RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2621/1

Decision and Discrete Mathematics 2

Thursday **22 MAY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

1 A fairground game offers 3 outcomes, £10, £1 or £0. A player has just been offered £1. She can accept or "go again", in which case the offer of £1 will be withdrawn and she will be offered £10, £1 or £0 with probabilities p, q and $1 - p - q$ respectively.

(i) Draw a decision tree for this game. [2]

(ii) If q is 0.5 what value must p exceed to tempt her to go again? [1]

In a similar game the player has again been offered £1, but gets two chances to go again. If she elects not to take the £1 and is subsequently offered £10, she will accept it. However, if the subsequent offer is £0 she will go again for the second and last time. If the offer is £1 then she must decide whether to accept it this time or to go again for the last time.

(iii) Draw a decision tree for this game. [4]

(iv) If $p = 0.02$ and $q = 0.5$, compute the emv at each node and give the player's best strategy. [3]

[Total 10]

2 (a) (i) The circuit shown in Fig. 2.1 is called a *half adder*.

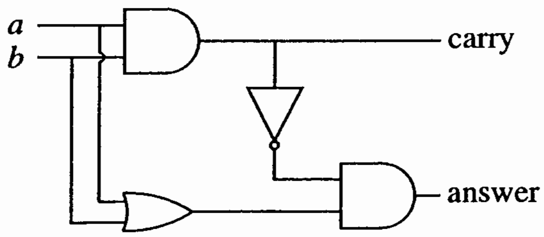


Fig. 2.1

Produce a table showing the outputs, carry and answer, in terms of the inputs, *a* and *b*. [3]

(ii) The circuit shown in Fig. 2.2 is called a *full adder*.

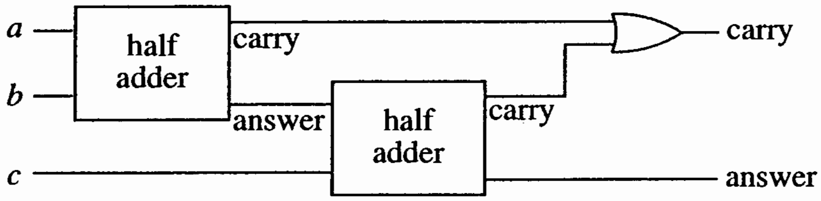


Fig. 2.2

Produce a table showing the outputs in terms of the inputs, *a*, *b* and *c*. [5]

(b) The cards shown in Fig. 2.3 are from a pack in which every card has a letter on one side and a number on the other.

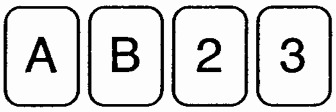


Fig. 2.3

It is claimed that behind a vowel there is always an odd number. Which of the four cards needs to be turned over to check this claim? [2]

[Total 10]

3 Direct transport links between five cities are shown in Fig. 3.1. The weights on the arcs are the times in hours for moving along those links.

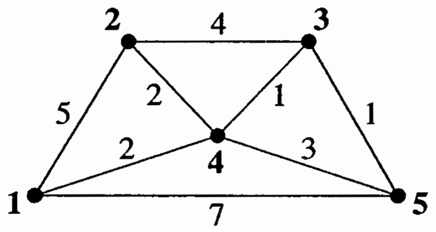


Fig. 3.1

(i) Set up initial time and route matrices for an application of Floyd's algorithm to find the complete network of shortest times between cities. [3]

Fig. 3.2 shows completed time and route matrices at the end of the second iteration of Floyd's algorithm, and incomplete third iteration matrices.

	1	2	3	4	5
1	10	5	9	2	7
2	5	10	4	2	12
3	9	4	8	1	1
4	2	2	1	4	3
5	7	12	1	3	14

	1	2	3	4	5
1	2	2	2	4	5
2	1	1	3	4	1
3	2	2	2	4	5
4	1	2	3	1	5
5	1	1	3	4	1

	1	2	3	4	5
1			9		
2			4		
3	9	4	8	1	1
4			1		
5			1		

	1	2	3	4	5
1			2		
2			3		
3	2	2	2	4	5
4			3		
5			3		

Fig. 3.2

(ii) Copy and complete the matrices for the third iteration. [6]

The final time and route matrices after completion of the fifth iteration of Floyd's algorithm are shown in Fig. 3.3.

	1	2	3	4	5
1	4	4	3	2	4
2	4	4	3	2	4
3	3	3	2	1	1
4	2	2	1	2	2
5	4	4	1	2	2

	1	2	3	4	5
1	4	4	4	4	4
2	4	4	4	4	4
3	4	4	4	4	5
4	1	2	3	3	3
5	3	3	3	3	3

Fig. 3.3

(iii) Use the final matrices to find the fastest route and shortest time from city 5 to city 2. [2]

(iv) By temporarily deleting city 4 and its arcs, find a lower bound for the minimum duration Hamilton cycle in the complete network of shortest times, as given in Fig. 3.3. (You do not need to show the use of a minimum connector algorithm.) [2]

(v) Use the nearest neighbour algorithm, starting at city 1, to find a Hamilton cycle in the complete network of shortest times, as given in Fig. 3.3.

Give the time for your cycle. [2]

(vi) Use your answer to part **(v)** to construct a fast route for an individual wishing to leave city 4, visit each of the other cities, and return to city 4. [2]

(vii) Find the shortest time needed to traverse every link in the network in Fig. 3.1, starting and finishing at the same vertex. [3]

[Total 20]

4 Three chemical products, X, Y and Z are to be made. Product X will sell at 40 p per litre and costs 30 p per litre to produce. Product Y will sell at 40 p per litre and costs 30 p per litre to produce. Product Z will sell at 40 p per litre and costs 20 p per litre to produce.

Three additives are used in each product. Product X uses 5 g per litre of additive A, 2 kg per litre of additive B and 8 g per litre of additive C.

Product Y uses 2 g per litre of additive A, 4 g per litre of additive B and 3 g per litre of additive C.

Product Z uses 10 g per litre of additive A, 5 g per litre of additive B and 5 g per litre of additive C.

There are 10 kg of additive A available, 12 kg of additive B, and 8 kg of additive C.

(i) Explain how the initial feasible tableau shown in Fig. 4 models this problem. [6]

P	x	y	z	s_1	s_2	s_3	RHS
1	–10	–10	–20	0	0	0	0
0	5	2	10	1	0	0	10 000
0	2	4	5	0	1	0	12 000
0	8	3	5	0	0	1	8000

Fig. 4

(ii) Use the simplex algorithm to solve your LP, and interpret your solution. [8]

(iii) The optimal solution involves making two of the three products. By how much would the cost of making the third product have to fall to make it worth producing, assuming that the selling price is not changed? [1]

There is a contractual requirement to provide at least 500 litres of product X.

(iv) Show how to incorporate this constraint into an initial tableau ready for an application of the two-stage simplex method. [3]

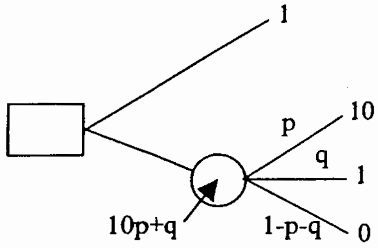
Briefly describe how the method works. You are NOT required to perform the iterations. [2]

[Total 20]

Mark Scheme

1.

(i)



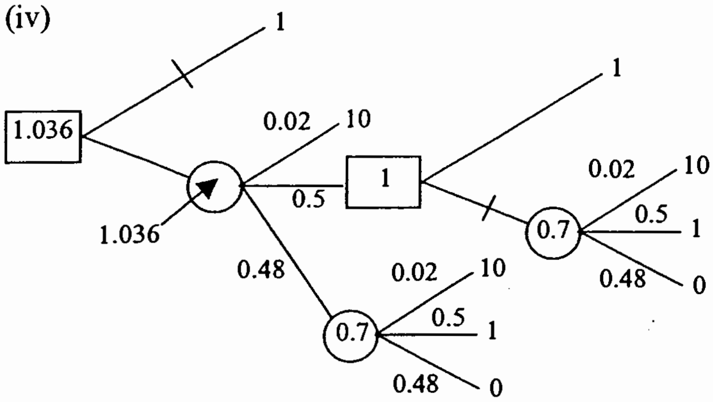
M1 probs and values not needed

A1

(ii) 0.05

B1

(iii) & (iv)



M1 A1 for extra □ bit

M1 A1 for extra ○ bit

M1 A1 emv's

Go again then accept the £1 if it is offered again.

B1

2.

(a)(i)

a	b	carry	answer
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

M1 4 rows
A1 carry
A1 answer

(ii)

a	b	c	carry	answer
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

M1 8 rows
A2 carry
A2 answer

(b) The “A” and the “2”

B1 “2”
B1 “A”
(but 0 if >2 cards turned)

3.

(i)

	1	2	3	4	5
1	∞	5	∞	2	7
2	5	∞	4	2	∞
3	∞	4	∞	1	1
4	2	2	1	∞	3
5	7	∞	1	3	∞

M1
A1

	1	2	3	4	5
1	1	2	3	4	5
2	1	2	3	4	5
3	1	2	3	4	5
4	1	2	3	4	5
5	1	2	3	4	5

B1

(ii)

	1	2	3	4	5
1	10	5	9	2	7
2	5	8	4	2	5
3	9	4	8	1	1
4	2	2	1	2	2
5	7	5	1	2	2

M1 A2

	1	2	3	4	5
1	2	2	2	4	5
2	1	3	3	4	3
3	2	2	2	4	5
4	1	2	3	3	3
5	1	3	3	3	3

M1 A2

(iii) **5 3 4 2** 4 hours — B1 B1

(iv) 1 + 2 + 7 (min conn) = 10 — B1 B1

(v) **1 4 3 5 2 1** 12 hours — B1 B1

(vi) **4 3 5 3 4 2 4 1 4** — M1 A1

(vii) 25 + 4 + 1 (best pairing) — B1 (25)
M1 A1

4.

(i)	Let x be the number of litres of X … Line 1 ⇔ max 10x+10y+20z (40−30=10 etc.) Line 2 ⇔ 5x+2y+10z ≤ 10000 (A's availability) Line 3 ⇔ 2x+4y+5z ≤ 12000 (B's availability) Line 4 ⇔ 8x+3y+5z ≤ 8000 (C's availability)	B1 variable defs. M1 A1 objective M1 constraints A1 LHS = used A1 RHS = available

(ii)

P	x	y	z	s1	s2	s3	RHS
1	-10	-10	-20	0	0	0	0
0	5	2	10	1	0	0	10000
0	2	4	5	0	1	0	12000
0	8	3	5	0	0	1	8000
1	0	-6	0	2	0	0	20000
0	0.5	0.2	1	0.1	0	0	1000
0	-0.5	3	0	-0.5	1	0	7000
0	5.5	2	0	-0.5	0	1	3000
1	16.5	0	0	0.5	0	3	29000
0	-0.05	0	1	0.15	0	-0.1	700
0	-8.75	0	0	0.25	1	-1.5	2500
0	2.75	1	0	-0.25	0	0.5	1500

B1 pivot
M1 A1

B1 pivot
M1 A1

Make 1500 litres of Y and 700 litres of Z, — B1
at a profit of £290. — B1

(iii) 16.5 p per litre — B1

(iv)

Q	P	x	y	z	s1	s2	s3	s4	a	RHS
1	0	1	0	0	0	0	0	-1	0	500
0	1	-10	-10	-20	0	0	0	0	0	0
0	0	5	2	10	1	0	0	0	0	10000
0	0	2	4	5	0	1	0	0	0	12000
0	0	8	3	5	0	0	1	0	0	8000
0	0	1	0	0	0	0	0	-1	1	500

B1 new objective
B1 surplus+artificial
B1 new constraint

Minimise Q until 0 (if feasible). — B1
Then drop Q and a and proceed to optimum. — B1

Examiner’s Report

2621 Decision and Discrete Mathematics 2

General Comments

Candidates were generally well prepared for this paper and performances were good. In particular, answers to question 3 (Floyd's algorithm and networks) and to question 4 (the simplex algorithm and two-stage simplex) were particularly well grounded. A printing error on the latter evaded all checks, but fortunately had no effect on the mathematics, and no discernible effect on any candidate.

Comments on Individual Questions

Q.1 Decision Analysis

This question was well done by a majority of candidates. Candidates who did not have clear the distinction between a decision node and a chance node were not able to make much progress, but then that will always be the case with the decision analysis question.

(ii) 0.05
(iv) emv's of £0.70 (twice), £1 and £1.036. Go again and accept the £1 if offered again.

Q.2 Logic

It was anticipated that part (a) would be difficult. It certainly proved to be discriminating, with good candidates answering it accurately and efficiently, but with some not so good candidates making little progress. One or two candidates failed to produce a table of outputs, instead giving Boolean expressions.

Part (b) is an old chestnut, and it was very surprising just how completely it floored the candidature. Only a very small number of candidates produced the completely correct answer, which is rather surprising for a module with logic on its syllabus

(b) A and 2 need to be turned over.

Q.3 Networks

Candidates did very well on parts (i), (ii) and (iii), which covered Floyd's algorithm. In the subsequent miscellany, surprisingly few managed to produce a correct minimum connector for the reduced network in part (iv), part (v) was tackled well, but hardly anyone realised that in part (vi) the answer to part (v) had both to be reorganised and expanded. Most candidates knew what to do with part (vii), but not many were able to do it accurately.

(iii) 5-3-4-2 4 hours
(iv) 1 + 2 + 7 (minimum connector) = 10
(v) 1-4-3-5-2-1 12 hours
(vi) 4-3-5-3-4-2-4-1-4
(vii) 25 + 4 + 1

Q.4 LP

Candidates had clearly been very well prepared for this question. Some were able to romp through it almost faultlessly, and in general the standard was pleasingly good.

The worst aspect was the first mark, which hardly anyone registered. The very first thing to do in any modelling, and particularly in LP modelling, is to establish the precise meanings of the variables. In this case x is the number of litres produced of product X, etc.

The single mark in part (iii) for identifying the reduced cost of product X, was of similar apparent difficulty, with few candidates scoring it.

Apart from those two marks almost all candidates scored well, with just slips discriminating between them. In particular, knowledge of two-stage simplex was good.

(ii) Make 1500 litres of Y and 700 litres of Z at a profit of £290,
(iii) 16.5 p per litre.