OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Wednesday **23 JUNE 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

1 **(i)** State the dimensions of frequency and density. Show that the dimensions of pressure are $\mathrm{M\,L^{-1}\,T^{-2}}$. [4]

The frequency, f, of the note emitted by an organ pipe of length a when the air pressure is p and the air density is d is believed to obey a law of the form

$$f = k a^{\alpha} p^{\beta} d^{\gamma} ,$$

where k is a dimensionless constant.

(ii) Determine the values of α, β and γ. [6]

A particular organ pipe is tuned to a frequency of 440 Hz. Subsequently the air density falls by 2% and the air pressure rises by 0.5%. Any change in the length of the pipe is negligible.

(iii) Calculate the new frequency of the organ pipe. [4]

[Total 14]

2

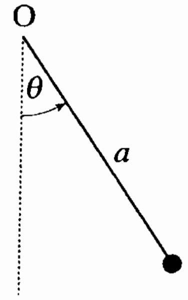


Fig. 2.1

Fig. 2.1 shows a particle of mass m suspended from a fixed point O by means of a light, inextensible string of length a. The particle moves in a vertical circle centre O. The speed of the particle is u at the lowest point of its motion. When the string makes an angle θ with the downward vertical the speed of the particle is v. Resistances to motion may be neglected.

(i) By considering the energy of the particle, show that

$$v^2 = u^2 - 2ag(1 - \cos\theta).$$ [3]

(ii) Find the tension in the string in terms of a, g, m, u and θ. [5]

(iii) Show that the condition for the particle to perform complete circles is $u^2 \geqslant 5ag$. [3]

The particle is now released from rest at the same level as O with the string taut. As the particle swings, the string hits a small peg a distance b vertically below O, as shown in Fig. 2.2, and wraps round it.

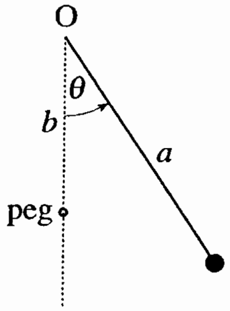


Fig. 2.2

(iv) The particle just completes a vertical circle after the string wraps round the peg. Find an expression for b in terms of a. [4]

[Total 15]

3 A sketch of the curve with equation $x = \sqrt{2y^2 - y^4 + 1}$ for $-1 \leqslant y \leqslant 1$ is shown in Fig. 3.1.

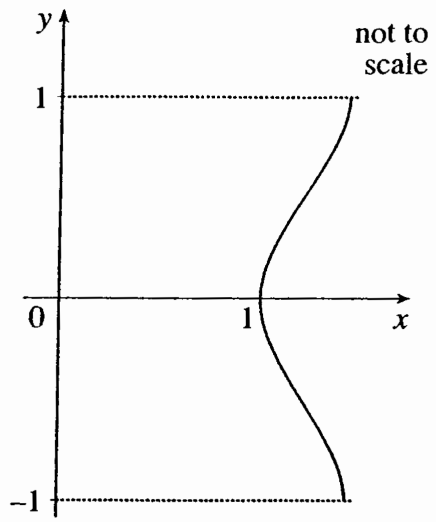


Fig. 3.1

A uniform solid, S, is in the shape formed by rotating the region bounded by the curve $x = \sqrt{2y^2 - y^4 + 1}$, the lines $y = \pm 1$ and the y-axis through 2π radians about the y-axis. The units of the axes are metres.

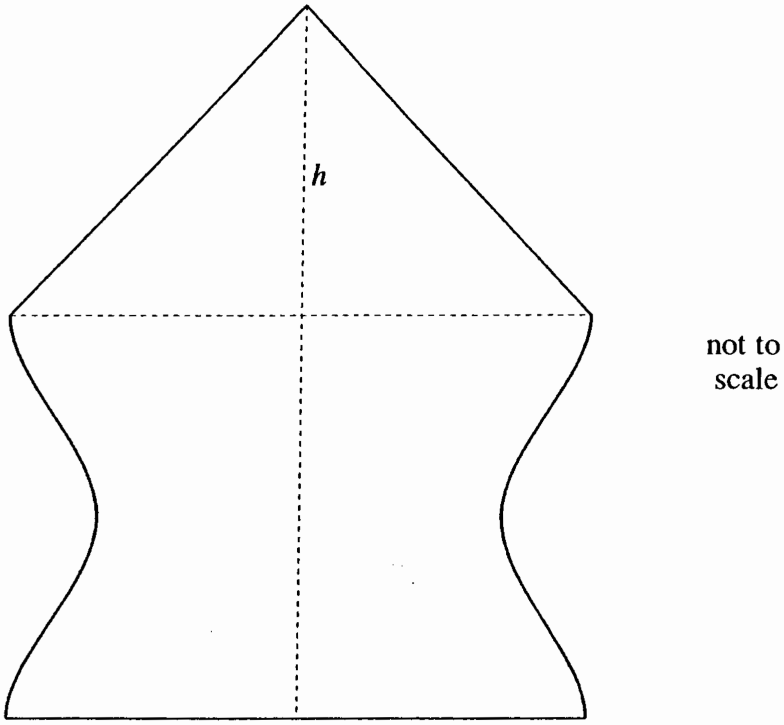
(i) Show that the volume of S is $\frac{44}{15}\pi$ m^3.

Show that the curve $x = \sqrt{2y^2 - y^4 + 1}$ is symmetrical about the line $y = 0$ and hence write down the coordinates of the centre of mass of S. [5]

(ii) Using integration, show that the centre of mass of a uniform right circular solid cone of vertical height h and base radius r is at a distance of $\frac{1}{4}h$ from the plane face along the axis of symmetry of the cone. [You may assume that the volume of this cone is $\frac{1}{3}\pi r^2 h$.] [5]

The question continues on page 5.

A right circular cone has height h and its plane face has the same radius as a plane face of S. A uniform solid, T, is formed by attaching this cone to S so that the plane faces meet and the axes of symmetry of S and the cone are in the same line, as shown in Fig. 3.2.



cross-section of T

Fig. 3.2

(iii) Calculate the value of h if the centre of mass of T lies in the plane of the join of S with the cone. [5]

[Total 15]

4 A light spring of stiffness $19.6\,\mathrm{N\,m^{-1}}$ is fixed at its bottom end and slides freely in a vertical tube. Initially, the spring is compressed 0.05 m by a disc of mass m kg resting on it in equilibrium, as shown in Fig. 4.1.

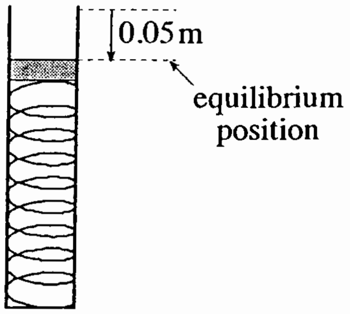


Fig. 4.1

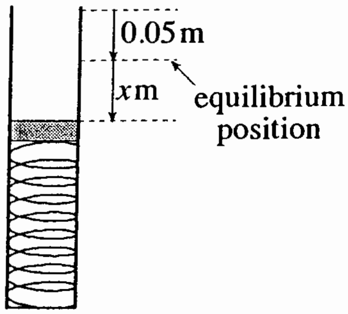


Fig. 4.2

(i) Calculate the value of m. [2]

The disc is pushed down a distance 0.1 m from the equilibrium position and released from rest.

(ii) Show that the disc begins to move in simple harmonic motion with equation $\ddot{x} + 196x = 0$, where x m is the displacement of the disc below its equilibrium position, as shown in Fig. 4.2. [5]

The disc is in contact with the spring until the spring reaches its natural length. At this instant, the spring is stopped from moving and the disc loses contact with it.

(iii) Calculate the speed of the disc as it leaves the spring.

Calculate also the time after its release for which the disc is in contact with the spring. [6]

With the disc resting on the spring in equilibrium, the disc is given a velocity of $v\,\mathrm{m\,s^{-1}}$ downwards. The system comes instantaneously to rest 0.2 m below its original rest position.

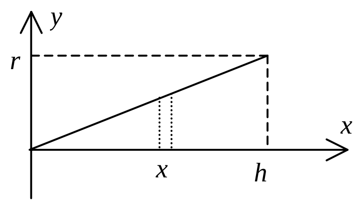
(iv) Calculate v. [3]

[Total 16]

Mark Scheme

Q 1		mark		sub
(i)	[frequency] = T^{-1}	B1		
	[density] = $M\ L^{-3}$	B1		
	[pressure] = [force] / [area]	B1	Award for pressure as force/area seen	
	= $M\ L^{-1}\ T^{-2}$	E1		4
(ii)	$T^{-1} = L^{\alpha}(M\ L^{-1}\ T^{-2})^{\beta}(M\ L^{-3})^{\gamma}$	M1 A1	Set up equation with all the terms Correct FT (i)	
	M: $0 = \beta + \gamma$	M1 A1	Comparing powers for at least 1 dimension One equation correct. FT (i)	
	L: $0 = \alpha - \beta - 3\gamma$	A1	Second equation correct. FT (i)	
	T: $-1 = -2\beta$			
	giving $\alpha = -1,\ \ \beta = \frac{1}{2},\ \ \gamma = -\frac{1}{2}$	A1	All correct. cao	6
(iii)	$\frac{f}{f'} = \frac{ka'\sqrt{\frac{p}{d}}}{ka\sqrt{\frac{p'}{d'}}} = \frac{a'}{a}\sqrt{\frac{p}{p'}.\frac{d'}{d}}$	M1	Attempt to eliminate *k*. Accept assigning values to *a*, *a'*, *p* and *d* for full credit here and below.	
	$a = a',\ \ \frac{p}{p'} = 1.005$ and $\frac{d}{d'} = 0.98$	B1	for either $\frac{p}{p'} = 1.005$ or $\frac{d}{d'} = 0.98$ or equivalents	
	so $f = 440\sqrt{\frac{1.005}{0.98}}$	A1	Seen or implied. FT (ii) only.	
	= 445.576… so 446 Hz (3 s. f.)	A1	Ignore wrong α used. cao	4
			total	14

Q 2		mark		sub
(i)	$0.5mu^2 = 0.5mv^2 + mga(1-\cos\theta)$	M1 B1	Attempt to use KE + PE = const $mga(1-\cos\theta)$	
	so $v^2 = u^2 - 2ag(1-\cos\theta)$	E1	Clearly shown. Accept consistent signs throughout.	3
(ii)	$T - mg\cos\theta = \dfrac{mv^2}{a}$ $\left(= ma\omega^2\right)$	M1	Radial EoM attempted. Accept *mg* not resolved. All terms present. Do not accept *T* resolved.	
		B1 B1	LHS RHS accept $u \leftrightarrow v$ and $a \leftrightarrow r$	
	so $T = mg\cos\theta + \dfrac{m}{a}\left(u^2 - 2ag(1-\cos\theta)\right)$	B1	Subst correct v^2 into **their** *T*.	
	$= \dfrac{m}{a}\left(3ag\cos\theta + u^2 - 2ag\right)$	A1	Accept any form with *T* the subject.	5
(iii)	Need $\left(3ag\cos\theta + u^2 - 2ag\right) \geq 0$ (for all θ)	M1	Accept =, > and given in words. Or start again.	
	Worst case is when $\theta = \pi$	B1	May be implied	
	giving $u^2 \geq 5ag$	E1	> must be justified	3
(iv)	$v = 0$ when $\theta = \dfrac{\pi}{2} \Rightarrow u^2 = 2ag$	M1 A1	Attempt to find *u* from initial conditions.	
	we need $2ag = 5(a-b)g$	M1	Attempt to relate result in (iii) to this situation. Accept $u^2 = 5bg$. Or start again.	
	so $5b = 3a$ or $b = \dfrac{3a}{5}$	A1		4
			total	15

Q 3		mark		sub
(i)	Vol is $\pi\int_{-1}^{1} x^2 dy$			
	$= \pi\int_{-1}^{1}\left(2y^2 - y^4 + 1\right)dy$	M1	Limits not required	
	$= \pi\left[\frac{2y^3}{3} - \frac{y^5}{5} + y\right]_{-1}^{1}$	A1	Limits not required	
		M1	Limits dealt with correctly	
	$= 2\pi\left(\frac{2}{3} - \frac{1}{5} + 1\right) = \frac{44\pi}{15}$ giving $\frac{44\pi}{15}$ m^3	E1	Clearly shown	
	$2(-y)^2 - (-y)^4 + 1 = 2y^2 - y^4 + 1$ so symmetrical about $y = 0$ so c. m. is at (0, 0)	B1	Must show symmetrical as well as give c.m.	5
(ii)	$\frac{1}{3}\pi r^2 h\bar{x} = \pi\int_0^h x\left(\frac{rx}{h}\right)^2 dx$	M1	Use of elementary discs radius rx/h or equivalent. Award for attempt to find $\int \pi x y^2 dx$ or equivalent	
		B1	Correct relation between x and y.	
	$= \left[\frac{\pi r^2}{h^2}.\frac{x^4}{4}\right]_0^h$	F1	Integration of **their** expression correct if at least quadratic. Neglect limits.	
	so $\frac{1}{3}\pi r^2 h\bar{x} = \frac{\pi r^2 h^2}{4}$	M1	Appropriate limits substituted and attempt to use M	
	$\bar{x} = \frac{3}{4}h$ so $\frac{1}{4}h$ from base.	E1	Clearly established	5
(iii)	Radius of cone is $\sqrt{2-1+1} = \sqrt{2}$	B1		
	Require $\frac{1}{3}\pi \times 2 \times h \times \frac{h}{4} = \frac{44\pi}{15} \times 1$	M1	Attempt to write equation using , e.g.,moments	
		A1	LHS or equivalent. FT **their** r.	
		A1	RHS or equivalent. FT **their** r.	
			[If three terms, A2 -1 each incorrect term. FT **their** r.]	
	$h = \sqrt{\frac{88}{5}}$ (= 4.1952…)	A1		5
			total	15

Q 4		**mark**		sub
(i)	$mg = 19.6 \times 0.05$ $m = 0.1$	M1 A1	Use of HL	2
(ii)	N2L $\downarrow$ $mg - F = m\ddot{x}$ $0.1g - 19.6(0.05 + x) = 0.1\ddot{x}$ so $\ddot{x} + 196x = 0$	M1 M1 A1 B1 E1	Use of N2L. Condone sign errors and *mg* omitted. Linear expression for *F* in terms of *x* $\lvert F \rvert$ correct All correct including signs. [Award up to 4/5 if *x* taken +ve upwards]	5
(iii)	**either** $v^2 = \omega^2 (a^2 - x^2)$ so $v^2 = 196(0.1^2 - (-0.05)^2) = 1.47$ so $v = 1.2124\ldots$ so 1.21 m s^{-1} (3 s. f.) **or** $\frac{1}{2} \times 0.1 \times v^2 + 0.1 \times 9.8 \times 0.15 = \frac{1}{2} \times 19.6 \times 0.15^2$ so $v = 1.2124\ldots$ so 1.21 m s^{-1} (3 s. f.) **or** First find $x = 0.1\cos 14t$ (see below) $v = -1.4\sin 14t$ sub $t = \frac{\pi}{21}$ (see below) $v = -1.2124...$ and speed is 1.21 m s^{-1} (3 s. f.) $x = 0.1\cos 14t$ we need $-0.05 = 0.1\cos 14t$ so $t = \frac{\pi}{21}$ (0.150 s (3 s. f.))	 M1 A1 A1 M1 A1 A1 B1 M1 A1 B1 M1 A1	 Award M1 even if *x* = 0 used below Correct substitution cao Equating KE + GPE to EPE All correct cao cao Obtaining expression for *x* in terms of *t* Equating *x* in terms of *t* to –0.05 cao. [Award 0 for ¼ period]	6
(iv)	**either** The motion is SHM, *a* = 0.2, $\omega = 14$ $v = v_{\max}$ of the SHM $v_{\max} = a\omega = 0.2 \times 14 = 2.8$ **or** $0.5 \times 0.1 \times v^2 + 0.1g \times 0.2 = \frac{19.6}{2}(0.25^2 - 0.05^2)$ $v = 2.8$	 M1 B1 A1 M1 M1 A1	 May be implied May be implied Equating KE + GPE to EPE Correct change in EPE [SC1 for $0.5 \times 0.1 \times v^2 = 0.5 \times k \times 0.2^2$]	3
			total	16

Examiner’s Report

2609 Mechanics 3

General Comments

Most of the candidates were able to make some progress with every question and very many of these attempts were substantial. Many completely correct solutions were seen to each question but even the most able candidates seemed to find some challenge in the paper. On the whole, candidates did better at Q1 than the others and the scores on Q3 were higher than those on Q2 and Q4. In some cases all the candidates from a centre did especially well or badly on particular questions and this was presumably due to unequal coverage of the syllabus.

The standard of presentation was generally reasonable but many candidates placed themselves under a considerable disadvantage by producing work that elevated untidiness to an art form and consequently misled and confused themselves. As always, others did not show sufficient working or give an adequate indication of method when establishing a given result

Comments on Individual Questions

Q.1 Almost all of the candidates knew exactly how to set about the first two parts and did so successfully. A few candidates did not know the dimensions of frequency but most of the other errors were slips. A small minority could not eliminate the dimensionless constant in the last part, presumably because proportional changes were given instead of numerical values. However, most candidates found some way of coping, the most common error being the use of 1.05 instead of 1.005 as the multiplier to increase by 0.5%.

Q.2 Most of the candidates knew how to establish the given result in part (i) but many failed to obtain full credit because they did not include sufficient working to *show* their derivation.

In part (ii), many candidates knew exactly what to do and did it neatly and effectively. A few did not know how to establish any equation for circular motion but rather more analysed the problem as if the motion was that of a conical pendulum instead of taking the radial direction to be that of the string.

In part (iii), some candidates immediately saw the need to establish that the *force* in the string was non-negative at any point of the motion if the string was to remain taut but others falsely thought that the condition depended only on energy considerations and so only required the *speed* to be positive everywhere but at the highest point. Some candidates did not use the result from part (iii) but, instead, established from first principles the equation for the motion at the top.

Again, in part (iv) some candidates did not see the connection with the previous part and started again. Many candidates did not really see what was going on but others efficiently applied the result from part (iii) to the new situation. A common error was to take the radius of the new circle to be *b* instead of $a - b$.

Q.3 In part (i), almost all of the candidates correctly found the volume of revolution but a few did not show enough detail when substituting the limits to obtain full credit for establishing a *given* answer. Quite a few candidates established the symmetry of the function about the x axis by correctly arguing that x was an even function of y: others wrongly considered the symmetry about the y axis or argued that the symmetry was proved by comparing one or more values above the axis with the corresponding ones below: others wrongly argued that the symmetry was shown because the volumes of revolution above and below the x axis were the same. Many candidates directly calculated the position of the centre of mass and did not answer the question set about the symmetry.

Part (ii) required the candidates to carry out a standard piece of bookwork, namely to find the centre of mass of a uniform right circular cone. Although this was done very well by many candidates, a surprisingly large number of candidates made a bit of a mess of it. Some could not find the equation of the generator and others could not use their equation properly in their integral. A quite common error was to consider the cone with x and y intercepts of r and h respectively and then to rotate this about the x-axis giving a cone of height r and base radius h; errors with the limits of the integration and the volume of the cone often falsely produced the given answer. A few candidates thought that the cone had $h = r$, despite the wording of the question; perhaps this was a wrong interpretation of the *right* circular cone. For some candidates, their poor presentation of the problem produced an alphabetical soup in which confusion was widespread.

Many candidates knew exactly what to do in part (iii) but did not always choose the best place for the origin to simplify the working. Common errors were either not to realize that r could be evaluated or to assume that $r = 1$.

Q.4 Virtually everyone correctly found the mass in part (i).

There were a lot of sound answers to the standard situation presented in part (ii) but, as always, many candidates omitted the weight term and then fiddled the tension term. Some candidates wrote an equation where x was taken to be positive upwards instead of downwards, as instructed in the question, and there were penalties for doing this.

Although there were many neat and correct solutions, there were also a lot of inefficient methods seen in part (iii) and many mistakes made. Common errors were: to think the amplitude was 0.15 m: to argue that the spring was at the centre of the simple harmonic motion when at its natural length: for those who used an energy approach, to forget the gravitational potential energy term. Many candidates tried to find the time by a sound method but many substituted 0.05 instead of –0.05. Surprisingly at this level, quite a few candidates used their v with an acceleration determined from the equation of motion for some x (or simply took the acceleration to be g) and then used $v = u + at$.

In part (iv), many candidates realized that this was again simple harmonic motion with a new amplitude but the old ω and obtained the answer quickly. Many of those who tried an energy method either omitted the gravitational potential energy term or did not consider the energy initially stored in the spring. Since it is true that, in these circumstances, if the stiffness of the spring is k and the distance dropped is h then $\frac{1}{2}mv^2 = \frac{1}{2}kh^2$, some credit was given for writing this down without justification.