OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2602/1

Pure Mathematics 2

Monday 24 MAY 2004 Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

HN/4

[Turn over

1 **(a)** Find the sum to infinity of the geometric series

$$2 + 0.5 + 0.125 + \dots .$$ [3]

(b) Fig. 1 shows the graph of $y = \ln x$.

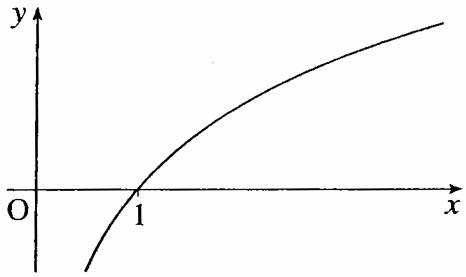


Fig. 1

Express $\ln(x^2)$ in terms of $\ln x$. Copy Fig. 1, and add a sketch of the graph of $y = \ln(x^2)$, making clear which is which. [3]

(c) Differentiate $\sqrt{1 + x^2}$. [2]

(d) Differentiate $\dfrac{1 + x}{1 - x}$. [4]

(e) Evaluate $\displaystyle\int_0^1 (2x + 1)^4 \, dx$. [4]

[Total 16]

2 A credit card company charges compound interest at 1.5% per month. This means that the amount owed is multiplied by a factor of 1.015 per month. So if a cardholder owes £a and no repayments are made, this debt is increased to £x after t months, where

$$x = a \times (1.015)^t.$$

(i) A cardholder wants to buy a TV costing £250. She can pay 6 monthly instalments of £46.50; or she can put it on her credit card, owe the full £250 for 6 months, and then pay off the debt. Which is the cheaper option, and by how much? [4]

(ii) Show that, if no repayments are made, a credit card debt goes up by about 19.6% in a year (12 months). This percentage is called the APR. [3]

(iii) Find how long, in completed months, it takes before a credit card debt is doubled, assuming no repayments are made. [4]

(iv) Another credit card company advertises an APR of 15%. Find the percentage compound interest rate per month charged on this credit card. [3]

[Total 14]

3 Fig. 3.1 shows a sketch of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where $\mathrm{f}(x) = 1 + e^{2x}$, and $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$. The graphs cross the axes at points P and Q respectively.

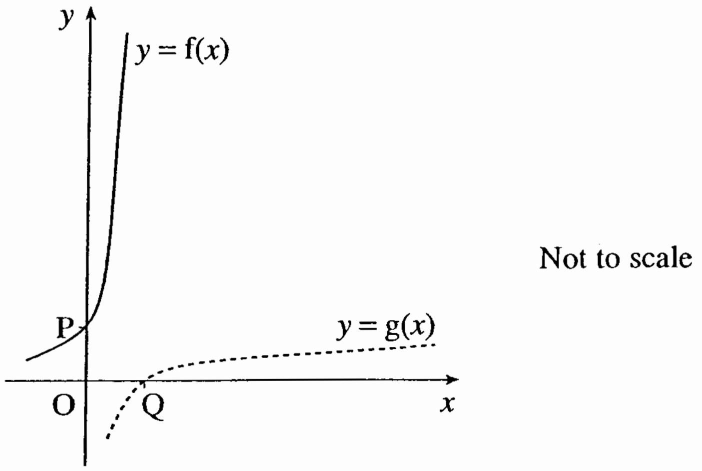


Fig. 3.1

(i) Find the coordinates of P and Q. [2]

(ii) Show that $\mathrm{g}(x) = \frac{1}{2}\ln(x-1)$. [3]

(iii) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$, and verify that $\mathrm{g}'(2) = \dfrac{1}{\mathrm{f}'(0)}$. Interpret this result in terms of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. [5]

The straight line $x = 1$ meets the x-axis at A and the curve $y = \mathrm{f}(x)$ at B. The straight line through B parallel to the x-axis meets the y-axis at C (see Fig. 3.2).

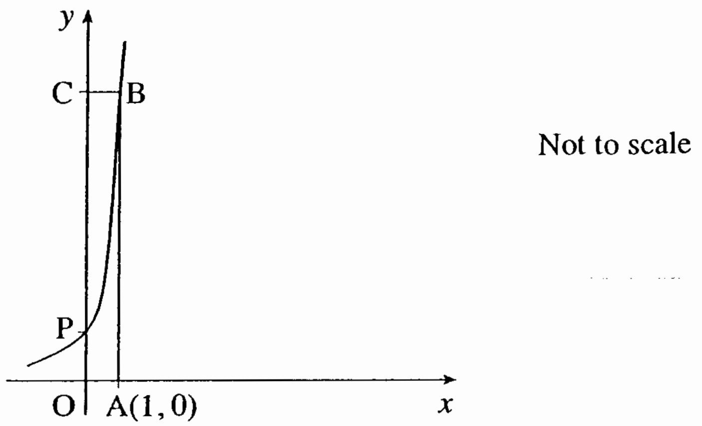


Fig. 3.2

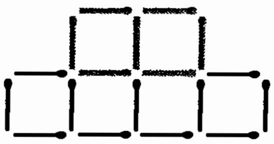
(iv) Show that the curve $y = \mathrm{f}(x)$ splits the area of the rectangle OABC into two equal parts. [5]

[Total 15]

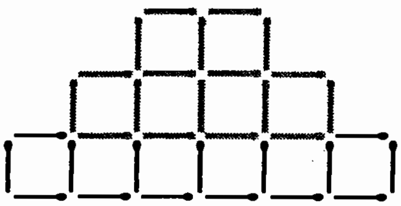
4 Jenny is making a pattern consisting of rows of matchstick squares.

She uses 7 matches to complete a first row of 2 squares.

She uses 11 matches to complete a second row of 4 squares.



She uses 15 matches to complete a third row of 6 squares.



She continues adding rows to the pattern in this way.

You may assume that the numbers of matches used to complete successive rows of the pattern form an arithmetic progression.

(i) Find how many additional matches Jenny needs to complete the fourth row. [1]

(ii) Suppose that Jenny has completed $(n-1)$ rows in the pattern. Find how many additional matches she needs to complete the nth row. [3]

(iii) Show that the **total** number of matches used in making a pattern with n rows is $n(5+2n)$.

Hence verify that, with 1000 matches, it is not possible to make more than 21 complete rows. [5]

Jenny, not surprisingly, runs out of matches after a certain number of complete rows of her pattern are made. She decides to leave in place all the matches forming the perimeter, but to remove all the matches **inside** the pattern.

(iv) Find, in terms of n, the number of matches in the perimeter of the pattern with n rows. Hence or otherwise show that the number of matches **inside** the pattern with n rows is $n(2n-1)$. [3]

(v) Jenny counts the number of matches she has removed, and finds there are 276. Find how many rows she made before she removed the matches. [3]

[Total 15]

Mark Scheme

General Instructions

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓
   - For error in follow through work, use ✗

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not be earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

   c.a.o. : correct answer only
   b.o.d. : benefit of doubt (where full work is not shown)
   X—X : work of no mark value between crosses
   s.o.i. : seen or implied
   s.c. : special case (as defined in the mark scheme)
   w.w.w : without wrong working

Procedure

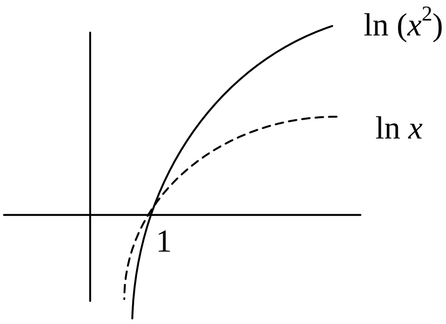
1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting, send a further sample of about 40 scripts, following the same procedure as in para 2.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

1(a) $a = 2$, $r = 0.25$ $S_\infty = \dfrac{a}{1-r} = \dfrac{2}{1-0.25}$ $= 2\dfrac{2}{3}$	M1 A1 A1 [3]	Use of GP S_∞ formula, or S_n formula and letting $n \to \infty$ $\dfrac{2}{1-0.25}$ allow 2.7 or better, but not 2.6, 2.66, etc.
(b) $\ln(x^2) = 2\ln x$ 	B1 B1 B1 [3]	 stretch s.f. 2 in y direction dep 1st B1 passes through (1, 0) (condone '1' label missing)
(c) $\dfrac{1}{2}(1+x^2)^{-1/2}.2x = \dfrac{x}{\sqrt{1+x^2}}$	B1 B1 [2]	½ $u^{-1/2}$ or $\dfrac{1}{2}\left(1+x^2\right)^{-1/2}$ their du/dx × 2x
(d) $y = \dfrac{1+x}{1-x}$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{(1-x).1-(1+x).(-1)}{(1-x)^2}$ $= \dfrac{2}{(1-x)^2}$ or using product rule: $y = (1 + x)(1 - x)^{-1}$ $\dfrac{dy}{dx} = (1 + x)(-1)(-1)(1 - x)^{-2} + 1.(1 - x)^{-1}$ $= (1 - x)^{-2}(1 + x + 1 - x)$ $= \dfrac{2}{(1-x)^2}$	M1 A1 B1 A1 cao M1 A1 A1 A1 [4]	Quotient rule soi (must be correct form) correct numerator – condone no brackets correct denominator or equivalent – mark final answer product rule soi with $u = 1 + x$ and $v = (1 - x)^{-1}$ or equivalent – mark final answer

(e) $\int_0^1 (2x+1)^4 \, dx$ let $u = 2x + 1$, $du = 2dx$ $= \int_1^3 \frac{1}{2}u^4 du = \left[\frac{1}{10}u^5\right]_1^3$ $= 24.2$ or $= \int_0^1 \left(16x^4 + 32x^3 + 24x^2 + 8x + 1\right)dx$ $= \left[\frac{16x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x\right]_0^1$ $= 24.2$	M1 A1 M1 A1 cao M1 A1 M1 A1 cao [4] [Total 16]	Integration to give $\frac{1}{5}u^5$ *or* $\frac{1}{5}\left(2x+1\right)^5$ × ½ substituting correct limits (for x or u) binomial expansion must be correct correctly integrated substituting limits

2(i) $6 \times 46.50 = £279$ $250 \times 1.015^6 = £273.36$ So credit card is cheaper by £5.64	B1 M1 A1 A1 cao [4]	Condone 273.4 for this A1
(ii) $1.015^{12} = 1.1956$ so increased by 19.56 ≈ 19.6%*	M1 A1 E1 [3]	 Must state % increase for E1
(iii) $1.015^t = 2$ $\Rightarrow$ $t \ln 1.015 = \ln 2$ $\Rightarrow$ $t = \ln 2 / \ln 1.015 = 46.56\ldots$ so doubles in 47 months Or trial and error: $1.015^{46} = 1.9835$ $1.015^{47} = 2.0133$ so doubles in 47 months	M1 M1 A1 A1cao [4] B1 B1 A1 cao [4]	Or equivalent, e.g. $250 \times 1.015^t = 500$ taking lns 46.56.. 47 (or, e.g., $250 \times 1.015^t = \ldots$ etc)
(iv) $1.15 = b^{12}$ $\Rightarrow$ $b = 1.15^{1/12} = 1.0117..$ $\Rightarrow$ 1.17% per month	M1 M1 A1 [3] [Total 14]	forming equation for b solving or 1.2%

3(i) P is (0,2) Q is (2,0)	B1 B1 [2]	Not '2' or $y = 2$ Not '2' or $x = 2$ If (2, 0) and (0, 2) without saying which is which, SCB1
(ii) $f(x) = 1 + e^{2x} = y \quad x \leftrightarrow y$ $x = 1 + e^{2y}$ $\Rightarrow x - 1 = e^{2y}$ $\Rightarrow \ln(x - 1) = 2y$ $\Rightarrow y = ½ \ln(x - 1)$ *	M1 M1 E1 [3]	Reasonable attempt to solve for x or y taking lns or $g(x) = ½ \ln(x - 1)$ www
(iii) $f'(x) = 2e^{2x}$ $f'(0) = 2$ $g'(x) = \dfrac{1}{2(x-1)}$ $g'(2) = ½$ $= \dfrac{1}{f'(0)}$ f and g are reflections in $y = x$.	M1 A1 M1 A1 B1 [5]	 $f'(0) = 2$ $g'(2) = ½$
(iii) Area under curve $= \int_0^1 (1 + e^{2x})dx$ $= \left[x + \frac{1}{2}e^{2x}\right]_0^1$ $= 1 + ½e^2 - ½$ $= ½(1 + e^2)$ Area of rectangle $= 1 \times (1 + e^2)$ So area under curve = ½ area of rectangle	M1 A1 A1 B1 E1 [5]	correct integral and limits correctly integrated allow correct numerical answers allow correct numerical answers but must be exact for final E1

4 (i) 19 matches	B1 [1]	
(ii) AP with $a = 7$, $d = 4$ nth term $= a + (n-1)d$ $= 7 + 4(n-1)$ $= 4n + 3$	M1 A1 A1 cao [3]	Must have simplified for final A1
(iii) $S_n = \frac{n}{2}(2a+[n-1]d)$ $= \frac{n}{2}(2\times 7+[n-1]4)$ $= \frac{n}{2}(14+4n-4)$ $= \frac{n}{2}(10+4n)$ $= n(5+2n)$ * When $n = 21$, $n(5+2n) = 987$ When $n = 22$, $n(5+2n) = 1078$ $\Rightarrow$ only 21 complete rows can be made with 1000 matches *or* $n(5+2n) = 1000$ $\Rightarrow 2n^2 + 5n - 1000 = 0$ $\Rightarrow n =$ $\frac{-5\pm\sqrt{25+4\times 2\times 1000}}{2\times 2}=\frac{-5\pm\sqrt{8025}}{2\times 2}=21.14...$ $\Rightarrow$ 21 complete rows	M1 A1 E1 B1 B1 M1 A1 [5]	sum formula used with $a = 7$ and $d = 4$ 987 1078 Formula or completing square (correct)
(iv) Perimeters have 6, 12, 18, …$6n$ matches So total matches inside pattern with n rows $= n(2n+5) - 6n$ $= n(2n-1)$ * or inside matches are 1, 5, 9, … So $S_n = \frac{n}{2}\left(2\times1+[n-1]4\right)$ $= n(2n-1)$ *	M1 M1 E1 M1 M1 E1 [3]	$6n$ subtracting perimeter matches [or by looking at inside matches in patterns, 1 6 15 … and a difference method]
(v) $n(2n-1) = 276$ $\Rightarrow 2n^2 - n - 276 = 0$ $\Rightarrow n = \frac{1\pm\sqrt{1+4\times 2\times 276}}{4}=\frac{1\pm\sqrt{2209}}{4}$ $= 12$ So she made 12 rows.	M1 M1 A1cao [3]	Equating solving by quadratic or trial and error

Examiner’s Report

2602 Pure Mathematics 2

General Comments

This proved to be an accessible paper with predominantly straightforward tests of syllabus content. Many students were well prepared and achieved high marks, and a substantial number gained full marks. Even weaker candidates still managed to score over 20. There was no reduction to linear form question, which has in the past been a reliable source of marks for weaker candidates. The contexts in Qs.2 and 4 required some comprehension skills; however, the marks for these two questions suggest that most candidates coped with them successfully.

All but a few candidates appeared to have sufficient time to answer all four questions. Algebraic weaknesses remain a concern, even with regard to well prepared, high scoring candidates. Lack of working led to some correct answers being penalised – it is really worth emphasising, especially to more able candidates, that they are required to show evidence of method, and that they should err on the side of showing too much rather than too little, especially in questions where they are using their calculators.

Comments on Individual Questions

Q.1 Virtually all candidates found something in this question which they could do, and many scored heavily. There was nothing here that should have worried well prepared students.

(a) This tended to be either 3 marks or none, depending upon whether students had mastered sums to infinity of geometric series. Errors included using the sum to n terms with no limiting process to follow, incorrect values for r, such as 4, and $r - 1$ instead of $1 - r$ in the formula.

(b) Weaker candidates did not know how to simplify $\ln(x^2)$. This lost them the second B1 as well as the first. Some very 'sketchy' graphs were accepted, provided they went through (1, 0), were 'stretched', and did not stop dead at the x-axis.

(c) This question could perhaps have done with an extra mark, which meant that some poor algebraic 'simplification' went unpenalised. The large majority scored full marks.

(d) The usual errors occurred in the quotient rule, for example $\frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$.

Some used the product rule, but generally with less success, especially when it came to simplifying. A surprising number of errors occurred in the easy algebra required for the last A1.

(e) This was a pretty easy integration by substitution, and many candidates scored full marks. They could also do this using the binomial expansion, but they 'burnt their boats' if they made errors with this. A surprising number of candidates converted the integral back to x's before substituting limits or, worse, did this and than substituted 1 and 3 as their values of x!

Q.2 This question was either done very well or rather badly – usually very well. Overall, the marks were high.

Part (i) was generally correct, although some candidates charged interest on the instalments. An accuracy mark was lost if the answer was not correct to the nearest penny.

Part (ii) was very well done. Many candidates chose a value such as £100, multiplied this by 1.015^{12}, and then verified that this represent a 19.6% increase.

In part (iii), some candidates had problems in establishing a correct equation, but once they had done this, most solved this correctly, either by taking logarithms or by trial and error. Unsupported correct answers were penalised; and to achieve full marks by trial and error, the results for both $t = 46$ and $t = 47$ was required.

Part (iv) was generally well done, although some candidates lost a mark by quoting the APR as 1.0117.

Q.3 This was the least productive question in terms of marks.

Part (i) was an easy starter for 2, although weaker candidates arrived at Q by somewhat circuitous routes. P = 2 or (2, 0), and Q = 2 or (0, 2) scored no marks.

In part (ii), candidates achieved a method mark for attempting to invert the formula, but many then 'burnt their boats' with $\ln(x - 1) = \ln x - \ln 1$. $g(x) = \frac{1}{1+e^{2x}}$ was quite a common error.

Many candidates made heavy weather of part (iii). The derivative of $1 + e^{2x}$ was fairly well done, but many candidates failed to find the derivative of $\frac{1}{2}\ln(x - 1)$ correctly. The reflection in $y = x$ was required to achieve the final B1, which was often lost.

Full marks in part (iv) was rarely achieved, even by sound candidates, as they approximated the areas. There were quite a few errors in the integral of $1 + e^{2x}$, including $\frac{1}{2}(1 + e^{2x})$ which fortuitously gave the correct area. Some candidates attempted fruitlessly to find the area PCB between the curve and the y-axis by integration.

Q.4 Most candidates scored well, although some were put off by the wordiness of the context. Some tried to establish quadratic formulae from a table of 1st and 2nd differences, but usually failed to show enough convincing working to 'show' the result.

Part (i) was usually correct, although some misunderstood the meaning of 'additional' and gave the answer '4'.

Part (ii) was well done, although quite a few candidates lost a mark by stopping at $7 + 4(n - 1)$.

Part (iii) was generally well answered, although weaker candidates just verified the result for a few values of n.

Part (iv) was less successful. Some correctly deduced the pattern of the perimeter matches (6, 12, 18, …) but then used the sum formula instead of $6n$.

Part (v) was well done, although some equated $n(5 + 2n)$ to 276, and some failed to solve the quadratic. Unsupported answers were condoned.

Coursework: Pure Mathematics 2

The proportion of centres whose marks was changed was 12% (36 out of 303), around 5% less than recent sessions. There were just 3 changes of 4 marks required. It has been the most accurate marking session to date.

- The majority of centres provide helpful comments and annotations which helps considerably with the moderation. Lack of comments tends to go with poorer marking as does incompletely filled out details at the top.
- The use of technology is expanding and improving with many impressive scripts seen.
- There are still very few assessors applying penalties for notation, but the moderators' comments continue to prepare the way for the new set of criteria.
- There are still cases of incorrect work being ticked. Ticking should only occur when the marker is sure the work is correct, i.e. calculations should have been checked for a tick to be entered.
- Many candidates think that stating the condition for convergence in $x = g(x)$ is sufficient, rather than a brief discussion.

Most frequently occurring sources of difficulty

- Notational errors of the type "I will solve $y = f(x)$". After some improvement last session if anything there was regression this time.
- Lack of illustration or decent explanation for change of sign method.
- Iterate values bearing no relation to illustrations.
- Trivial examples of failure.
- Stating that Newton-Raphson has failed when there is no root to find.
- "Failures" which find the root in the given table of values.
- "Failures" which actually converge eventually after some initial oscillation.
- Lack of iterate values for failures.
- Error bounds not established, just quoted.
- In $x = g(x)$ a different equation used for failure.
- Quoting $|g'(x)| < 1$ condition and not providing comparison of the gradient with that of $y = x$.
- General bookwork.
- Different starting points and different degrees of accuracy in the comparison section. Not stating how many iterates required to achieve a particular degree of accuracy.
- Inadequate comparisons in the hardware/software section.