OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2607**

Mechanics 1

Tuesday **3 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1

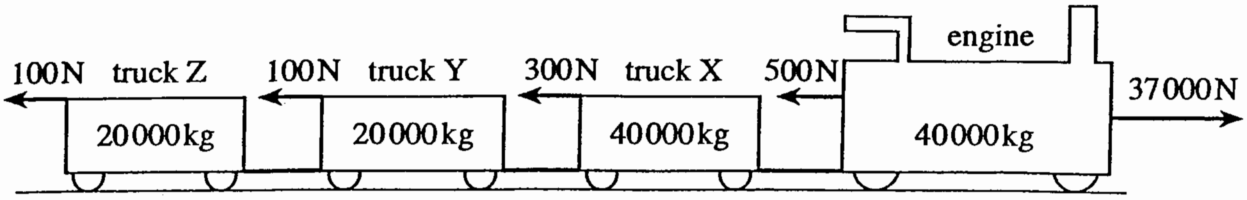


Fig. 1

A train consists of an engine and three trucks with masses and resistances to motion as shown in Fig. 1. There is also a driving force of 37 000 N. All the couplings are light, rigid and horizontal.

(i) Show that the acceleration of the train is $0.3\,\mathrm{m\,s^{-2}}$. [3]

(ii) Draw a diagram showing all the forces acting on truck Z in the line of its motion.

Calculate the force in the coupling between trucks Y and Z. [4]

With the driving force *removed*, brakes are applied, so adding a further resistance of 11 000 N to the total of the resistances shown in Fig. 1.

(iii) Calculate the new acceleration of the train. [2]

(iv) Calculate the new force in the coupling between trucks Y and Z if the brakes are applied

(*A*) to the engine,

(*B*) to truck Z.

In each case state whether the force is a tension or a thrust. [6]

2 **(a)** A particle accelerates uniformly from $7\,\mathrm{m\,s^{-1}}$ to $21\,\mathrm{m\,s^{-1}}$ in 8 s.

How far does it travel in this time? [2]

(b) Lewis is travelling in a car along a straight road. He wonders whether the car is accelerating uniformly.

Lewis estimates that the car takes

5 s to travel a distance of 75 m from A to B,

15 s to travel a distance of 315 m from A to C.

Lewis models the acceleration as a constant $a\,\mathrm{m\,s^{-2}}$. He also takes the speed of the car at A to be $u\,\mathrm{m\,s^{-1}}$, as shown in Fig. 2.

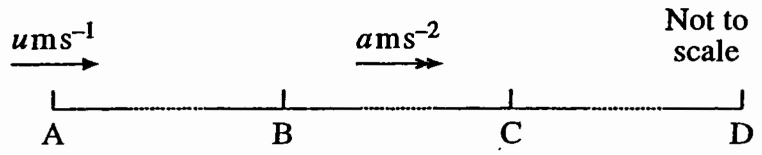


Fig. 2

(i) By considering the motion from A to B, show that $75 = 5u + 12.5a$. [3]

(ii) Find a second equation involving u and a. [4]

(iii) Hence find the value of u and show that $a = 1.2$. [3]

Lewis decides to check whether his assumption of constant acceleration is consistent with the motion of the car after reaching C. He notes that when the car reaches D, the distance CD is 200 m and the car's speed is $36.5\,\mathrm{m\,s^{-1}}$.

(iv) Does the extra information suggest that the constant acceleration model is reasonable? [3]

3 **(a)** A force of $(20\mathbf{i} - 15\mathbf{j})$ N acts on a particle at O. The vectors **i** and **j** are the standard unit vectors.

(i) Calculate the magnitude of this force and the angle it makes with the **i** direction. [3]

A second force $(5\mathbf{i} - 40\mathbf{j})$ N also acts on the particle at O.

(ii) What third force is required if the particle at O is in equilibrium with all three forces acting on it? [2]

(b) As shown in Fig. 3, a block of mass 12 kg is on a smooth plane inclined at 30° to the horizontal. The block is held in equilibrium by means of a light string at 20° to the plane.

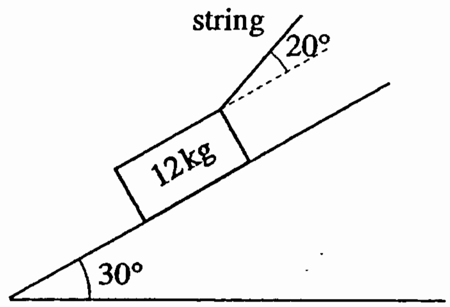


Fig. 3

(i) Draw a force diagram showing the tension in the string, the weight of the block and the normal reaction of the plane on the block. [2]

(ii) By considering the components of the forces parallel to the plane, show that the tension in the string is 62.6 N (correct to three significant figures).

Calculate also the normal reaction of the plane on the block. [5]

A further *horizontal* force of 40 N is applied to the block in an attempt to push it up the plane. The block remains in equilibrium with the angle of the string unchanged.

(iii) Calculate the new tension in the string. [3]

4 Air resistance should be neglected in this question.

A bottle of champagne is held with its cork 1.5 m above a level floor. The cork leaves the bottle at 60° to the horizontal. The cork has a *vertical* component of velocity of $9\,\text{m s}^{-1}$, as shown in Fig. 4.

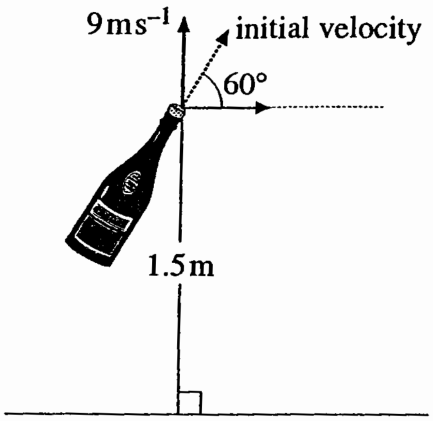


Fig. 4

(i) Show that the initial horizontal component of velocity is $5.20\,\text{m s}^{-1}$, correct to three significant figures. [2]

(ii) Find the maximum height above the floor reached by the cork. [3]

(iii) Write down an expression in terms of t for the height of the cork above the floor t seconds after projection. [2]

After projection, the cork is in the air for T seconds before it hits the floor.

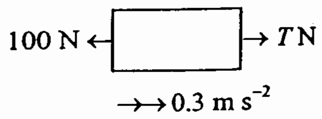
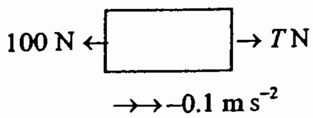
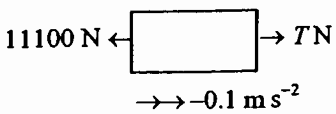
(iv) Show that T satisfies the equation $49T^2 - 90T - 15 = 0$.

Hence show that the cork is in the air for 1.99 s, correct to three significant figures.

Calculate the horizontal distance travelled by the cork before it hits the floor. [5]

(v) Calculate the speed with which the cork hits the floor. [3]

Mark Scheme

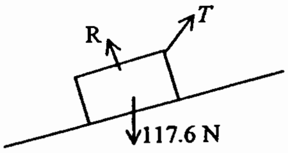
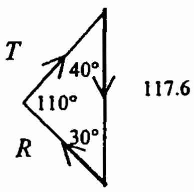
Q 1		Mark		
(i)	N2L → 36 000=120 000a $a = 0.3$ so 0.3 m s^{-2}	M1 B1 E1	Use of N2L with attempt at 120 000(g)a LHS (36 000 or equiv)	3
(ii)		B1	Accept thrust if used consistently below. Ignore forces given in other directions and on other trucks. Force In coupling must be labelled (accept **their** 6100 N).	
	$T - 100 = 20000 \times 0.3$ $T = 6100$ so 6100 N	M1 A1 A1	N2L with **their** $a = 0.3$ + attempted 20 000(g)a Condone missing 100 N. No extra forces. Mass correct, T and 100 N present. Accept wrong sign cao	4
(iii)	N2L → $-12000 = 120000a$ $a = -0.1$	M1 A1	Use of N2L with correct mass and ± 12000. No extra forces Deceleration explicit. Accept diagram or - 0.1.	2
(iv) (A)				
	N2L → $T - 100 = 20000 \times -0.1$ $T = -1900$ so 1900 N compression	M1 A1 B1	N2L with correct mass, ± 100 N and T; allow ± **their** a from (iii). No extra forces Accept only –1900 N **or** 1900 N with a clear sign convention **or** FT only error in magnitude of a from (iii) Thrust. FT. Reason must follow a calculated force.	
(B)				
	N2L → $T - 11100 = 20000 \times -0.1$ $T = 9100$ so 9100 N tension	M1 A1 B1	N2L with T, correct mass and ± 11100; allow ± **their** a from (iii). No extra forces. Accept only 9100 N **or** -9100 N with a clear sign convention **or** FT only error in magnitude of a from (iii) Tension. FT. Reason must follow a calculated force. [If 37 000 N included award max of (iii) M0A0 (iv) M1 A1 B1 M1 A1 B0]	
		Tot 15		6

Solutions and mark scheme

Q 2		Mark		
(a)	$s = \frac{(7+21)\times 8}{2} = 112$	M1	Use of appropriate '*uvast*'	
	so 112 m	A1		2
(b) (i)	$75 = 5u + 0.5\times a \times 25$	M1 A1	Use of $s = ut + 0.5at^2$ or equiv Correct subst. [Must see $0.5\times a\times 25$ or $s = 75$ and $t = 5$ stated]	
	$75 = 5u + 12.5a$	E1	Clearly shown	3
(ii)	Motion A to C	M1 M1	Consider motion A to C using $s = ut + 0.5at^2$ Use a and u. Dependent on 1[st] M1	
	$315 = 15u + 0.5\times a\times 225$	A1	Subst $s = 315$, $t = 15$	
	$(42 = 2u + 15a)$	A1	All correct. Any form [If B to C used M1 $s = ut + 0.5at^2$ M1 using $u' = u + 5a$. Dep on 1[st] M1 A1 subst u', $t = 10$, $s = 240$ A1 All correct. Any form]	4
(iii)	Solving simultaneously $a = 1.2$ $u = 12$	M1 E1 B1	Method leading to both values May be awarded without the M given FT **their** equation in (ii) [If given value subst in given equation, SC1]	3
(iv)		M1	Proper use of two of a, v, s to calculate the third in AD or CD or BD and compare. The comparison may be implied, for instance by a conclusion.	
	see table below	B1	Value of v, s or a. Or equivalent (e.g. u) cao	
			[NB. Using $v^2 = u^2 + 2as$ they may calculate and compare v^2 for full credit]	
	Quite close. Approx figures. Seems reasonable.	F1	Reasoned conclusion. Accept not close enough. Dep on M. Do not award for circular arguments. Expect to see a direct numerical comparison made or equivalent.	3
		Tot 15		

	a	s	v
For AD	1.15 (1.2)	495 (515)	37.1 (36.5)
For CD	1.08 (1.2)	180 (200)	37.1 (36.5)
For BD	1.15 (1.2)	420 (440)	37.1 (36.5)

Solutions and mark scheme

Q 3		Mark		
(a) (i)	$\sqrt{20^2+15^2}=25\text{ N}$	B1		
	$\arctan\left(\frac{-15}{20}\right)$ so $-36.9°$ or $323°$ (3 s. f.)	M1	Use of arctan or equiv. Accept $\pm\frac{15}{20}$ or $\pm\frac{20}{15}$ or equivalent	
		A1	Must indicate 'below' **i** direction e.g. on diagram. Accept -36.9° or 323° as indication.	3
(ii)	$\begin{pmatrix}20\\-15\end{pmatrix}+\begin{pmatrix}5\\-40\end{pmatrix}+\mathbf{F}=\mathbf{0}$ so $\mathbf{F}=\begin{pmatrix}-25\\55\end{pmatrix}$	B1 B1	Each compt. SC1 for $\mathbf{F}=\begin{pmatrix}25\\-55\end{pmatrix}$	2
(b) (i)		B1 B1	Correct forces with arrows. No extra forces. Correctly labelled. No extra forces. Accept no angles. Accept W, mg etc	2
(ii)	$117.6\sin 30=T\cos 20$	M1	Attempt at resolution of both forces and equating. No extra forces. Accept $\sin\leftrightarrow\cos$ and 12 for 117.6	
	$T=62.5736\ldots$ so 62.6 N (3 s. f.)	E1	Clearly shown	
	$R+T\sin 20=117.6\cos 30$	M1	Resolving T and W perp to plane. All terms present. No extras. Accept $\sin\leftrightarrow\cos$. Must be 117.6.	
		A1	Both resolutions correct. Allow wrong signs.	
	$R=80.443\ldots$ so 80.4 N (3 s. f.) **or**	A1	cao	5
	Triangle of forces or Lami's Theorem 	M1 A1 E1 M1 A1	Use of sine rule or Lami's Theorem Angles correct; may be awarded with 2nd M $T=62.6$ cao Use of sine rule or Lami's Theorem $R=80.4$ cao	
(iii)	$T\cos 20+40\cos 30=117.6\sin 30$	M1	All terms present; no extras; resolution of all terms attempted. Accept wrong signs. Accept $\sin\leftrightarrow\cos$.	
		A1	All correct	
	$T=25.709\ldots$ so 25.7 N (3 s.f.)	A1	cao [If 40 N taken parallel to plane SC M1A1A0 max]	3
		tot 15		

Solutions and mark scheme

Q 4		Mark		
(i)	$v = \dfrac{9}{\tan 60}$	M1	Or equivalent method finding projection speed first. Allow $\sin \leftrightarrow \cos$. (Speed of projection is 10.392...)	
	$= 5.196\ldots$ so 5.20 m s^{-1} (3 s. f.)	E1		2
(ii)	$0^2 = 81 - 2 \times 9.8 \times s$	M1	Appropriate '*uvast*' with $u = 0$ or $v = 0$. Allow use of $u = 10.4$ but not 5.2	
		A1	Correct substitution. Allow sign errors.	
	$s = 4.1326\ldots$ so height above the floor is 5.63 m (3 s. f.)	A1	Clearly established. cao	3
(iii)	$y = 9t - 4.9t^2 + 1.5$	M1	Use of vertical component with ± 9.8, ± 10 The 1.5 may be omitted	
		A1	All terms present and correct	2
(iv)	$9T - 4.9T^2 + 1.5 = 0$	M1	Equating **their** y to zero or equivalent	
	so $49T^2 - 90T - 15 = 0$	E1	Shown clearly.	
	Solving for the + ve root $t = 1.9905\ldots$ so 1.99 s (3 s. f.)	B1	Evidence of solving. If substitution used comparison required. Accept no reference to – ve root. [WW accept 1.990... or 1.991 as evidence]	
	$x = 5.196\ldots \times 1.9905\ldots$	M1	Use of horizontal component	
	$= 10.343\ldots$ (using 5.20 and 1.99 = 10.348...)			
	so 10.3 m (3 s. f.)	A1	FT **their** (i) only	5
(v)	$\dot{y}(1.99) = 9 - 9.8 \times 1.99 = -10.50\ldots$	B1	Speed in y direction. Condone +10.5.	
	we require $\sqrt{5.196\ldots^2 + 10.50\ldots^2}$	M1	Use of Pythagoras with **their** $\dot{x}$ and $\dot{y}$.	
	$= 11.717\ldots$ so 11.7 m s^{-1} (3 s. f.)	A1	cao	3
		Tot 15		

Examiner’s Report

2607 Mechanics 1

General Comments

Many candidates scored well and clearly found every question approachable. However, there was a minority who had little idea of the principles involved in any question and who scored very few marks.

The errors of the candidates who did not score well were often clearly due to poor basic technique. There was a lack of clear diagrams and when a situation changed a new diagram was not always attempted. For instance, in Q3 many candidates did not attempt a diagram for part (iii) and took the new force to be parallel to the plane in their calculations; this mistake was less commonly seen when there was an accompanying diagram. When starting a solution, it was clear that many candidates were not asking themselves what the question was about and what principles were involved but were instead trying the given information into their calculators in different combinations until something like the given answer came out.

Comments on Individual Questions

Q.1 Newton's second law applied to a train

Although there were many nice answers to this question, the idea of applying Newton's second law to parts of a system was not well understood by many of the candidates.

Most got part (i) right but a number omitted the resistance, used 37 000 / 120 000 and said the answer was about 0.3.

In part (ii) the diagram often included the driving force. Although there were many correct solutions, some candidates did not know that they had to apply Newton's second law and used spurious methods based on false calculations of notional forces acting on the trucks. Others clearly did not know what to do and so did nothing. In this and later parts, some candidates wrote down the equations of motion for *all* of the trucks and some of these then solved them from the engine back to the penultimate truck.

In part (iii) most got the new acceleration in modulus but many gave no regard to the sign. Those who gave the answer as positive usually showed in the next part that they did not realize that they had found a deceleration.

In part (iv), few drew clear diagrams to help them and there was much muddling of signs. It was clear that many candidates selected tension or thrust according to what they felt must be happening and not according to the sign of the force they had calculated as acting on truck Z in the coupling.

(ii) 6100 N; (iii) 0.1 m s^{-2} in opposite direction to motion;
(iv)(A) 1900 N thrust (B) 9100 N tension.

Q.2 Constant acceleration problem

(a) Many of the candidates obtained the correct answer but quite common was the use of $0.5 \times (v-u)t$ and many found the acceleration first.

(b) Most candidates successfully obtained the first equation in part (i) and many the second in part (ii) but others used the same information again and a chain of *uvast* formulae, only getting a different result because of their errors.

In part (iii) those candidates with two valid equations could usually solve them. Those without usually obtained one mark by substituting the given result into the given equation.

Many candidates knew exactly what they should do but failed to make a clear comparison and statement at the end. A common error was to consider the motion from C to D but use the value of u at A. It was acceptable to say that the acceleration was constant or not.

(a) 112 m; (b)(iii) $u = 12$.

Q.3 (a) vector problem (b) static equilibrium

(a)(i) was often done quite well. The most common mistake was not to give the direction fully but some candidates were clearly unfamiliar with the term magnitude.

Part (ii) was done correctly for full marks by many. 1 mark was allowed for the negative of the answer.

(b) Although there were many good answers, a lot of candidates showed little understanding of statics and, in particular, could not equate resolved parts in a particular direction.

A lot of the diagrams were wrong, lacking arrows or labels or with weight perpendicular to the plane or with the normal reaction vertical. Surprisingly many candidates failed to resolve correctly in the *direction given* in part (i) and a lot had no idea how to find the normal reaction; even otherwise strong candidates who knew that they should resolve perpendicular to the plane often neglected the component of the tension in the string. There were some nice solutions using Lami's theorem but others who used this technique could not complete the question.

In part (iv), some candidates made no attempt at all and few drew a diagram. Many candidates took the 40 N to be parallel with the plane and, as stated above, this was less common when a clear diagram was given.

(a)(i) 25 N, -36.9°; (b)(ii) 80.4 N (3 s.f.) (iii) 25.7 N (3 s. f.).

Q.4 Projectile problem

Despite giving both the initial horizontal and vertical components of velocity, many candidates took the initial speed to be 9 in part (i) and the components to be 9sin60 and 9cos60.

In part (ii) quite a few candidates forgot the height above the floor but many recovered in part (iii) where the answer was accepted without working. A common error was to give an equation for a *specific* rather than a *general* height; e.g. $5.63 = 9t - 4.9t^2 + 1.5$.

In part (iv) candidates were expected to equate their expression in (iii) to zero and show some manipulation to obtain the stated result. Some started again and obtained the correct answer here when they had gone wrong in part (iii), presumably because the answer was given. Many solved the quadratic equation correctly and many others verified the given result by substitution. A common error was to show no evidence of a calculation but just make the statement that the expression was approximately zero. Most candidates who remembered to answer the question got the horizontal distance but some used 5.2cos60.

There were a lot of good answers to part (v) from candidates who knew just what to do but there were many who omitted the part or became confused and calculated some vertical component of speed but did not combine it with a horizontal component.

(ii) 5.63 m (3 s. f.) (iv) 10.3 m (3 s. f.) (v) 11.7 m s^{-1}.