OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Monday 24 MAY 2004 Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (30 marks)

1 Express 120° in radians, leaving π in your answer. [2]

2 Expand $(2-x)^3$, simplifying the coefficients. [3]

3 Solve the inequality $|2x+1| < 5$. [3]

4 Fig. 4 is a sketch of the graph of a cubic polynomial $y = \mathrm{f}(x)$.

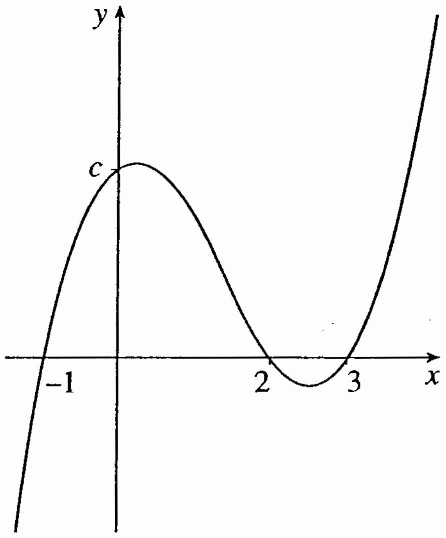


Fig. 4

Given that the coefficient of x^3 is 1, express $\mathrm{f}(x)$ in factorised form.

Hence find the value of c. [3]

5 Find the range of values of k for which the equation $2x^2 - kx + 8 = 0$ has no real roots. [3]

6 Sketch the graph of $y = \tan x$ for $0° < x < 360°$.

Solve the equation $\tan x = -3$ for $0° < x < 360°$. Give your answers correct to 1 decimal place. [4]

7 Solve the simultaneous equations $y = x^2 - 4x + 1$ and $2x + y = 4$. [4]

8 Fig. 8 shows a sector of a circle with centre O and radius 6.2 cm. The sector angle is 1.2 radians.

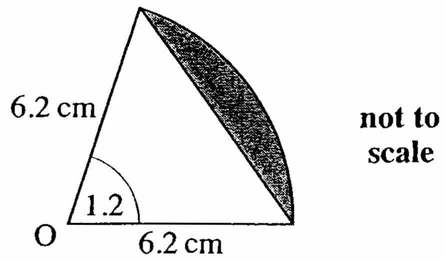


Fig. 8

Show that the area of the triangle is 17.9 cm^2, correct to 3 significant figures.

Calculate the area of the shaded segment of the circle. [4]

9 Show that the equation

$$x^2 + y^2 + 8x - 12y + 20 = 0$$

represents a circle with centre $(-4, 6)$, and find its radius. [4]

TURN OVER FOR SECTION B

Section B (30 marks)

10

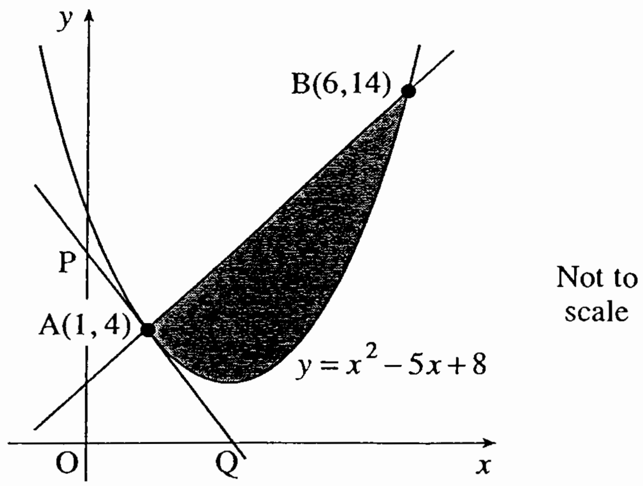


Fig. 10

The point $A(1, 4)$ lies on the curve $y = x^2 - 5x + 8$, as shown in Fig. 10.

(i) Find the equation of the tangent at A.

The tangent crosses the axes at P and Q. Show that the area of triangle OPQ is $\frac{49}{6}$ square units. [6]

The point $B(6, 14)$ also lies on the curve.

(ii) Find the equation of line AB in the form $y = mx + c$. [3]

(iii) Find the area between the curve and AB, shaded in Fig. 10. [5]

[Total 14]

11 The rate of flow of oil into a tank is measured each half hour over a 3-hour period.

Time (t hours)	0	0.5	1	1.5	2	2.5	3
Rate of flow (r m^3 per hour)	10.0	9.6	8.4	7.0	6.3	7.3	10.0

(i) Use the trapezium rule with 6 strips to estimate the volume of oil entering the tank during this period. [3]

The rate of flow of oil, r m^3 per hour, into the tank at time t hours is modelled by

$$r = t^3 - 3t^2 + 10 \qquad \text{for } 0 \leqslant t \leqslant 3.$$

(ii) Calculate, correct to 2 significant figures, the relative error in the rate of flow at $t = 1$ when using this model. [3]

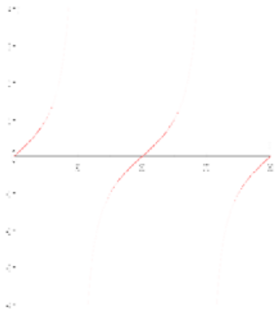
(iii) Use calculus to find the minimum rate of flow given by this model, and the time at which it occurs. [5]

(iv) Evaluate $\displaystyle\int_0^3 (t^3 - 3t^2 + 10)\,\mathrm{d}t$. [3]

(v) By considering your answers to parts **(i)** to **(iv)**, make two distinct comments on the suitability or otherwise of this model. [2]

[Total 16]

Mark Scheme

	Section A			
1	$\frac{2\pi}{3}$ or $\frac{2}{3}\pi$ o.e. in form $k\pi$	B2	allow 120 × π/180 or 120 × 2π/360 isw; condone 0.66π or better B1 for 120 ÷ 180/π o.e. or 2.09 to 2.1	2
2.	$8 - 12x + 6x^2 - x^3$	B3	condone $-1\ x^3$ B2 for 3 terms correct or for correct digits, wrong signs or for wrong working following fully correct ans. or B1 for 1,3,3,1 soi	3
3.	$x < 2$ $x > -3$	B1 B2	deduct 1 in qn. for $\le$ etc M1 for $2x + 1 > -5$ or following B0, B0, SCB1 for (e.g. $x = 2$ and –3 or $(2x + 1)^2 < 25$	3
4.	$(x + 1)(x - 2)(x - 3)$ [c =]6	B2 B1	B1 for $k(x + 1)(x - 2)(x - 3)$ $k \ne 1$ correct or ft; condone (0, 6)	3
5.	$-8 < k < 8$	B3	B2 for 8 and –8 seen or M1 for $k^2 - 4 \times 2 \times 8$ soi, condone $-k^2$	3
6.	 108.4 and 288.4	G2 B2	no numbers required on axes unless more sections shown. G1 for at least one ‘period’ of correct shape [ignore numbers on axes for G1] B1 for one soln or for both with extras in range or not to 1 dp; ignore extras outside range	4
7.	eg $2x + x^2 - 4x + 1 = 4$ $x^2 - 2x - 3$ [= 0] and attempt at soln (–1, 6) or (3, –2) [condone separate x and y values]	M1 M1 A2	eliminate y by subst or subtraction or subst for x attempt at rearrangement to quadratic = 0 and soln. of this, or completing the square; dep on first M1 or M2 for e.g. graphical method with domain at least [–1, 3] A1 for one pair of x and y or for both x or both y values; SCB2 for one or both pairs without evidence of method for second M mark	4
8.	0.5 × 6.2 × 6.2 × sin 1.2 [ans given] 0.5 × 6.2 × 6.2 × 1.2 o.e. or 23.0(64) their attempt at sector – 17.9(...) =5.14 to 5.2	M1 M1 M1 A1	or longer method for area of triangle accept 23.04 to 23.08 or 23.1 or B3	4

	Section A			
9.	$(x + 4)^2 + (y - 6)^2$ seen $(x + 4)^2 + (y - 6)^2 = 32$ correctly obtained by completing square; <u>or</u> A1 for $x^2 + 8x + 16 + y^2 - 12y + 36 = r^2$ then A1 for correctly obtaining $r^2 = 32$ radius = $\sqrt{32}$ or 5.6 to 5.7	M1 A2 B1	B1 for correct expn. of $(x + 4)^2$ or $(y - 6)^2$ o.e. eg $x^2 + 8x = (x + 4)^2 - 16$ <u>or</u> M1 for clear use of $x^2 + y^2 + 2fx$... A1 for centre is $(-f, -g)$ NB ans given M1 for use of $r = \sqrt{(f^2 + g^2 - c)}$	4

		Section B			
10	(i)	$y' = 2x - 5$ c.a.o. At A, $y' = -3$ tgt is $y - 4 = -3(x - 1)$ o.e. e.g. $y = 7 - 3x$ OP = 7, or (0, 7) seen OQ = 7/3 or (7/3, 0) seen Area = 0.5 × 7 × 7/3 [= 49/6]	M1 A1 M1 ft M1 M1 B1	 ft their y' used or SC grad = –0.5 or B3 ft their tgt NB mark method – answer given	6
	(ii)	grad AB = (14 – 4) ÷ (6 – 1) [= 2] $y - 4$ = their 2 $(x - 1)$ or $y - 14$ = their2 $(x - 6)$ $y = 2x + 2$	M1 M1 A1	or M2 for $\frac{y-4}{14-4} = \frac{x-1}{6-1}$ o.e., or B3	3
	(iii)	$\int [(2x+2)-(x^2-5x+8)][\mathrm{d}x]$ o.e. $(x^2+2x)-(\frac{x^3}{3}-\frac{5x^2}{2}+8x)$ o.e. value at 6 – value at 1 $20\frac{5}{6}$ or 125/6 or 20.8 to 20.84	M2 M1 M1 A1	M1 for integral of line or parabola seen; condone (parabola – line) or no brackets for M2; allow ft of their (ii) for line or parabola correct; condone one error if combined may be implied by area under line = 45 or area under curve = $24\frac{1}{6}$ or M3 for $24\frac{1}{6}$, B1 for 45 from trap. or for final subtraction forgotten	5
11	(i)	24.3	B3	M2 for 0.5×0.5×[20+2(9.6+8.4+7.0+6.3+7.3)] o.e.; M1 for one error [wrong h or brackets missing etc] or 4 trapezia correct	3
	(ii)	error = 0.4 soi their 0.4 /8.4 or their 8 / 8.4 s.o.i. 0.048 or 4.8% c.a.o.	B1 M1 A1	ignore any neg. sign for error or rel. error or B3	3
	(iii)	$3t^2 - 6t$ $r' = 0$ s.o.i. $3t(t - 2)$ t = [0 or] 2 min flow = 6	M1 M1 M1 A1 A1	condone one error attempt at factorising /formula etc soi both As dep. on previous Ms earned e.g. 0 for qn. if no calculus seen	5

	(iv)	integration attempted $\frac{t^4}{4} - t^3 + 10t$ 23.25 o.e.	M1 A1 A1	at least one term correct; condone *x* used at least 2 terms correct	3
	(v)	two comments from min at same value of *t* about right size area under curve fits at $t = 0$ and / or 3 less than 5% error at $t = 1$	1+1	comments must ft from their (i) to (iv) e.g. not allow ‘area results are close’ if they are 24.3 and 16.75 and so not close; if more than two comments, mark best two	2

Examiner’s Report

2601 Pure Mathematics 1

General Comments

The majority of centres seemed to have candidates who were well prepared for the examination and have a good idea of what was expected of them. However, some centres had a significant proportion of their candidates scoring under 20 marks.

In spite of the fact that some candidates used long methods on occasion, particularly in section A, time did not appear to be a problem, with such candidates going on to attempt both long questions in section B.

Comments on Individual Questions

Q.1 This conversion of degrees to radians was generally well done, although some did not simplify their answer correctly.

Q.2 Many candidates multiplied out the brackets, rather than using the binomial theorem. A lack of organisation led to many errors in signs by candidates.

Q.3 The result $x < 2$ was often obtained, but poor understanding of the modulus meant that the other part of the inequality was not dealt with correctly.

Q.4 Many obtained two marks for expressing the function in factorised form, but many multiplied out to obtain c rather than simply substitute 0 for x.

Q.5 As in the past, candidates' handling of the discriminant was poor. Many did not know what to do, and many were unable to progress from using $k^2 - 64$, in the quadratic formula.

Q.6 Some excellent sketches of tan graphs were seen, but many candidates did not produce curves of the correct shape, or had the period wrong. Most candidates were able to obtain at least one of the solutions to the equation, although some did not give their solutions to the required accuracy.

Q.7 Most used the correct method for eliminating y, but many candidates forgot to go back and obtain the final accuracy mark by obtaining the y values.

Q.8 Most candidates used the formula $^1/_2\, ab \sin C$ correctly for the area of the triangle, although needed to convert to degrees to use their calculators, and some long-winded methods were seen, some of which were successful. Most knew that the area of the segment was the sector – the triangle, but some had the wrong formula for the area of a sector.

Q.9 Most realised that the circle equation involved $(x+4)^2 + (y-6)^2$, but expanding the brackets or working from the given equation and handling the 20 correctly was done much better in some centres than others.

Q.10 This question tested several skills and many candidates gained full marks on the question. The examiners accept that it was unfortunate that the diagram may have encouraged some students to think that PQ was perpendicular to AB – a special case was allowed on the mark scheme for this error, to limit the marks lost. The majority, however, differentiated correctly in (i) and found the equation of the tangent. Most went on to obtain the intersections with the axes and hence the area of the triangle, but some omitted this part.

In part (ii), most found the equation correctly, although an inverted gradient was seen occasionally.

Most candidates were able to integrate correctly for the area of the region under the curve, although there were errors in arithmetic fairly frequently. Some failed to go on to find the area under the line. An encouraging number handled both integrals together, although some made life harder for themselves by failing to simplify their expression before integrating. Some candidates integrated from 4 to 14 instead of 1 to 6.

Q.11 There were the usual errors in using the trapezium rule formula in part (i), but there was possibly a slight improvement compared to past years.

In part (ii), some candidates compared the value of 8 to 24.3 (from part (i)) instead of the 8.4 in the table. The common errors, however, were to use the absolute error, 0.4 as the relative error or to find 0.4 as a percentage of 8 instead of the true value 8.4.

In spite of the instruction in part (iii), many candidates failed to use calculus, using instead the minimum shown in the table of measured values. Those who did use calculus were not very impressive when solving $3t^2 - 6t = 0$; many used the quadratic formula or failed to factorise correctly. Many candidates, having found $t = 2$, forgot to go back and find the corresponding value of r given by the model.

The integration in part (iv) was very well done.

In part (v), many believed that the model provided the true or accurate answers. Many failed to make clear which quantities they were comparing. Relatively few gave good clear reasons for suitability.