must be obtuse .Accept 1.89 radians
(iii) $\begin{pmatrix}2\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\0\\2\end{pmatrix} = 2\times1 + 2\times0 + (-1)\times2 = 0$ $\begin{pmatrix}2\\2\\-1\end{pmatrix}.\begin{pmatrix}-1\\1\\0\end{pmatrix} = 2\times(-1) + 2\times1 + (-1)\times0 = 0$ so **n** is perpendicular to l_1 and l_2. Equation of plane is $2x + 2y - z = d$ Substituting $x = 1, y = 1, z = 2$, $\Rightarrow 2 + 2 - 2 = d \Rightarrow d = 2$ So plane is $2x + 2y - z = 2$.	B1 B1 M1 A1 [4] Total [14]	May also use **CB** = $\begin{pmatrix}-4\\4\\0\end{pmatrix}$ or **AB**= $\begin{pmatrix}-5\\4\\-2\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\end{pmatrix}-\begin{pmatrix}0\\2\\2\end{pmatrix}=\begin{pmatrix}0\\-1\\-2\end{pmatrix}$ (or use of the vector equation of the plane (in a correct form and elimination of the (parameters (or the correct evaluation of the vector (product of two appropriate vectors

4 (i) $2x - 8\dfrac{dy}{dx} + 8y\dfrac{dy}{dx} = 0$ $\Rightarrow 2x = 8\dfrac{dy}{dx} - 8y\dfrac{dy}{dx} = (8-8y)\dfrac{dy}{dx}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{2x}{8-8y} = \dfrac{x}{4(1-y)}$ **or** $x = \pm\sqrt{8y-4y^2} \Rightarrow \dfrac{dx}{dy} = \pm\dfrac{1}{2}\dfrac{8-8y}{\sqrt{8y-4y^2}}$ $= \dfrac{4-4y}{x} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{x}{4-4y}$	M1 M1 E1 M1 M1 E1	Differentiating implicitly Collecting terms If the result is verified by integration give M1 for separating the variables <u>and</u> integrating both sides, and M1 for including an arbitrary constant. Then E1 for a justification that the constant can be zero Applying the chain rule Using $\dfrac{dy}{dx} = 1/\dfrac{dx}{dy}$ Result
(ii) $\dfrac{dy}{dx} = 0$ when $x = 0$ $\Rightarrow -8y + 4y^2 = 0$ $\Rightarrow 4y(-2 + y) = 0$ $\Rightarrow y = 0$ or $y = 2$ so (0, 0) and (0, 2) $\dfrac{dy}{dx}$ is 'infinite' when $y = 1$ $\Rightarrow x^2 - 8 + 4 = 0, \Rightarrow x^2 = 4, x = \pm 2$ so (2, 1) and (–2, 1)	M1 A1, A1 [3} M1 A1, A1 [3]	Special cases; Both points correct and no wrong working B3 One correct point without any working B0 Both points correct but with wrong working is B1 Special cases as above
(iii) $x^2 - 8y + 4y^2$ $= 4\cos^2\theta - 8(1 + \sin\theta) + 4(1 + \sin\theta)^2$ $= 4\cos^2\theta - 8 - 8\sin\theta + 4 + 8\sin\theta + 4\sin^2\theta$ $= 4 - 8 + 4 = 0$ *	M1 E1 [2]	Substituting $x = 2\cos\theta$, $y = 1 + \sin\theta$ into the equation.(condone one slip) or the use of $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ
(iv) D is an ellipse C is D translated 1 unit in y-direction 	B1 B1 B1 B1 [4]	Accept 'oval' 'shift ' or 'move up' are B0 but accept 'vector D→C is $\begin{pmatrix}0\\1\end{pmatrix}$' or 'vector C→D is $\begin{pmatrix}0\\-1\end{pmatrix}$ Assume that the translation applies to D unless otherwise stated sketch of D, labelled sketch of C, labelled SC both curves correct but not labelled or labelled incorrectly B!

2603 B Comprehension Question

What is the shape of an egg ?

1.

x	-1	-0.5	0	0.5	1
Scale factor	1.5	1.25	1.0	0.75	0.5

One correct value **B1**
The rest **B1**

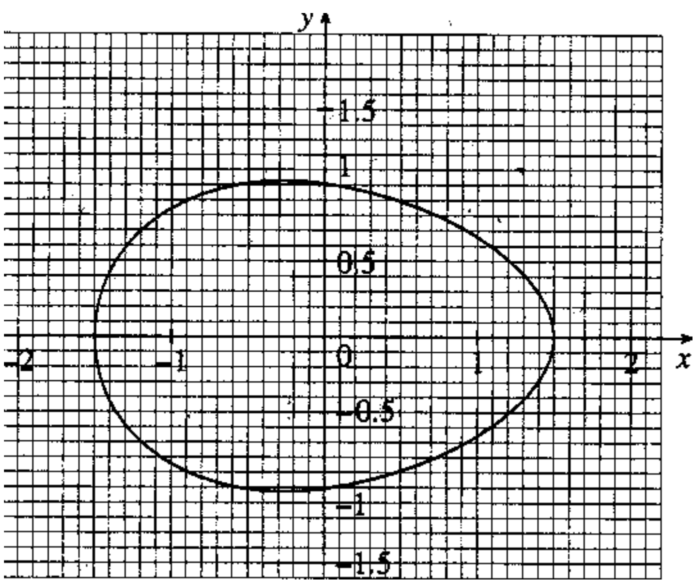
2. (i) (*A*) The factor $\sqrt{1-\left(\frac{x}{a}\right)^2}$ gives the ellipse shape **B1**

(*B*) The factor $(1 - k\frac{x}{a})$ stretches the ellipse (variably) in the *y* direction
Accept a comment referring to pointedness **B1**

(C) The $\pm$ sign gives both the top and the bottom of the curve
Accept a reference to symmetry about the *x*-axis . **B1**

(ii) $a = 1$ and $k = 0$ **B1**

3. In the diagram OA = 1.5 units, OB = 1.0 units and OD = 0.3 units



$a = \frac{\text{OA}}{\text{OB}} = 1.5$ **B1**

$p = \frac{\text{OD}}{\text{OA}} \approx \frac{0.3}{1.5} = 0.2$ Condone $p = -\ldots$ **B1**

$k = \frac{p}{1-2p^2} \approx 0.2$ *k* must be positive for the A1 **M1** **A1**

4. (i) The volume of revolution $= \pi \int_{-a}^{a} y^2 \mathrm{d}x$

$= \pi \int_{-a}^{a} (1 - \frac{x^2}{a^2}) \mathrm{d}x$ **M1**

A1

$= \pi \left[x - \frac{x^3}{3a^2} \right]_{-a}^{a}$

$= \frac{4\pi a}{3}$ **E1**

(ii) The radius of the original sphere was 1 unit and so its volume was $\frac{4\pi}{3}$.

Since $a \neq 1$, a>1, and so $\frac{4\pi a}{3} > \frac{4\pi}{3}$.

$\therefore$ This egg has greater volume than the spherical egg. **B1**

The resulting chicks are bigger and so more likely to survive. **B1**

Possible values for Question 3.

OD	p	k	
0.2	2/15 = 0.13	0.138	Accept 0.1
0.25	1/6 = 0.17	0.176	0.2
0.3	0.2	0.217	0.2
0.35	7/30 = 0.23	0.26	0.3
0.4	4/15 =0.27	0.310	0.3

Examiner’s Report

2603 Pure Mathematics 3

General Comments

The performances on this paper compared well with last year's. Perhaps there were fewer very high marks, greater than, say 70, but a very good number scoring in the 60+ range. Also, the number of very weak scripts, below 10, was low. There were plenty of marks accessible to poorer candidates and enough work to differentiate at the upper end. There was some evidence that candidates of all abilities ran out of time on Section A, but this may also have been accounted for by the fact that many candidates found question 4 the most difficult.

Only a small number, of generally the more able candidates, scored high marks, 13 to 15 say, on the Comprehension paper but the majority scored in the middle range of marks, the first three questions being very well answered.

Comments on Individual Questions

Q.1 (a) Generally well answered although some candidates, who were not clear on the method, got α = 36.87 or –53.13. Others failed to complete the solution of the equation, stopping after writing $5\sin(\theta - 53.13) = 5$.

(b) Most candidates recognized the need for integration by parts and made the correct choice of functions, but very many were unable to integrate e^{2x} correctly, e^{2x} and $2e^{x}$ being common errors. Another common error occurred when substituting the limits for the final answer,

$\frac{1}{2}e^2 - \left[\frac{1}{4}e^{2x}\right]_0^1 = \frac{e^2}{2} - \frac{e^2}{4} - \frac{1}{4}$, or similar.

(c) A question on which the majority of candidates obtained at least 3 out of the 4 marks available. The only common error was the omission of the arbitrary constant. A small number of candidates, on separating the variables, had $\int y\,dy$ instead of $\int \frac{1}{y}dy$.

(d) This question was badly done, the great majority of candidates making very heavy weather of it by using double angle formulae, and then having to use both the product rule <u>and</u> differentiation by function of a function.

$y = \sin^3 x = \sin x \sin^2 x = \sin x\left(\frac{1}{2}(1 - \cos 2x)\right)$ was very common indeed and although some completed the subsequent differentiation successfully, at some cost in terms of time, most made errors along the way, or failed to differentiate at all. A small number of candidates who differentiated $\sin^3 x$ by the chain rule, still used a double angle formula to obtain the required result instead of using the identity $\sin^2 x + \cos^2 x = 1$;

$\frac{d}{dx}\sin^3 x = 3\sin^2 x\cos x = 3\cos x.\frac{1}{2}(1 - \cos 2x) = 3\cos x.\frac{1}{2}\left(1 - \left(2\cos^2 x - 1\right)\right)$

and hence to the correct result, providing no basic algebraic errors were made, and again at a cost in terms of time.

Q.2 (i) The equations of the two asymptotes were found correctly by almost all the candidates.

(ii) Again, the question was very well done with only an occasional error in the constants A and B.

(iii) Yet again, the binomial series for $(1+x)^{-1}$was found correctly by most candidates. The first error of any consequence in question 2 was in expanding $(2–x)^{-1}$, a number of candidates expressing this as $2\left(1-\frac{x}{2}\right)^{-1}$ or $\frac{1}{2}(1-x)^{-1}$.

(iv) Most candidates were able to find f(0) and g(0) and hence show that they were equal but $f'(x)$ presented a few problems, perhaps the most common error was $\frac{d}{dx}(2-x)^{-1} = -(2-x)^{-2}$ although $f'(x) = 4\ln|1+x|–\ln|2–x|$ was also seen from time to time .

Q.3 (i) Most candidates made a good start to question 3 and, generally, only arithmetic errors marred the calculation of the coordinates of A and B and the distance AB. The only exception was that some candidates confused the vector **AB** and its modulus and failed to find the latter.

(ii) A variety of methods was used to show that C lies on both lines l_1 and l_2. Where the method involved the statement or calculation of the values of λ and μ it was usually possible to follow the candidates argument but, otherwise, the lack of explanation made marking difficult.

Angle ACB was calculated correctly by many candidates. Problems here included the choice and the sense of the vectors used; some candidates used the wrong pair of vectors often including **AB** as one of them; others used **CA** and **CB** calculated from their wrong values of A and B, or incorrectly calculated from correct values; still others used one of these vectors either deliberately or accidentally in the wrong sense thus obtaining an acute angle .The obvious choice of the direction vectors given in the preamble to the question escaped most candidates.

(iii) A surprising number of candidates did not understand that it is necessary to show that the given vector is perpendicular to two non-parallel vectors in the plane to prove that it is the normal to the plane. Others were given no credit here because they failed to show any working for their scalar products, $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 0$ was not sufficient.

Most candidates writing down the Cartesian form of the equation of the plane using this normal did so correctly although some using **n.a** to find the value of *d* often gave it the wrong sign.

Many candidates still prefer to start from a vector form of the equation of the plane and in this question this proved to be a reasonable method. The elimination of the parameters was quite easy and the whole of the question could be answered. Full marks were possible provided that a reasonable explanation was given.

Q.4 Candidates found this the most difficult question on the paper and, whether due to lack of knowledge or lack of time, solutions were often incomplete.

(i) The majority of solutions started with an intrusive $\frac{dy}{dx}$ and, more importantly, the omission of the appropriate right hand side of the equation, thus, $\frac{dy}{dx} = 2x - 8\frac{dy}{dx} + 8y\frac{dy}{dx}$.

Provided that the L.H.S of the above was not included in subsequent work this very poor notation was condoned and $2x = 8\frac{dy}{dx} - 8y\frac{dy}{dx}$ was accepted as the outcome. For candidates reaching this point, the other possible error was the omission of brackets from $\Rightarrow 2x = (8-8y)\frac{dy}{dx}$.

(ii) There was a small number of immaculate solutions to this part from the most able candidates, in the main, though solutions were often confused and incomplete, often lacking any explanation of the candidates intention. A number of candidates distinguished between the two parts and put x=0 in the one case and y=1 in the other but found only one point for each with no explanation of how they had obtained the second coordinate. Perhaps the most disappointing were those solutions which commenced $\frac{dy}{dx} = 0$ when x=0 or y=1 and which then proceeded to obtain the four points (0,0), (0,2), (2,1), and (–2,1), without any reference to points where the tangent is parallel to the y-axis. Other candidates started the second part with $\frac{dy}{dx} = 1$, and a few attempted to find the equation of a tangent to the curve at some unspecified point

(iii) Solutions in which candidates substituted the parametric coordinates into the given equation were frequently subject to two, even sometimes three, errors, and even from the more able candidates, thus,
$x^2 - 8y + 4y^2 = 2\cos^2 x - 8 + 8\sin\theta + 4 + 4\sin^2\theta.$

Those candidates who substituted $\sin\theta = y-1$ and $\cos\theta = \frac{x}{2}$ into $\sin^2\theta + \cos^2\theta = 1$ seemed less likely to make the corresponding errors in squaring (y–1) and $\frac{x}{2}$.

(iv) This last part of question 4 was done very badly and only the very best candidates scored all four marks. Many candidates recognized the equations $x = 2\cos\theta$, $y = \sin\theta$ as the parametric equations of an ellipse, or they eliminated θ to produce the Cartesian equation, but ellipse turned up very frequently indeed as elipse or even eclipse. Others thought that the curve was a circle, a parabola, a hyperbola, or a ‘trig curve’. The transformation was rarely described correctly as a translation, more often as a movement or shift up, or down the y-axis, or even a shear or a stretch. Finally, with the exception of a small number of excellent sketches from high scoring candidates, the sketches of the curves were very poor, often not labelled and often with little information to position the curves on the axes.

Section B (Comprehension)

Q.1 Almost all the candidates completed the table correctly.

Q.2 (i) Methods of describing the contribution made by each part of the equation to the shape of the curve varied but most candidates had the right idea; perhaps the mark which was lost most frequently was that for the ± sign, some candidates making only a comment about sign ambiguity instead of referring to the top and bottom of the curve or symmetry about the x-axis. A few candidates referred to an ambiguity of sign in both x and y.

(ii) Almost always correct. Just a few candidates made the slip, a=0 or k=1.

Q.3 Also very often correct. There were some errors in the calculation of k and a number of candidates made rather heavy weather of the calculations by starting from the equation for the x-coordinate of the point D instead of using the transformed equation for k given in the text. A common error was to give the value of k as negative.

Q.4 (i) This question was not well done, only a minority of candidates realized that the volume of a solid of revolution by integration was required and only about half of those who wrote down the correct integral managed to integrate correctly. Perhaps the most common error was $\int\left(1-\left(\frac{x}{a}\right)^2\right)dx=\left[x-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$, but also some candidates omitted π from their integral or had the wrong limits,1 and –1 instead of a and $-a$.

(ii) Although an appreciable number of candidates referred to the fact that the greater length of an elongated egg resulted in an increased volume, many of them did not know the formula for the volume of a sphere and this appeared variously as $\frac{4}{3}\pi r$, $\frac{1}{3}\pi r^3$, $\frac{4}{3}\pi r^2 h$, even $4\pi r^2$. Of those who had this formula correct, put r =1, and stated $\frac{4}{3}\pi a>\frac{4}{3}\pi$ not many stated that this was so because $a>1$**.**

The final mark was scored by some candidates quite independently of any of the above.