OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2608/1

Mechanics 2

Wednesday **9 JUNE 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 In this question about a spacecraft, you may assume that the resistance to motion and the gravitational attraction are negligible, that all the motion is in the same straight line and that all the collisions are direct. The initial direction of motion is taken as positive.

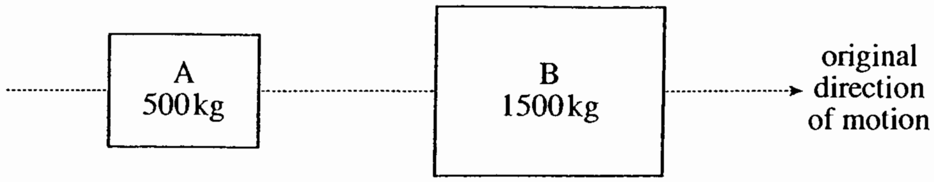


Fig. 1

A spacecraft of mass 2000 kg travelling at $100\,\mathrm{m\,s^{-1}}$ separates into parts A and B. As shown in Fig. 1, A has a mass of 500 kg and B a mass of 1500 kg.

After separation the speed of B is $150\,\mathrm{m\,s^{-1}}$ in the original direction of motion.

(i) Calculate the impulse acting on B in the separation. [2]

(ii) Calculate the velocity of A after the separation. [3]

(iii) Given that the separation takes place over 75 seconds, calculate the average separating force. [2]

B meets and attaches to a satellite C of mass 500 kg travelling in the same direction as B with speed $130\,\mathrm{m\,s^{-1}}$. The combined B and C is referred to as D.

(iv) Show that the velocity of D is $145\,\mathrm{m\,s^{-1}}$ in the original direction of motion. [3]

An impulse now acts on A so that it approaches D at a relative speed of $10\,\mathrm{m\,s^{-1}}$. There is then a perfectly elastic collision between A and D (that is, a collision with $e = 1$).

(v) Calculate the velocity of A and the velocity of D after the collision. [6]

[Total 16]

2 **(a)** A grandfather clock is driven for two days by a 4 kg mass of brass falling through a vertical distance of 80 cm.

What power is required to drive the clock? [3]

(b)

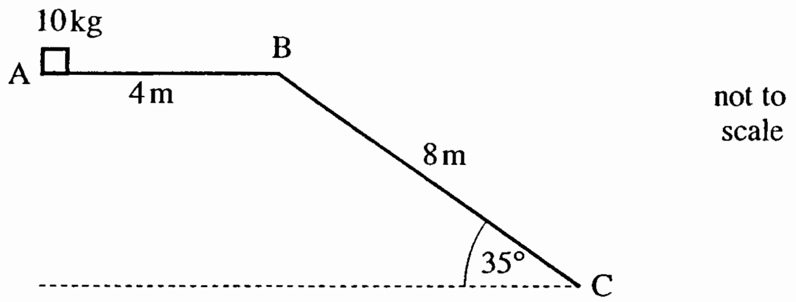


Fig. 2

Fig. 2 shows a small block of mass 10 kg set in motion along a path ABC. The block first travels a distance of 4 m horizontally from A to B against a constant resistance of 40 N. Its speed at A is $v\,\text{m s}^{-1}$.

(i) Calculate the least possible value of v. [3]

Without losing contact, the block now slides 8 m down a ramp from B to C. The ramp is inclined at 35° to the horizontal.

(ii) Calculate the gravitational potential energy lost by the block as it slides from A to C. [2]

In a particular case, $v = 6$. From B to C the block moves against a constant resistance of 50 N.

(iii) Calculate the speed of the block at C. [6]

[Total 14]

3 **(a)** A cuboid of weight W N is in equilibrium on a horizontal table. The cuboid is a m long and the centre of mass is b m from end P. A vertical force, T N, acts on it at end Q. The reaction force, R N, of the table on the cuboid acts x m from end P, as shown in Fig. 3.

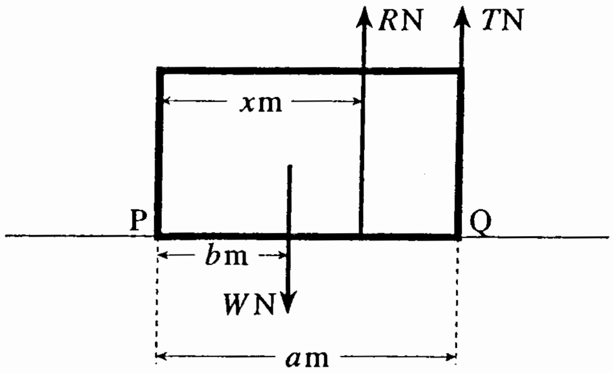


Fig. 3

(i) By resolving and taking moments, write down two equations for the equilibrium of the cuboid.

Hence show that $x = \dfrac{Wb - Ta}{W - T}$. [5]

(ii) Find an expression for R in terms of W, a and b when the cuboid is on the point of turning about the edge through P. [2]

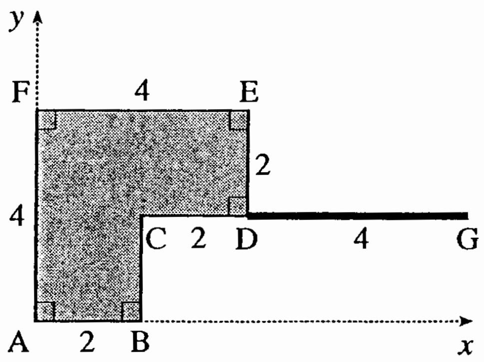
(b) A uniform ladder rests against a smooth vertical wall and on rough horizontal ground. It is inclined at 72° to the horizontal when it is about to slip.

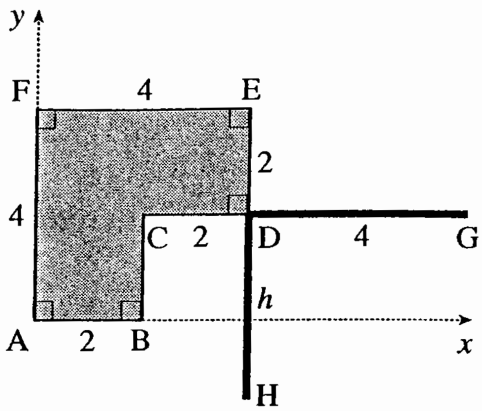
Draw a diagram showing the forces acting on the ladder.

Calculate the coefficient of friction between the ladder and the ground. [8]

[Total 15]

4 Fig. 4.1 shows a uniform lamina ABCDEF. The units are metres. The lamina has a weight of 1 N per unit area. The rod DG is 4 m long and rigidly attached to the lamina so that CDG is a straight line. The weight of the rod is 1 N per unit length.





not to scale

Fig. 4.1 **Fig. 4.2**

(i) Calculate the coordinates of the centre of mass of the lamina and rod, referred to the axes shown in Fig. 4.1. [5]

A further rod, DH, of length h m is made of the same material as DG. This rod is rigidly attached to the lamina so that EDH is a straight line, as shown in Fig. 4.2.

(ii) Referred to the axes shown in Fig. 4.2, show that the coordinates of the centre of mass of the lamina and two rods are

$$\left(\frac{44 + 4h}{16 + h}, \frac{72 + h(4 - h)}{2(16 + h)}\right).$$ [6]

The composite figure shown in Fig. 4.2 is freely suspended from M, the mid-point of AF, and hangs in equilibrium with B vertically below M.

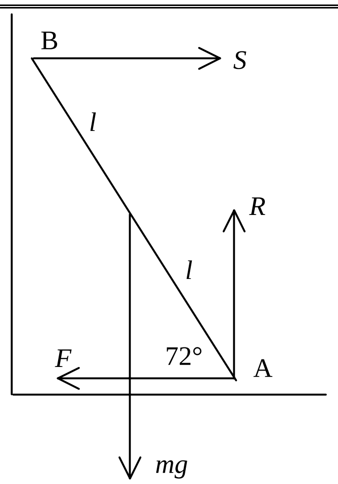
(iii) Show that, referred to the axes shown in Fig. 4.2, its centre of mass must lie on the line $y = 2 - x$. Calculate the value of h. [4]

[Total 15]

Mark Scheme

Q 1		mark		sub
(i)	$\rightarrow$ $1500(150-100) = 75000$ N s in original direction	M1 A1	$m\mathbf{v} - m\mathbf{u}$ Accept +ve answer without direction	2
(ii)	**either** $\rightarrow$ $500 \times 100 - 75000 = 500v$ $v = -50$ so 50 m s^{-1} in opposite direction to original motion **or** $\rightarrow$ $2000 \times 100 = 500v + 1500 \times 150$ $v = -50$ so 50 m s^{-1} in opposite direction to original motion	M1 B1 A1 M1 A1 A1	Impulse-momentum, both terms Use of **their** –75000 or equivalent Accept –ve answer without direction Use of PCLM	3
(iii)	$F \times 75 = 75000$ $F = 1000$ so 1000 N	M1 A1	Use of $\mathbf{I} = t\mathbf{F}$ FT **their** 75 000	2
(iv)	$\rightarrow$ $1500 \times 150 + 500 \times 130 = 2000V$ $V = 145$ so 145 m s^{-1}	M1 B1 E1	PCLM Deal with coalescence	3
(v)	$\rightarrow$ $u_A = 155$ $500 \times 155 + 2000 \times 145 = 500v_A + 2000v_D$ so $735 = v_A + 4v_D$ $v_D - v_A = 10$ Solving $v_D = 149$ so 149 m s^{-1} $v_A = 139$ so 139 m s^{-1}	B1 M1 A1 B1 A1 A1	 PCLM Any form Any form; condone wrong sign. cao cao	6
			total	16

Q 2		mark		sub
(a)	$\frac{4\times 9.8\times 0.8}{2\times 24\times 3600}=181\,\mu\text{ W}$ (3 s. f.)	M1 M1 A1	Use of *mgh* for GPE Use of WD/time	3
(b) (i)	**either** Must not stop before the start of the ramp so $0.5\times 10\times v^2 \geq 4\times 40$ least v is $4\sqrt{2}$ m s^{-1} or 5.66 m s^{-1} (3 s. f.) **or** Use of N2L to give $a=-4$ $v_B{}^2 = v^2 + 2\times -4\times 4$ $v_B \geq 0$ least v is $4\sqrt{2}$ m s^{-1} or 5.66 m s^{-1} (3 s. f.)	M1 A1 A1 B1 M1 A1	Expression attempted involving WD and KE Correct. Accept =. Accept ≥ Allow use of **their** *a* in appropriate *uvast* Accept ≥	3
(ii)	$10\times 9.8\times 8\sin 35$ = 449.6839… so 450 J (3 s. f.)	M1 A1	Use of cpt in GPE change cao	2
(iii)	**either** $0.5\times 10\times 36 + 449.6839... - 4\times 40 - 8\times 50$ $=0.5\times 10V^2$ V = 3.73319… so 3.73 m s^{-1} (3 s. f.) **or** $v_B = 2$ N2L on BC $10g\sin 35 - 50 = 10a$ a = 0.62 $V^2 = 2^2 + 2\times 0.62\times 8$ V = 3.73319… so 3.73 m s^{-1} (3 s. f.)	M1 B1 B1 A1 B1 A1 B1 M1 B1 A1 M1 A1	W-E equation with KE, GPE and WD Change in KE (neglect sign) Either work done against resistance term (neglect sign) All terms correct (condone sign errors) All signs correct cao Use of N2L Weight component Appropriate *uvast* with **their** v_B and *a* cao	6
			total	14

Q 3		mark		sub
(a) (i)	$\widehat{\text{P}}$ $Wb = Rx + Ta$	M1 A1	Moments equation attempted	
	$\uparrow$ $W = R + T$	B1	Resolve or moments again	
	Eliminate R $Wb = (W-T)x + Ta$	M1	Substitute to eliminate R	
	so $x = \dfrac{Wb - Ta}{W - T}$	E1	Clearly shown	5
(ii)	**either**			
	Need x = 0 giving $T = \dfrac{Wb}{a}$	B1	Both required	
	$R = W - \dfrac{Wb}{a} = \dfrac{W}{a}(a-b)$	B1	Either form	
			[Award B2 for answer seen]	
	or Moments about edge through Q	M1	Moments about edge through Q. R acting at edge thro' P	
	Result	A1		2
(b)		B1	All forces labelled correctly with arrows	
	$\uparrow$ $R = mg$	B1		
	$\rightarrow S = F$	B1		
	$\widehat{\text{A}}$ $mgl\cos 72 = S \times 2l\sin 72$	M1 A1	Moments about a suitable point (if B used then F term receives the $S = F$ B1mark above)	
	so $F = \dfrac{mg\cos 72}{2\sin 72}$	A1	May be implied	
	$F = \mu R$ so $\mu = \dfrac{mg\cos 72}{2\sin 72} \div mg$	M1	Subst of **their** R and F required	
	$= \dfrac{1}{2}\cot 72$ or 0.162 (3 s. f.)	A1	Any form and any reasonable accuracy	8
			total	15

Q 4		mark		sub
(i)	$16\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=8\begin{pmatrix}1\\2\end{pmatrix}+4\begin{pmatrix}3\\3\end{pmatrix}+4\begin{pmatrix}6\\2\end{pmatrix}$	M1 B1 A1	Correct method Masses correct At least one c.m. correct (if x, y done separately, at least two c. m. cpts correct)	
	$\bar{x}=\frac{11}{4}$,	A1		
	$\bar{y}=\frac{9}{4}$	A1		5
(ii)	$(16+h)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}44\\36\end{pmatrix}+h\begin{pmatrix}4\\ 2-\frac{h}{2}\end{pmatrix}$	M1 B1 M1 A1	Correct method $16+h$ Attempt at $\begin{pmatrix}4\\ 2-\frac{h}{2}\end{pmatrix}$ Correct	
	$\bar{x}=\frac{44+4h}{16+h}$	E1	Clearly shown	
	$\bar{y}=\frac{36+h\left(2-\frac{h}{2}\right)}{16+h}=\frac{72+h(4-h)}{2(16+h)}$	E1	Clearly shown	6
(iii)	B lies vertically below M so c. m. lies on BM which has equation $y=2-x$	E1	Full explanation but allow equation of BM not derived	
	$\frac{72+h(4-h)}{2(16+h)}=2-\frac{44+4h}{16+h}$	M1	Substituting for $\bar{x}$ and $\bar{y}$	
	$\Rightarrow 72+4h-h^2=64+4h-88-8h$ $\Rightarrow h^2-8h-96=0$	A1		
	giving h = 14.6 (3 s. f.)	A1	Accept only positive root given	4
				15

Examiner’s Report

2608 Mechanics 2

General Comments

This paper appeared to be accessible to the vast majority of the candidates with almost all of them being able to gain credit in part of every question. No question appeared to give higher scores than the others nor did any question seem to cause more difficulties than the others. While a large number of excellent scripts were seen, the standard of presentation of a significant minority of candidates was poor. Diagrams were badly drawn or omitted altogether; working was unclear or badly sequenced; variables were not always defined clearly and, in a small number of cases, the same variable was used to describe more than one quantity. Some candidates also failed to show enough of the relevant working when trying to establish given answers.

Comments on Individual Questions

Q.1 The first part of this question caused difficulties for a large number of candidates. While all of them appeared to know that impulse is 'change in momentum', many did not appreciate exactly what this meant and it was common to see (1500x150 – 2000 x 100) given as the answer. The next three parts of the question caused very few problems and the majority of candidates were able to score highly on them. A small number of candidates did not give a direction in part (ii), specifying only the speed of A, not the velocity requested. In the last part of the question, most of the candidates understood that conservation of momentum was involved as well as Newton's experimental law. Mistakes were mainly due to sign errors in the application of Newton's experimental law or the assumption that A was travelling at 10m s^{-1} *slower* than D as it approached D. A small number of candidates did not appear to understand the description of the situation and assumed A to be travelling at 10m s^{-1} and D at145m s^{-1}.

Q.2 This question was tackled much more successfully than similar questions set in recent sessions. Most of the candidates could score quite highly on the first three parts of the question with errors mainly being arithmetic rather than conceptual. The last part posed some difficulties to some of the candidates although there were a high number of correct responses to it. The difficulties encountered usually centred on the omission of one of the terms in the work energy equation (most frequently one of the work done terms). The candidates that attempted to solve the problem using Newton's laws and *uvast* formulae were usually successful with the main source of error being the assumption that the speed at B was 6 ms^{-1}.

Q.3 There were very many correct responses to the first part of the question with almost all of the candidates being able to set up the two equations requested. Candidates who took moments twice (instead of resolving and taking moments as requested) were not penalised if their equations were correct. A very small minority found it difficult to manipulate the equations to obtain the given answer. Part (ii) was poorly done by the majority of the candidates with only the most able appreciating that if the cuboid turned about the edge through P then $x = 0$ and R acted through that edge. It was common to see $R = 0$ stated as the answer or for a candidate just to rearrange one of the expressions obtained in part (i). Attempts at part (b) were less successful than those at part (a). A small number of candidates did not draw the diagram requested; others drew a diagram that was poorly labelled or had one or more forces missing. Those who did draw the diagram could usually write down equilibrium equations for the forces in both horizontal and vertical directions but fewer could construct a correct equation involving moments. It was common to see μ = tan 72 stated as the answer with little working to support this.

Q.4 Almost all of the candidates gained full credit in part (i) but part (ii) posed more difficulties. Establishing the given answer for the *x* co-ordinate was done successfully but establishing that for the *y* co-ordinate was less so. Many candidates appeared not to realise that being asked to show a given answer means showing all of the relevant steps in the working and giving a clear indication of the method being used. Working back from the given answer is not usually a strategy on which it is worth wasting time. The final part of the question caused difficulties for all but the most able, not so much in the concepts involved but in obtaining and solving a suitable quadratic equation. Overall the biggest problem encountered by a significant minority in this question was that they could not work at the level of algebraic manipulation required.

Coursework: Mechanics 2

Once again we were very impressed by some of the work produced. Many of the projects had been clearly marked and annotated and this was very helpful. In particular it is helpful if some of the numerical predictions generated from the model are checked for accuracy and if incorrect work is indicated.

Unfortunately, the main problems moderators encountered were ones which were mentioned in last year's report and in previous reports.

The criteria require a model to be derived, capable of generating a set of numerical predictions which can then be tested by measuring real life values either collected from a practical experiment or quoted from a recognised source. Where this was not evident, it was difficult to justify the marks awarded in domains giving credit for comparison and revision.

Marks awarded in the first domain were sometimes too generous. The assumptions listed should be relevant to the task and candidates should include some discussion to indicate that they are aware of the reason for making the assumptions. The second mark requires the candidate to show the relevance of the assumptions to the equations used in developing the model and also to the design of the experiment.

Many projects submitted involved friction. Much of this work would have been improved if candidates had carried out an initial independent experiment to determine an estimate for μ. Guessing a value is not very convincing, and using the outcomes of the experiment which are being collected to verify the theoretical predictions leads to a circular argument.

Credit in the revision domain should be given for proposals to improve the match between theoretical and practical values. Marks were sometimes awarded for simply indicating possible causes of error.