RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Monday **21 JUNE 2004** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- There is an **insert** for use in Question **3**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages, 2 blank pages and an insert.

Section A

1 Six ports are connected by ferry services. In Fig. 1 the vertices represent the ports, and each directed arc represents one scheduled ferry trip.

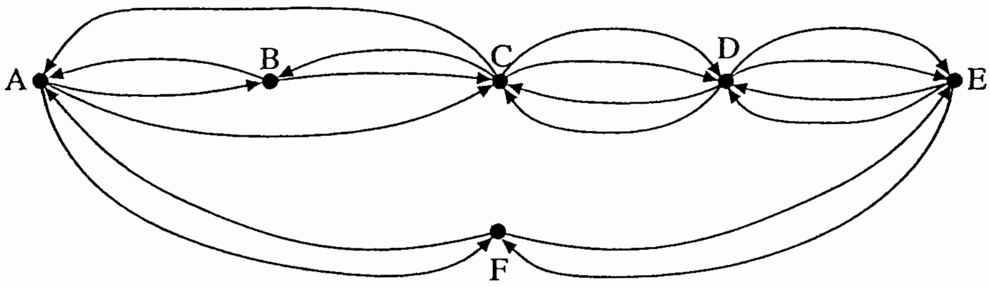

Fig. 1

One day the ferry "Sofia" starts at F, crosses to E, and returns to F. It then travels to A and on to D via B and C.

(i) Draw a graph representing the Sofia's journey. [1]

(ii) Draw the complement of your graph in part **(i)**, i.e. the graph of the journeys which have to be made by other ferries. [1]

(iii) Explain why the complement graph shows that at least one other ferry must start and end the day at different ports. [1]

(iv) Prove that, whatever the Sofia's journey, if it starts and ends the day at different ports then at least one other ferry must start and end the day at different ports. [2]

2 (i) Use Kruskal's algorithm to find a minimum connector for the complete network on five nodes shown in Fig. 2. Give the order in which you select arcs. Draw your connector and give its total weight. [3]

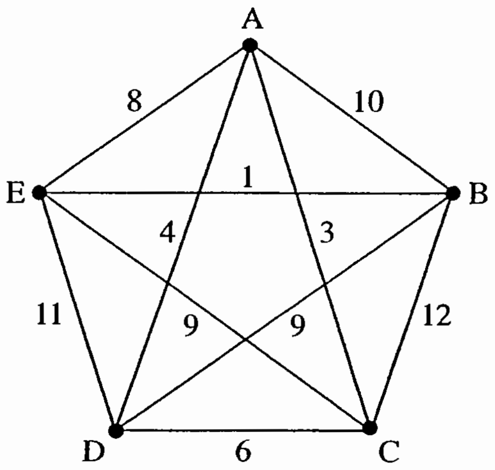

Fig. 2

(ii) If part (i) took you 3 minutes to complete, approximately how long would it take you to complete the same task on a complete network on 25 nodes? [2]

3 There is an insert provided for this question.

The following algorithm is to be applied to the positive integers from 1 to 12.

Step 1:	Cross out every even number.
Step 2:	Change the state of every multiple of 3 (including 3) – i.e. for every multiple of 3, if it is crossed out then remove the crossing out, and if it is not crossed out then cross it out.
Step 3:	Change the state of every multiple of 4 (including 4).
Step 4:	Change the state of every multiple of 5 (including 5).
...	
...	
Step 11:	Change the state of every multiple of 12 (including 12).

(i) Use Table 3 on the insert to apply the algorithm. Use a row of the table for each step. In each step the crossings-out should be the same as in the previous step except for those numbers whose state is changed. [4]

(ii) Say what the algorithm achieves. [1]

Section B

4 A ferry transports cars and lorries. The charge for transporting a car is €65. The charge for transporting a lorry is €275. Each lorry takes up the space of 4 cars, and the ferry has space for 200 cars, or equivalent. Lorries weigh on average 25 tonnes, and cars weigh on average 1.5 tonnes. The ferry can safely transport vehicles weighing 600 tonnes in total.

The ferry company wants to find the numbers of cars and lorries which generate the maximum income while satisfying the space and weight constraints.

(i) Define appropriate variables and formulate an LP to find the optimal mix of vehicles. [4]

(ii) Use a graphical approach to solve your LP, and interpret the solution. [8]

New legislation is introduced, allowing heavier lorries. The average weight of a lorry goes up to 31 tonnes.

(iii) By how much will the charge for transporting a lorry have to increase if the new optimal mix of vehicles is to produce the same income as before? [3]

5 Pedestrians arrive at a crossing controlled by traffic lights at intervals given by the distribution shown in Table 5.1.

Inter-arrival time (seconds)	0	5	10	15	20	25	30	35	40
Probability	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$

Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate intervals between pedestrians arriving at the lights. [3]

(ii) Use two-digit random numbers from the list below to simulate the arrival times of five pedestrians at the lights. The first pedestrian arrives at the time given by the first inter-arrival interval.

Random numbers: 12, 48, 64, 75, 99, 52 [3]

On arrival at the lights, pedestrians immediately press a button to turn the lights red. The lights will not turn red until 45 seconds after they last turned red, or 5 seconds after the button was pressed, whichever is the later. Any pedestrian already waiting, and any pedestrian arriving up to and including 10 seconds after the lights turned red, is allowed to cross the road.

Five simulated pedestrian arrival times are 20, 20, 55, 80, 120. Table 5.2 shows the beginning of a simulation of what happens.

Crossing start time	25	70	...
Crossing end time	35	80	...
Number crossing	2	2	...

Table 5.2

(iii) Add 120 seconds to each of your five simulated arrival times from part **(ii)** so that there are ten simulated pedestrian arrival times in total (20, 20, 55, 80, 120, and the five times that you have calculated).

Copy and complete the simulation for all ten pedestrians. [6]

For every crossing interval of 10 seconds traffic is held up for 15 seconds.

(iv) Find the percentage of the first five minutes of simulated time for which the traffic is held up.

Find the average time for which your ten pedestrians had to wait before being allowed to cross the road. [3]

6 Honor wants to clean and prepare her two holiday guest rooms for new guests. The activities, precedences and durations are shown in Table 6. Honor has staff to help her with these tasks.

	Activity	Immediate predecessors	Duration (minutes)
A	Wash and dry crockery, room 1	–	10
B	Strip room 1 of linen, towels, etc.	A	5
C	Tidy and put away, room 1	A	5
D	Clean floors and surfaces, room 1	C	15
E	Wash and dry crockery, room 2	–	10
F	Strip room 2 of linen, towels, etc.	E	5
G	Tidy and put away, room 2	E	5
H	Clean floors and surfaces, room 2	G	15
I	Wash and dry linen and towels	B, F	30
J	Make bed and supply fresh towels, room 1	B, C	10
K	Make bed and supply fresh towels, room 2	F, G	10
L	Put away clean linen and towels	I	5

Table 6

(i) Draw an activity-on-arc network for the activities shown in Table 6. [4]

(ii) Perform forward and backward passes to determine early and late event times. Give the critical activities and the minimum duration. [6]

Honor cannot start preparing a room until the departing guests have left. The guests from room 1 leave at 09:00, and the guests from room 2 leave at 09:15.

(iii) Draw a cascade chart for preparing the rooms assuming that each activity starts at its earliest start time, but subject to the constraint imposed by the guests' leaving times.

By what time will the rooms be ready for the next guests? [4]

(iv) All of the activities require one person except for activity I, which requires no-one (since Honor has a washer/dryer). If Honor has only one helper by what time can the rooms be ready? [1]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS **2620/1**

Decision and Discrete Mathematics 1

INSERT

Monday **21 JUNE 2004** Morning 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **3.**
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

Insert for question 3.

(i) Table 3

Numbers	1	2	3	4	5	6	7	8	9	10	11	12
Step 1	1	2	3	4	5	6	7	8	9	10	11	12
Step 2	1	2	3	4	5	6	7	8	9	10	11	12
Step 3	1	2	3	4	5	6	7	8	9	10	11	12
Step 4	1	2	3	4	5	6	7	8	9	10	11	12
Step 5	1	2	3	4	5	6	7	8	9	10	11	12
Step 6	1	2	3	4	5	6	7	8	9	10	11	12
Step 7	1	2	3	4	5	6	7	8	9	10	11	12
Step 8	1	2	3	4	5	6	7	8	9	10	11	12
Step 9	1	2	3	4	5	6	7	8	9	10	11	12
Step 10	1	2	3	4	5	6	7	8	9	10	11	12
Step 11	1	2	3	4	5	6	7	8	9	10	11	12

(ii)

Spare copy of Table 3. (You do not need to use this.)

Numbers	1	2	3	4	5	6	7	8	9	10	11	12
Step 1	1	2	3	4	5	6	7	8	9	10	11	12
Step 2	1	2	3	4	5	6	7	8	9	10	11	12
Step 3	1	2	3	4	5	6	7	8	9	10	11	12
Step 4	1	2	3	4	5	6	7	8	9	10	11	12
Step 5	1	2	3	4	5	6	7	8	9	10	11	12
Step 6	1	2	3	4	5	6	7	8	9	10	11	12
Step 7	1	2	3	4	5	6	7	8	9	10	11	12
Step 8	1	2	3	4	5	6	7	8	9	10	11	12
Step 9	1	2	3	4	5	6	7	8	9	10	11	12
Step 10	1	2	3	4	5	6	7	8	9	10	11	12
Step 11	1	2	3	4	5	6	7	8	9	10	11	12

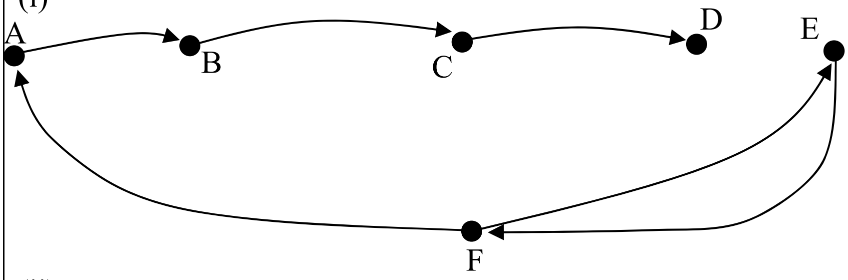
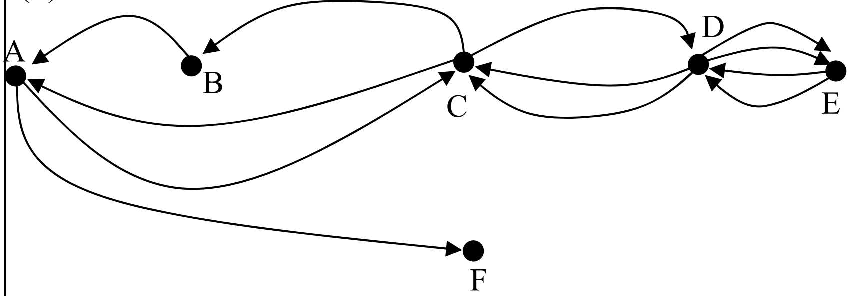
Mark Scheme

2620/01 MEI Decision and Discrete Mathematics 1

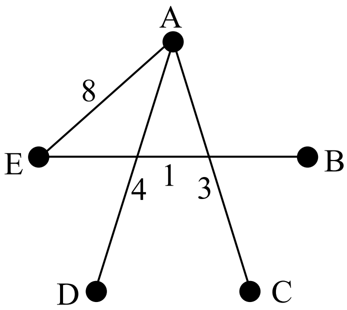
June 2004

Mark Scheme

1.

(i) 	B1
(ii) 	M1 A1
(iii) e.g. A ferry must end at F, but no ferry can start there.	B1
(iv) All of the nodes of the original graph have number of "in" arcs = number of "out" arcs. If the Sofia starts and ends at different ports then the complement graph does not have that property. Therefore it is not Eulerian.	B1

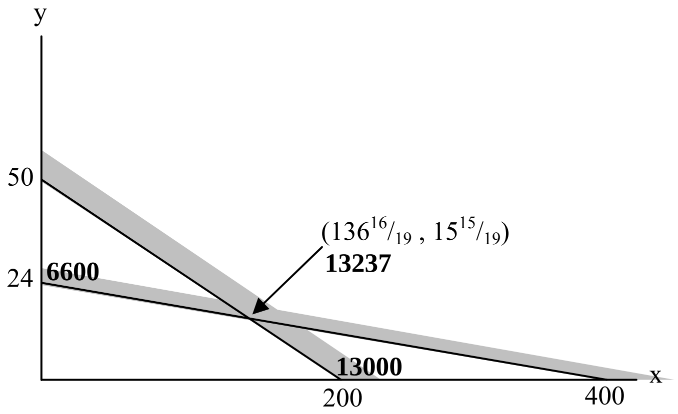
2.

(i) BE, AC, AD, AE	B1 (cao)
	B1 (cao)
Total weight = 16	B1 (cao)
(ii) Cubic complexity, so $3 \times 5^3 = 375$ minutes	M1 A1 (quadratic complexity − 75 mins − SC1)

3.

(i) step													
1	1		3		5		7		9		11		M1
2	1				5	6	7				11	12	A1 step 2
3	1			4	5	6	7	8			11		
4	1			4		6	7	8		10	11		
5	1			4			7	8		10	11	12	A1 steps 3 to 5
6	1			4				8		10	11	12	
7	1			4						10	11	12	
8	1			4					9	10	11	12	
9	1			4					9		11	12	
10	1			4					9			12	
11	1			4					9				A1 steps 8 to 11
(ii) square numbers													B1

4.

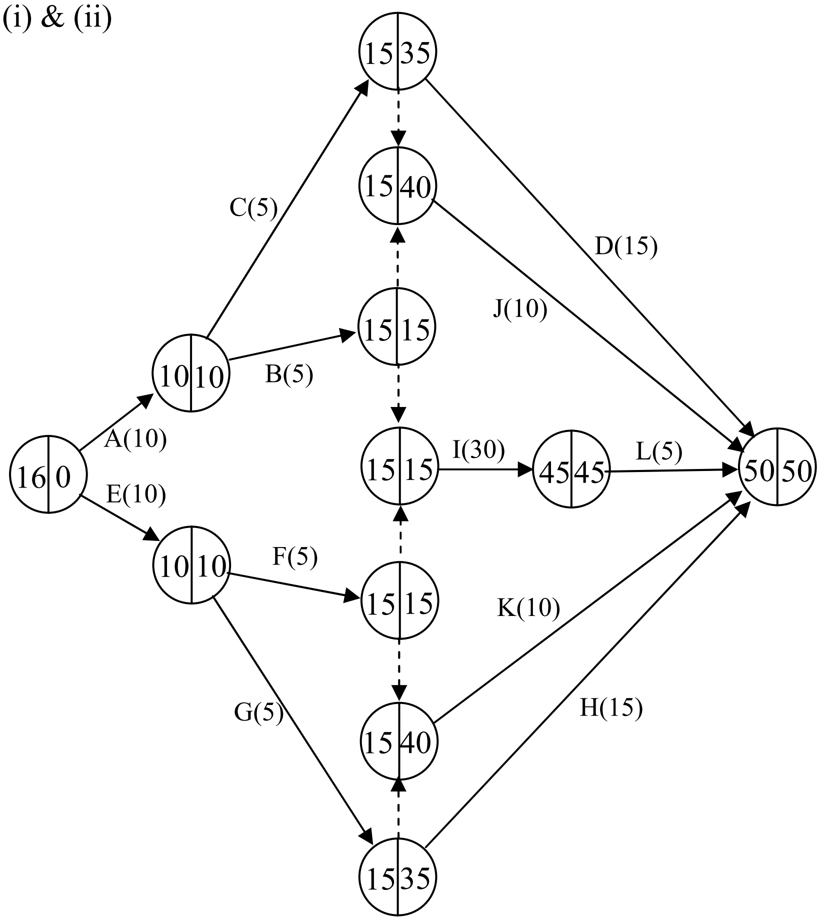
(i)	Let x be the number of cars loaded on to the ferry. Let y be the number of lorries.	B1 must be explicit
	Max $65x + 275y$ st $x + 4y \le 200$ $1.5x + 25y \le 600$	B1 B1 B1
(ii)		B1 lines B1 B1 shading B1 correct vertex
	$(136, (600–1.5\times136)/25) = (136, 15.84)$ $(137, (200–137)/4) = (137, 15.75)$ $(200–4\times15, 15) = (140, 15)$ $((600–25\times16)/1.5, 16) = (133^1/_3, 16)$	M1 integer point
	$(136, 15) \rightarrow$ **12965** $(137, 15) \rightarrow$ **13030** $(140, 15) \rightarrow$ **13225** $(133, 16) \rightarrow$ **13045**	A1 15 and 16 lorries
	So optimum mix is 140 cars and 15 lorries, giving an income of €13225	M1 A1 (cao)
(iii)	New optimal point given by $x+4y=200$ and $1.5x+31y=600$, ie. (152, 12) Require $152\times65 + 12\times\text{cost} = 13225$, giving €278.75. This is an increase of only €3.75	B1 1.5x+31y B1 152 or 12 B1√ new value or increase

5.

(i) eg.(this applies to whole question) 00–02 → 0 03–05 → 5 06–11 → 10 12–17 → 15 18–26 → 20 27–35 → 25 36–44 → 30 45–62 → 35 63–89 → 40 90–99 → ignore	 B1 sca at proportions M1 some ignored A1
(ii) 15 35 40 40 35 15 50 90 130 165	B1√ intervals M1 times A1
(iii) arrival times 20 20 55 80 120 135 170 210 250 285 start 25 70 125 175 220 265 310 end 35 80 135 185 230 275 320 number 2 2 2 1 1 1 1	 B1 implied OK M1 A1 any correct red light interval A1 any interval correctly delayed to 45s after start of previous interval A1 any interval with arriving pedestrians correctly crossing A1√ numbers crossing
(iv) % time traffic held up = $(6\times15\times100/300) = 30$ Average pedestrian wait = $(5+5+15+0+5+0+5+10+15+25)/10 = 8.5$ seconds	B1√ M1 A1

6.

(i) & (ii)

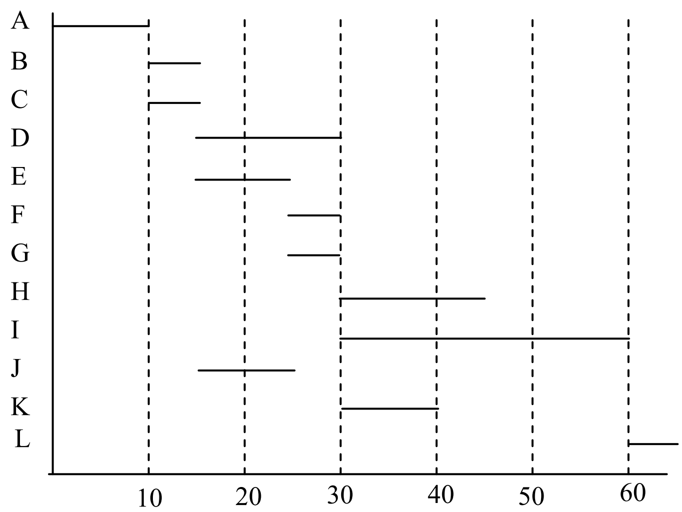


M1 activity-on-arc
A1 D or H
A1 J or K
A1 I
A1 rest

M1 forward pass
A1 (cao)
B1 backward pass

Critical: A, B, E, F, I, L — B1
Duration: 50 minutes — B1√

(iii)



M1
A1 E, F, G, H
A1 rest (cao)

Duration: 65 minutes, so ready by 10:05 — B1 10:05

(iv) Still 65 mins (plenty of time available for D, J, K and H), so 10:05 again. — B1 (65 OK)

Examiner’s Report

2620 Decision and Discrete Mathematics I

General Comments

The level of performance on this paper was again disappointing. There was clear evidence of candidates being entered who were not at all ready for it.

There was evidence that the paper was too long. This may have affected scores in the upper attainment range.

Comments on Individual Questions

Q.1 Parts (i) and (ii) were intended to be easy starters, and most candidates scored 3, sometimes 2, of the available 3 marks. Again, part (iii) should have been easy, with node F standing out in the complement graph like a sore thumb.

The final part was a little more difficult. Many candidates were unable to transfer into graph theory, often offering fanciful answers concerning the mechanics of ferry operations.

Note that the complement graph is directed, so that it is not strictly appropriate to refer to the order of nodes in mounting an argument in part (iv). However, the corresponding undirected graph is not Eulerian, so the directed graph certainly cannot be. Therefore arguments referring to node orders were accepted.

Q.2 This question revealed severe weaknesses in the preparedness of many candidates. There were many cases in which what was offered as a minimum connector was not a tree.

Very few candidates seemed to have heard of "complexity".

Q.3 A number of candidates were not able to follow the step-by-step algorithmic instructions. Not all candidates who ended with 1, 4 and 9 were able to say that the algorithm identifies square numbers.

Q.4 In contrast with some other parts of the paper, most candidates were able to demonstrate some idea of how to proceed in this question. The usual weaknesses were still to be seen, including a refusal to define variables and the consumption of much paper and effort in achieving plots of two straight lines. Weak candidates were often seen struggling to solve simultaneous equations, when reading off a decent graph would have been acceptable. Centres should note that graph paper will be required.

Q.5 Again, many candidates seemed to not have seen before what is required of them in achieving realisations of a discrete random variable. An even more substantial hurdle seemed to be using inter-arrival times to construct arrival times. The simulation itself was more difficult, but candidates who arrived at it without a decent set of arrival times did not give themselves the best of chances.

Q.6 This question required candidates to have sufficient clarity of vision to see the network in terms of the two rooms plus common activities. Some were better able to do this with the cascade chart than with the activity network. There were many cases of candidates constructing activity-on-node networks. These are not acceptable.

Coursework: Decision and Discrete Mathematics 1

Work was submitted from 3449 candidates at 204 centres, very similar to last summer. There were some large entries, almost all from experienced centres. The proportion of candidates who had their marks adjusted as the result of moderation was higher than last summer – about one fifth. Almost all of the adjustments were downward. The most frequent problem was over-marking in the first two domains.

The quality of the entry was lower than in January 2004 but similar to that in June 2003.

There were many aspects of this summer's coursework which were the same as last summer's. Several of the points in this report have been made before. They are repeated either to report continuing features, or as reminders, or for the benefit of new centres.

Marking

Most centres had taken the time to mark the work very carefully and accurately, annotating the students' work appropriately. However too many candidates were given marks of 3 or 4 in the first domain for a fairly basic model where there was no evidence that the moderator could see that the work met any of the criteria for higher marks. Regrettably there were still some cases where there was no marking evidence at all. In such cases the unfortunate moderator has to mark the work as well as moderate it. In some cases a mark of zero was incorrectly reported – absentees should be marked as absent.

Content

General
As usual, the majority of the projects were on the CPA or Simulation areas, with rather more on CPA. Unfortunately, we had yet again a handful of cases of off-syllabus work; in each case this time the problem identified was a TSP problem. Centres are reminded of this possibility particularly if candidates are attempting networks projects. The problem identified must not be a 2621 problem. There may be some cases where a candidate identifies a 2620 problem, applies 2620 techniques, and then suggests an extension which goes into the realms of 2621. That is acceptable. The difficulty occurs where the problem itself and the techniques used belong in 2621.

There were again some cases where all of the pieces from a centre were very similar. The candidates had clearly been given very prescriptive advice, or in some cases partly or even fully specified problems. Such candidates cannot score as highly for their problem identification and modelling. Similarly some candidates still do not explain how they obtained or justified their data (for example CPA activity durations and/or precedences). Simulation again was the best in this respect and LP the worst. In some cases source data had in fact been taken from web sites or textbooks. It needs to be repeated that all such sources should be acknowledged. The mark for Problem Identification and Modelling needs to reflect the fact that such candidates (and those who produce their data out of thin air) have generally done little of their own modelling. Candidates could improve their projects by consideration of the quality of their data and the implications of any inaccuracies.

There is clearly still some confusion between refinement and extension. Refinement changes the model. Extension changes the problem. Again this time some claimed extensions were in fact simplifications - for example a removal of an LP constraint. This really adds nothing of value to the work.

Many candidates do not examine the assumptions made when setting up their model (and sadly few seem to have the modelling cycle in mind when working on their problem; there are few references to it in scripts). For example the activity durations used in CPA are usually estimates. Investigation of maximum/minimum durations could lead to a changed critical path, and to some awareness of the sensitivity of the model.

Another issue was spelling and grammar. Much work is word-processed, but few candidates seem to use their spelling/grammar checkers properly – either not using them at all, or allowing them to make incorrect alterations. Some spelling was so poor that it obscured much of the meaning. The apostrophe was widely used to indicate plurals, not possessives, and omitted in possessives.

Pleasingly, the standard of presentation was often high.

Networks
There were fewer projects this time in this area, and most of them were rather straightforward. Again there were some pleasing exceptions including good extensions involving modifications of Dijkstra's algorithm, and thoughtful modelling of times. However, too often there is no explanation at all of the origin of the arc lengths.

In this area particularly, candidates often omit evidence of their use of algorithms. We do not want to see pages of textbook theory quoted. What the moderator is looking for is correct application of algorithms in the context of the chosen problem. The reader cannot see if Prim's algorithm has been correctly applied if all that is presented is the final MST - even if it is correct. Some claimed uses of Dijkstra looked like inspection to the moderator. The inclusion of all temporary and permanent labels clarifies this.

Several candidates stated that they were using Prim or Kruskal to find a route. Markers did not always comment on this.

It is worth reiterating that quite large networks are necessary to provide the complexity needed for earning high marks in this area.

Linear Programming
As previously there were some notable exceptions, but again many of the problems chosen were quite unconvincing. There were too many problems of the cake-and-buns type, and many of these were no more complex than the simplest textbook examples – with the products sometimes even just described as cake A and cake B. In some cases there were only two very straightforward constraints.

Data such as prices and quantities tended just to be quoted, and rarely was there any attempt to consider the figures used - for example for selling prices. Few went beyond a basic solution. Far too often the “refinement”, where it existed, was simply changing a constraint, sometimes for flimsy reasons such as finding another bag of flour or dropping some of the butter. This is not refinement; it is a change to the problem and is therefore extension; moreover it is often extension which does not add anything to the mathematics. Real extensions included making better use of slack resources.

Simulation
Much of the work in this area was very good indeed, with some impressive use of spreadsheets. The use of spreadsheets is to be encouraged, as it allows several reasonably lengthy repetitions to be made easily. Some candidates do not display the formulae that they have used. Those candidates who do not use spreadsheets often carry out quite short simulations, and often do not repeat them. The reliability of the solutions is often not considered.

Most candidates collected data to inform their simulation, grouped it sensibly, and allocated appropriate random numbers to reflect their distributions. Some candidates did not include their raw data, merely providing the claimed grouped distribution; we need to see the data.

Again some, regrettably, merely hypothesised their distributions. The results of the initial simulation should be compared with the data.

Few candidates gave compact, tabular summaries of their results. It is far easier to assimilate comparison information if all of the summary data are presented in a table than if they are all on separate pages and described in writing.

There was a wide variety of refinements, some original, for example suggesting different queuing disciplines.

CPA

Much of the work in this area was good and some of it was very good indeed. Too much, however, was based on artificial scenarios. Most networks contained between fifteen and thirty activities related in a complex way. In too many networks arrows were not included on all activities. Some of the problems identified are trivial; some of them are taken directly from textbooks (sometimes even the MEI textbook). It needs to be noted that precedence tables which lead to predominantly linear networks are not considered complex enough to warrant the highest marks.

Some candidates carried out careful modelling, with data sources and precedences justified. Many candidates, however, just produced a precedence table. Some of these were given full marks for problem identification and modelling, yet there was no evidence at all of modelling.

Most candidates carried out CPA right through to resource levelling. However, too often resource histograms appeared without explanation of how the activities had been moved.

Genuine extensions or refinements might involve, for example, an analysis of some of the activities on the critical path to see if there was a practical way of reducing the time taken.

Graphs

Again a small handful of projects were on graphs – again too few to make any observations or give any generalised advice.