RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MEI STRUCTURED MATHEMATICS **2621/1**

Decision and Discrete Mathematics 2

Friday **28 MAY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

**TIME** 1 hour 20 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

**This question paper consists of 6 printed pages and 2 blank pages.**



[Turn over

Jun04/erratum29

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# MEI STRUCTURED MATHEMATICS 2621/1

**Decision and Discrete Mathematics 2**

Friday **28 MAY 2004** Afternoon 1 hour 20 minutes

## ERRATUM NOTICE

**One copy to be given to each candidate**

There is an error in the table in Question **2** part **(ii)**.

The tableau in part **(ii)** must be crossed out.

**You must use the replacement tableau below when answering part (ii) of the question.**

**You must also note that the reference in the first line of part (iii) refers to the replacement tableau, on which a further pivot is required.**

| $P$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | RHS |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\frac{1}{6}$ | 0 | $\frac{7}{3}$ |
| 0 | 0 | 0 | 1 | $-\frac{3}{4}$ | $\frac{1}{4}$ | 1 |
| 0 | 0 | 1 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 2 |
| 0 | 1 | 0 | 0 | $-\frac{1}{4}$ | $\frac{3}{4}$ | 4 |

1 **(a)** In this question $n$ is a positive integer.

Write down the contrapositive of the following statement:

"If $n^2$ is even then $n$ is even."

Prove that if $n^2$ is even then $n$ is even. [4]

**(b)** Use Boolean algebra to prove that $\sim a \vee (a \wedge b) \vee (a \wedge c) \Leftrightarrow \sim a \vee b \vee c$.

(Do not use a truth table.) [6]

2 Despina can invest her savings of £1000 in bonds or in equities. Bonds are generally reckoned to be safer than equities. Equities have potentially higher returns (measured in % per annum), but may suffer losses (again measured in % per annum).

From past history the probability of equity returns being +15% is 0.6, but the probability of equity returns being –10% is 0.4. For bonds the probability of a 5% increase is 0.8 and the probability of a 3% increase is 0.2.

**(i)** What will be the value of Despina's savings after one year under each of the 4 possible scenarios, i.e.

if she invests in equities and they increase in value by 15%;
if she invests in equities and they decrease in value by 10%;
if she invests in bonds and they increase in value by 5%;
if she invests in bonds and they increase in value by 3%. [2]

**(ii)** Draw a decision tree for Despina. Use the EMV criterion to advise her where to invest her money. [5]

Despina's utility function for her savings is given by

$$Utility = (Monetary\ Value)^p \text{, where } p < 1 .$$

**(iii)** Using expected utility, show that the value of $p$ which will make Despina indifferent between investing in equities and investing in bonds is 0.5 (correct to 1 decimal place). [3]

**3** The graph in Fig. 3 shows the feasible region for the following LP problem.

$$\begin{aligned} \text{Maximise} \quad & P = \tfrac{1}{3}x + \tfrac{1}{2}y \\ \text{subject to} \quad & x + 2y \leqslant 9 \\ & 2x + 3y \leqslant 14 \\ & 2x + y \leqslant 10 \\ & x \geqslant 0 \text{ and } y \geqslant 0 \end{aligned}$$

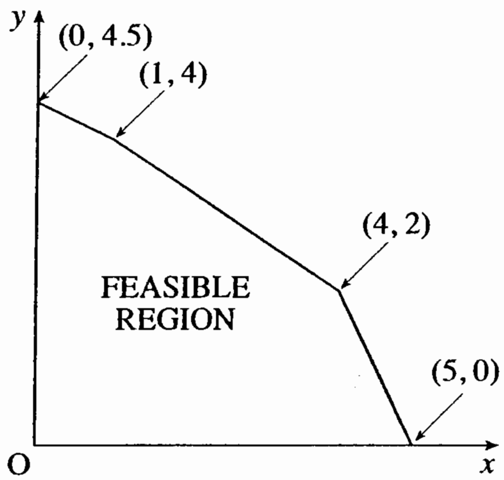



**Fig. 3**

**(i)** Use the graph to solve the LP problem. [4]

**(ii)** Solve the problem using the simplex algorithm. Start with pivoting on the $x$-column.

Show that the final tableau is

| $P$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | RHS |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\frac{1}{6}$ | 0 | $\frac{7}{3}$ |
| 0 | 0 | 0 | 1 | $-\frac{3}{4}$ | $\frac{1}{4}$ | 1 |
| 0 | 1 | 0 | 0 | $-\frac{1}{4}$ | $\frac{3}{4}$ | 4 |
| 0 | 0 | 1 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 2 |

Interpret this solution. [7]

**(iii)** From the final tableau given in part **(ii)**, perform another pivot using the $\frac{1}{4}$ as pivot element. Comment on the result. [4]

**Question 3 continued on Page 5**

The simplex algorithm is applied to the problem

$$\begin{aligned} \text{Maximise} \quad & Q = \tfrac{1}{3}x + \tfrac{1}{2}y + \tfrac{7}{12}z \\ \text{subject to} \quad & 4x + 8y + 9z \leqslant 36 \\ & 4x + 6y + 7z \leqslant 28 \\ & 4x + 2y + 5z \leqslant 20 \\ & x \geqslant 0, y \geqslant 0 \text{ and } z \geqslant 0 \end{aligned}$$

The following tableau is produced

| $Q$ | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | $\frac{1}{12}$ | 0 | $\frac{7}{3}$ |
| 0 | $-\frac{8}{7}$ | $\frac{2}{7}$ | 0 | 1 | $-\frac{9}{7}$ | 0 | 0 |
| 0 | $\frac{4}{7}$ | $\frac{6}{7}$ | 1 | 0 | $\frac{1}{7}$ | 0 | 4 |
| 0 | $\frac{8}{7}$ | $-\frac{16}{7}$ | 0 | 0 | $-\frac{5}{7}$ | 1 | 0 |

**(iv)** Interpret this tableau.

Say how you know that there are other optimal solutions, and find the two other optimal vertices.

(Hint: Put $z = 0$ in the statement of the problem.) [5]

4 The map in Fig. 4 shows a small island. The labelled points are beaches. Tracks are marked, with distances shown in km. The total length of the tracks is 32 km.

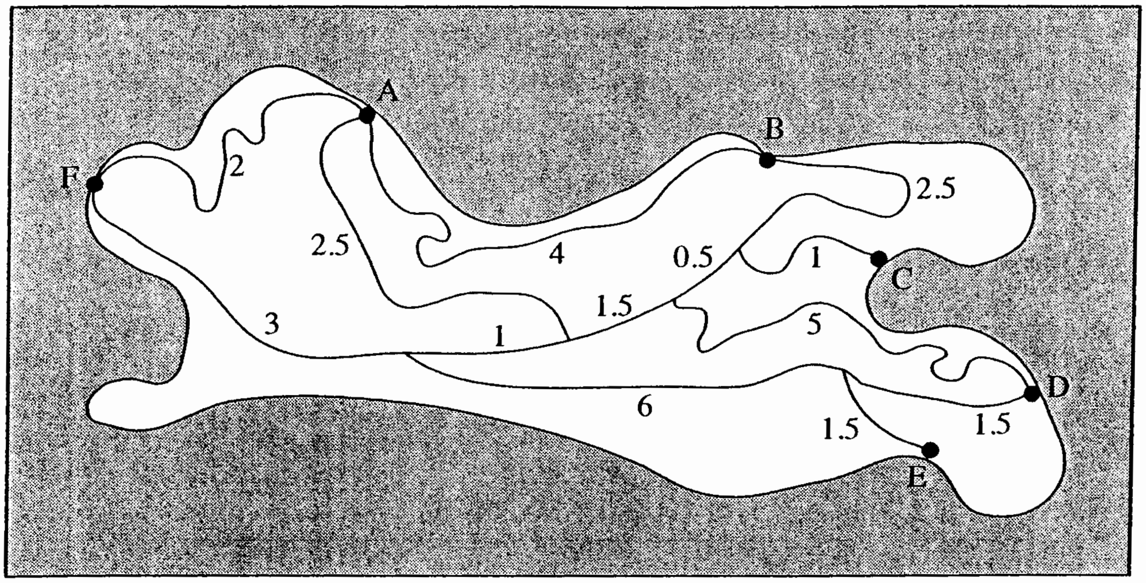



**Fig. 4**

Floyd's algorithm is applied to a six-node network extracted from this map, giving the following final matrices.

Distance Matrix

| | **A** | **B** | **C** | **D** | **E** | **F** |
|---|---|---|---|---|---|---|
| **A** | 4 | 4 | 5.5 | 9 | 11 | 2 |
| **B** | 4 | 7 | 3.5 | 8 | 11 | 6 |
| **C** | 5.5 | 3.5 | 7 | 6.5 | 9.5 | 7 |
| **D** | 9 | 8 | 6.5 | 6 | 3 | 10.5 |
| **E** | 11 | 11 | 9.5 | 3 | 6 | 10.5 |
| **F** | 2 | 6 | 7 | 10.5 | 10.5 | 4 |

Route Matrix

| | **A** | **B** | **C** | **D** | **E** | **F** |
|---|---|---|---|---|---|---|
| **A** | F | B | C | D | E | F |
| **B** | A | C | C | D | D | A |
| **C** | A | B | B | D | D | F |
| **D** | A | B | C | E | E | F |
| **E** | A | D | D | D | D | F |
| **F** | A | A | C | D | E | A |

(i) Explain how to use the matrices to find the shortest distance and route from E to B. Give the route and the distance. [4]

**Question 4 continued on Page 7**

Joan and Keith spend a few days on the island. Joan wants to visit every beach.

**(ii)** Use the nearest neighbour algorithm on the shortest distance network, given by the final distance matrix, to find an efficient order for Joan to visit all of the beaches, starting and finishing at beach A. (Note that this may give a route which involves re-visiting a vertex in the original network.)

Give the distance travelled, and use the route matrix to give the route. [5]

**(iii)** Give a shorter route starting and finishing at A. [2]

Keith would like to walk along every track on the island.

**(iv)** Explain why the six-node network would not be a suitable model for finding a good route for Keith.

Give the number of nodes on an appropriate network.

Which of the nodes A, B, C, D, E and F would not be needed? [3]

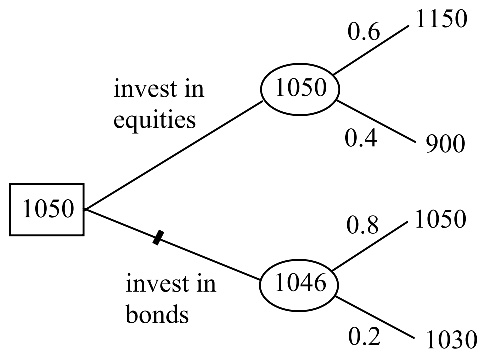
**(v)** Obtain an efficient route for Keith starting and finishing at A, and give its length. [6]

# Mark Scheme

1.

| | |
|---|---|
| (a) Contrapositive: n odd $\Rightarrow$ $n^2$ odd<br><br>Assume n is odd.<br>But then $n^2$ will be odd (since $(2n+1)^2 = 2(2n^2+2n)+1$)<br>So n odd $\Rightarrow$ $n^2$ odd<br>Contrapositive: $n^2$ even $\Rightarrow$ n even | B1<br><br>M1<br>A1<br><br>B1 contrapositive equivalent to original |
| (b) $\sim a\vee(a\wedge b)\vee(a\wedge c) \Leftrightarrow \sim a\vee(a\wedge(b\vee c))$<br>$\Leftrightarrow (\sim a\vee a)\wedge(\sim a\vee(b\vee c))$<br>$\Leftrightarrow \sim a\vee(b\vee c)$<br>$\Leftrightarrow \sim a\vee b\vee c$<br>or<br>$\sim a\vee(a\wedge b)\vee(a\wedge c) \Leftrightarrow ((\sim a\vee a)\wedge(\sim a\vee b))\vee(a\wedge c)$<br>$\Leftrightarrow (\sim a\vee b)\vee(a\wedge c)$<br>$\Leftrightarrow (\sim a\vee b\vee a)\wedge(\sim a\vee b\vee c)$<br>$\Leftrightarrow \sim a\vee b\vee c$ | M1 distributive rule<br>A1<br>M1 distributive again<br>A1<br>M1 identity rule<br>A1 |

2.

| | |
|---|---|
| (i) 1150; 900; 1050; 1030 | M1 A1 |
| (ii)   | M1 chance nodes<br>A1<br>A1<br><br>B1 decision node<br>B1 advice – invest in equities |
| (iii) Require p such that<br>$0.6\times1150^p + 0.4\times900^p = 0.8\times1050^p + 0.2\times1030^p$<br>Letting f(p) =<br>$0.6\times1150^p + 0.4\times900^p - 0.8\times1050^p - 0.2\times1030^p$<br>f(0.5) = 0.0053<br>f(0.45) = –0.0003 | M1 implied OK<br><br>A1 a correct evaluation<br>A1 bracketing solution |

3.

(i) (0, 4.5) → **2.25** M1
(1, 4) → **7/3**
(4, 2) → **7/3** A2 (–1 each error)
(5, 0) → **5/3**

So **7/3** at either (1, 4) or (4, 2) (or anywhere on the line segment joining them). A1

(ii)

| P | x | y | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|
| 1 | −1/3 | −1/2 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 1 | 0 | 0 | 9 |
| 0 | 2 | 3 | 0 | 1 | 0 | 14 |
| 0 | 2 | 1 | 0 | 0 | 1 | 10 |
| 1 | 0 | −1/3 | 0 | 0 | 1/6 | 5/3 |
| 0 | 0 | 3/2 | 1 | 0 | −1/2 | 4 |
| 0 | 0 | 2 | 0 | 1 | −1 | 4 |
| 0 | 1 | 1/2 | 0 | 0 | 1/2 | 5 |
| 1 | 0 | 0 | 0 | 1/6 | 0 | 7/3 |
| 0 | 0 | 0 | 1 | −3/4 | 1/4 | 1 |
| 0 | 0 | 1 | 0 | 1/2 | −1/2 | 2 |
| 0 | 1 | 0 | 0 | −1/4 | 3/4 | 4 |

M1 initial tableau
A1

B4 (1 for each row)

(given)

Solution is **7/3** at (4, 2). B1

(iii)

| Q | x | y | s1 | s2 | s3 | RHS |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1/6 | 0 | 7/3 |
| 0 | 0 | 0 | 4 | −3 | 1 | 4 |
| 0 | 0 | 1 | 2 | −1 | 0 | 4 |
| 0 | 1 | 0 | −3 | 2 | 0 | 1 |

M1
A2 (–1 each error in a row)

This represents the other optimal vertex, (1, 4) B1

(iv) Tableau is optimal at (0, 0, 4) with value **7/3**. B1 B1

Other solutions exist since there is (are) a non-basic variable(s) with zero in the objective row. B1

(1, 4, 0) and (4, 2, 0) B1 B1

4.

| | | |
|---|---|---|
| (i) | Entry in row E and column B of distance matrix is 11 | B1 B1 |
| | Entry in row E and column B of route matrix is D, so route is ED … | B1 explanation |
| | Entry in row D and column B of route matrix is B, so route is EDB. | B1 EDB |
| (ii) | **A**(2)**F**(6)**B**(3.5)**C**(6.5)**D**(3)**E**(11)**A** – 32 | M1 A1<br>B1 (32) |
| | Route: AFABCDEA | M1 A1 |
| (iii) | **A**(4)**B**(3.5)**C**(6.5)**D**(3)**E**(10.5)**F**(2)**A** – 29.5 | B1 B1 |
| (iv) | Tracks are not defined by beaches – need to have nodes for track junctions. | B1 |
| | Either 6+5 = 11 or 3+5 = 8 | B1 |
| | (the latter by cutting out the 2-nodes at B, D and F) | B1 |
| (v) | Need to repeat 1 + 1.5 + 2.5 + 1 + 1.5 = 7.5 | M1 A1 Implied OK |
| | e.g.<br>**A**(5.5)**C**(3.5)**B**(4)**A**(2)**F**(10.5)**D**(3)**E**(11)**A** | M1 A1 |
| | Distance = 32 + 7.5 = 39.5 | M1 A1 |

# Examiner’s Report

## 2621/01 Decision and Discrete Mathematics 2

### General Comments

Overall scores were down compared to usual, partly because candidates performed badly on question 1. Candidates were well prepared for the LP work in question 3. They seemed particularly to enjoy successfully completing the manipulations within the simplex algorithm.

A late change to question 3 in the production process led to some confusion, for which the Board apologises. Two inequalities in the LP, and the corresponding two lines in the final tableau were to be swapped, to make them line up better with the diagram. In the event only the inequalities were swapped. Whilst this did not invalidate the tableau, it was felt to be best to issue an erratum slip to correct it. Unfortunately this slip was in error, referring to question 2 instead of to question 3.

### Comments on Individual Questions

Q.1 The beginning of part (a) was intended to help the candidates realise that all that was required to prove that "$n^2$ even $\Rightarrow$ n even" was to prove that "n odd $\Rightarrow$ $n^2$ odd". Few candidates scored marks on this section.

Part (b) was a good discriminating question. Able candidates did well on it. The less able found it to be difficult.

Q.2 Candidates perform well on decision analysis. Parts (i) and (ii) were no exception to this rule, but part (iii) was more difficult. Two of the three marks here were awarded for the computation of two expected utilities. Since utility functions are nonlinear these are not the same as the utilities of the expectations. The final mark, which was very difficult, was awarded only to those few candidates who realised that the relevant values of p to be investigated were p = 0.45 and p = 0.55 (or, in the event, p = 0.5 and p = 0.45).

Q.3 This was very well answered. Candidates were well prepared for applying the simplex algorithm, and showed considerable skill in using it. The final three marks were more challenging. Many scored two of them by using the earlier solutions to produce Q = 2/3 at (1, 4, 0) and at (4, 2, 0). The third mark was only for the very best candidates, requiring them to note that in the final tableau there were non-basic variables with zeros in the objective row.

Q.4 The Floyd/TSP work in parts (i) to (iii) was done well. The route inspection work in parts (iv) and (v) was discriminating. In particular the first mark of part (iv) required the application of modelling skills. Candidates needed to note that extra nodes were needed at track intersections.

### Coursework: Decision and Discrete Mathematics 2

Work was submitted from 518 candidates at 66 centres, a small increase in candidates on last summer.

Most of the centres had entered candidates for 2621 in the past. Moderating adjustments were made to about one seventh of the marks, a higher proportion than last summer. Most of these adjustments related to over-generous marking in the first two domains.

**<u>Marking</u>**

The majority of centres had taken the time to mark the work very carefully and helpfully, annotating the students' work appropriately. As previously, in a few cases work that was clearly wrong had been marked as correct.

**Content**

About two thirds of the projects were on the Networks section. There were substantially more than last summer on Decision Analysis, with only a few centres entering work on Linear Programming and Logic.

As previously, not all centres encouraged their candidates to use appendices for details of calculations, with summary tabulations in the main report. Those that did produced much more readable reports.

*Logic*
The few pieces seen were appropriate and original. Candidates undertaking projects in this area tend to be confident and competent.

*Decision Analysis*
In the best work, original and individual problems were identified, with clear modelling and interpretation demonstrated. At the other extreme were invented text-book type problems, often involving game shows and often too simple for A2 projects, with little evidence of any modelling.

*Linear Programming*
Again several of the pieces submitted this time were, in the initial model, only standard two-variable problems which could easily be solved using 2620 methods. Problems identified should normally have at least three variables. Candidates should first obtain real solutions, and then - if appropriate - they should search for nearby integer solutions. Some candidates still did not seem to appreciate that whilst such integer solutions are likely to be good, they are not necessarily optimal.

Refinements tended to be rather contrived additional constraints in order to force the two-stage simplex algorithm.

Computer software is readily available to carry out LP. Use of such software cannot substitute for manual demonstration of the relevant methods, but it can be helpful as a check.

*Networks*
Some candidates paid appropriate attention to data sources, but others did not.

The few Route Inspection projects were generally competent, with suitably complex networks. Candidates generally appreciated the need for a systematic approach to the pairing of odd nodes.

TSP was again overwhelmingly the most popular choice. Many of the tasks were interesting and original, although too many were standard (and unrealistic) tasks such as tours of several prospective universities.

Most candidates this time were clear about the distinction between the classical problem and the practical problem. Candidates' problems are nearly always practical, with initially incomplete networks and with no constraint on revisiting vertices. In the classical problem, the network is complete, and no vertex may be revisited.

The approach adopted in the syllabus is to convert the practical problem into the classical problem, with a complete network of shortest distances (typically some of these shortest distances will be via other nodes). The method for producing a lower bound is by deleting a vertex and its edges, finding the length of the minimum spanning tree of the remaining network, and then adding in the lengths of the two shortest deleted edges. The method for producing an upper bound is by the use of the nearest neighbour algorithm from each vertex in turn, and not by using twice the minimum spanning tree. Too many of the latter reappeared this time, after an improvement last year. The best tour found using nearest neighbour should be interpreted in the context of the original problem, stating clearly the route to be taken

It remains a matter of some concern that some candidates still attempt to obtain bounds from incomplete networks. Of those who did obtain complete networks of shortest distances, some appeared to have used inspection, and some of the lengths obtained were not in fact shortest distances. Other candidates used internet route finders. Many made good use of Floyd's algorithm. It should be noted that the use of Floyd is very time-consuming when applied to large networks. Candidates should be advised accordingly.

Not enough attention is paid to interpretation. When interpreting the classical solution back to the practical problem, candidates should note any differences between the classical and practical solution. Many did not consider such practicalities as duration and accommodation. Many of the tours obtained would need to be split over several days in practice.