OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Monday **13 JANUARY 2003** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

Section A (30 marks)

1 Solve the equation $\sin x = -0.9$ for values of x between 0° and 360°. [2]

2 Find the coordinates of the point of intersection of the lines with equations

$$3x + y = 1 \text{ and } y = 7x - 2.$$ [3]

3 Find the equation of the line joining the points (3, 2) and (5, 12). Give your answer in the form $y = ax + b$. [3]

4

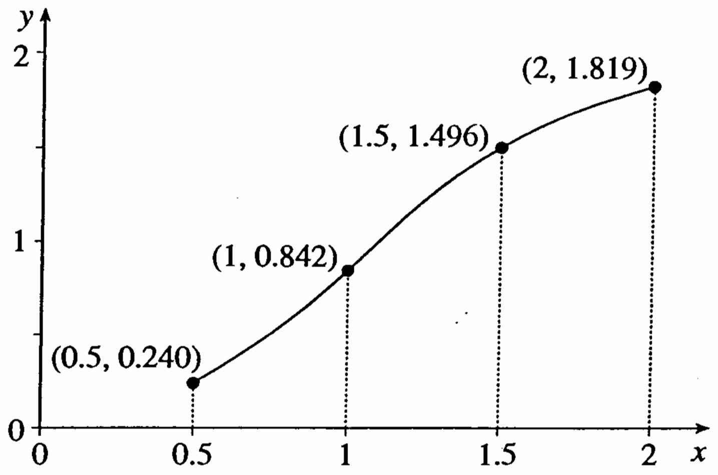

Fig. 4

Using the trapezium rule with three strips, calculate an estimate of the area between the curve and the x-axis from $x = 0.5$ to $x = 2$, as shown in Fig. 4. [3]

5 Express $3x^2 + 24x + 1$ in the form $a(x + b)^2 + c$. [3]

6 Sketch the graph of the function $y = |2x + 5|$.

State the coordinates of the points where the graph meets the axes. [4]

7 Starting from $\sin^2\theta + \cos^2\theta = 1$, show how to obtain the result $\tan^2\theta + 1 = \sec^2\theta$.

Find the possible values of $\tan\theta$ when $\sec\theta = 5$. Give your answers in an exact form. [4]

8

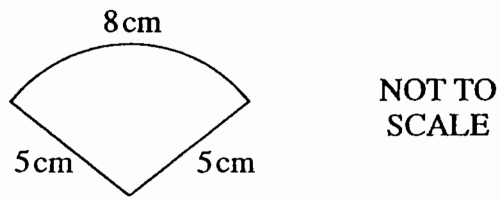

Fig. 8

Fig. 8 shows a sector of a circle. Find, in radians, the angle of the sector.

Find also the area of the sector. [4]

9

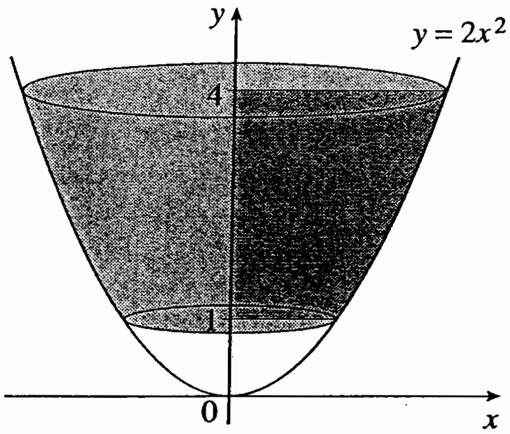

Fig. 9

A region in the first quadrant is bounded by the curve $y = 2x^2$, the lines $y = 1$ and $y = 4$, and the y-axis.

This region is rotated through 360° about the y-axis, as shown in Fig. 9.

Calculate the volume of revolution which is generated. [4]

Section B (30 marks)

10

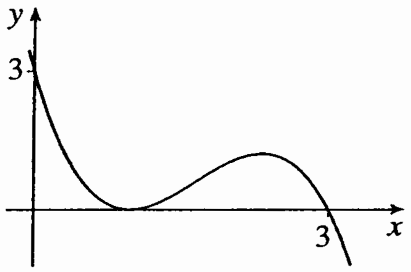

Fig. 10

Fig. 10 shows the graph of the curve with equation $y = (x-1)^2(3-x)$.

(i) Show that the equation may be written as $y = 3 - 7x + 5x^2 - x^3$. [2]

(ii) Find $\frac{dy}{dx}$ and hence find the gradient of the curve at the point (0, 3).

Find also the equation of the normal to the curve at this point. [4]

(iii) Find the coordinates of the turning points on the curve. [5]

(iv) Calculate the area between the curve and the x-axis for values of x from 1 to 3. [4]

11

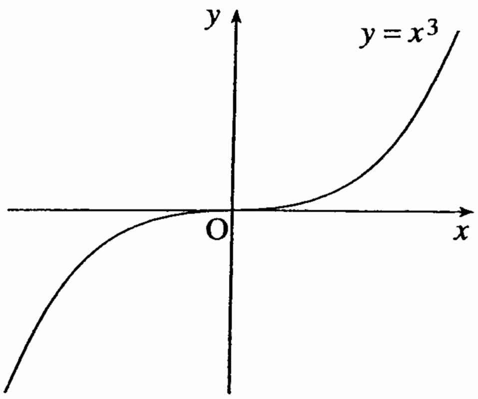

Fig. 11

Fig. 11 shows a sketch of the curve with equation $y = x^3$.

(i) Find the equation of the tangent to the curve at the point where $x = 2$. Give your answer in the form $y = ax + b$. [4]

(ii) Calculate the gradients of the chords joining the points on the curve $y = x^3$ for which

(A) $x = 2$ and $x = 2.1$,

(B) $x = 2$ and $x = 2.01$. [4]

(iii) (A) Expand $(2 + h)^3$.

(B) Simplify $\dfrac{(2+h)^3 - 2^3}{h}$.

(C) Show how your result in part **(iii)**(B) can be used to find the gradient of $y = x^3$ at the point where $x = 2$. [7]

Mark Scheme

	Section A			
1.	244(.1...) and 295(.8...)	B2	1 each; no extras in range for B2 but ignore if just B1 gained B1 for grads 251(.2..) and 288(.7...) or rads 181(.1..) and 358(.8..)	2
2	$3x+7x-2=1$ $x=0.3$ and $y=0.1$	M1 A1+ A1 or B3	or alt method eg graphs drawn; for elimination method must attempt appropriate subtn or addn. [2 correct terms out of 3]	3
3.	$\frac{y-2}{12-2}=\frac{x-3}{5-3}$ any correct eqn $y=5x-13$ c.a.o.	M2 A1 or B3	or M1 $m=5$ or $(12-2)\div(5-3)$ M1 subst of 1 of given pts in $y=$ their $mx+c$ or in $y-y_1=$ their $m(x-x_1)$ etc	3
4.	$0.5/2\times[0.240+1.819+2(0.842+1.496)]$ [o.e. for separate traps] 1.68...	M2 A1 or B3	M1 if error in e.g. brackets or height or for 2 correct traps out of 3 (0.2705, 0.5845, 0.82875)	3
5.	$a=3$ $b=4$ $c=-47$	B1 B1 B1	implied by $3(x^2+8x+...)$ etc	3
6.	line seg for $y=2x+5$ line seg for $y=-2x-5$ fully correct and V shape identified 5 and -2.5 indicated on axes or (0,5) and (-2.5, 0) seen	G1 G1 G1 G1	bod if no labels SC2 for fully correct labelled graph of $y=\|2x-5\|$, SC1 if unlabelled	4
7.	division throughout by $\cos^2\theta$ $\sin\theta/\cos\theta=\tan\theta$ and $1/\cos\theta=\sec$ $\pm\sqrt{24}$ or exact equiv. as final answers [no extras]	M1 M1 B2	Ms may be earned in either order may be implied by alignment of terms in solution B1 for one answer or for $\tan^2\theta=24$ or $\tan\theta=\sqrt{24}$ seen	4
8.	$\theta=1.6$ or 8/5 as final answer Area = 20 [cm^2] (allow $\pm$ 0.05)	B2 B2	or 0.51π or better; M1 for $5\theta=8$ oe M1 for $\frac{1}{2}\times5^2\times$ [their] θ in rads or for $\frac{A}{\pi\times5^2}=\frac{8}{2\pi\times5}$ o.e.	4

9.	$\int \frac{\pi y}{2}[dy]$	M2	condone without limits M1 for integral of πx^2 or πy^2	
	$\frac{[\pi]y^2}{4}$	A1	A1 for $\frac{4}{5}[\pi]x^5$ SC1 for $y^2/4$ or SC2 for 15/4	
	$\frac{15\pi}{4}$ o.e. or 11.78.. to 3 sf or more	A1	following M0 for omission of π isw for 'doubling'	4
			Total Section A	30

Section B					
10.	(i)	$(x^2-2x+1)(3-x)$ $3x^2-6x+3-x^3+2x^2-x$ (answer given)	B1 B1	1 each for 2 correct constructive steps or B2 for $x^3(-1)+x^2(3+1+1)$ $+x(-3-3-1)+3$	2
	(ii)	$-7+10x-3x^2$ At D, grad.of tgt = −7 grad of normal = 1/7 $y=\frac{1}{7}x+3$ o.e.	M1 A1 B1 B1	condone one error ft their gradient; may be implicit ft their gradient of tgt or normal	4
	(iii)	$0=-7+10x-3x^2$ $0=(x-1)(3x-7)$ $x=1$ or $\frac{7}{3}$ $(1, 0)$ or $(\frac{7}{3}, \frac{32}{27})$	M1 M1 A1 A1+ A1	ft their gradient; condone implied by attempt at solving quadratic. attempt at factorisation or subst in formula, ft their gradient <u>alternative scheme:</u> B2 for (1, 0), B3 for $(\frac{7}{3}, \frac{32}{27})$ or (2.33.., 1.18-1.19); condone *y*s found but coords not stated	5
	(iv)	integral of $3-7x+5x^2-x^3$ attempted $3x-\frac{7}{2}x^2+\frac{5}{3}x^3-\frac{1}{4}x^4$ $\frac{4}{3}$ or 1.3^r; allow 1.33(3...)	M1 A1 A2	condone wrong or no limits condone one error; ignore + *c* M1 for attempt at difference of [value of integral at 3 and value at 1] eg $2.25-0.916^r$	4
11.	(i)	$dy/dx=3x^2$ $=12$ at $x=2$ $(y-8)=12(x-2)$ $y=12x-16$ cao	M1 A1 M1 A1	 or subst of (2,8) in $y=$ their $12x+c$, ft their evaluated dy/dx.	4
	(ii)	*A* 12.6(1) cao, isw *B* 12.06(01) cao, isw	M1A1 M1A1	M1 for (9.26(1) − 8)/0.1 M1 for (8.120601 − 8)/0.01 etc [implied by 12.06(0) or 12.1 but not by 12 or 8.12 without better evidence] 12.615 and 12.06015 imply M0	4
	(iii)*A* *B* *C*	$8+12h+6h^2+h^3$ (may be seen simplified in *A* or *B*) [their (iii)*A* − 8] / *h* $12+6h+h^2$ This is gradient of chord As *h* tends to 0, gradient of tangent = 12	B3 M1 A1 E1 E1	B2 if unsimplified or for 3 terms correct or B1 for coeffts 1, 3, 3, 1 used ft from *A* only if B2 earned may be shown on a diagram condone 'at $x=2$, $h=0$'	7
				Total Section B	30
				Total for paper	60

Examiner’s Report

2601 Pure Mathematics 1

General Comments

This was a largely straightforward paper which caused good candidates no major problems. There were many very good scripts; few candidates appeared to have been entered prematurely for the examination and there were comparatively few very weak candidates.

A considerable number of candidates made little or no attempt at 11(iii)(*C*). The examiners felt that this was mostly due to the difficulty of the question rather than pressure of time at the end of the examination.

Comments on Individual Questions

Question 1

This was often completely correct, but weaker candidates sometimes produced only one of the solutions and some extra solutions within the range.

244, 296

Question 2

This was mostly well-done, but errors such as $x = 10/3$ from $10x = 3$ were not uncommon. Those who used elimination rather than substitution made errors in manipulation more frequently.

(0.3, 0.1)

Question 3

This was usually correct, although some inverted the gradient..

$y = 5x - 13$

Question 4

There were few errors in calculation. Most errors were from producing garbled versions of the formula. Some candidates still do not realise that it appears in the formula book.

1.68375, given to 3 s.f. or more

Question 5

This question on completing the square was not often completely correct. Weaker candidates often only managed a. Even the better candidates had trouble with c.

$a = 3$, $b = 4$, $c = -47$

Question 6

Sketching a modulus graph produced the usual variety of responses, including curves. Not many candidates scored full marks.

sketch graph of $y = |2x + 5|$

Question 7

The first part of question 7 was completed successfully by an encouraging number of candidates but strong candidates especially tended to think that the relationships $\tan\theta = \sin\theta / \cos\theta$ and $\sec\theta = 1/\cos\theta$ too trivial to write down – they obtained both marks only if their work as set out clearly implied their awareness and use of these relationships. Many candidates did not attempt the second part of the question, suggesting little experience of using trigonometric identities. Of those that did, most gave only one exact answer in spite of the plural invited in the question. A worrying number of candidates went from $\tan^2\theta + 1 = 25$ to $\tan\theta + 1 = 5$.

$\tan\theta = \pm\sqrt{24}$

Question 8

Many candidates knew the radian formulae required and used them appropriately. A common mistake was to proceed from $5\theta = 8$ to $\theta = 1.6\pi$ radians.

1.6; 20

Question 9

Relatively few candidates were familiar with the method for finding the volume of revolution about the y-axis. Some rotated about the wrong axis. Some of those who knew what was required made mistakes in substituting for x. Fully correct answers were seen fairly rarely.

Question 10

This question was a good source of marks for most candidates.

(i) Most gained both marks, but a few were not able to expand the brackets correctly, usually because they attempted all 3 at once.

(ii) Differentiation was usually correct and followed by the correct gradient, although a few used $x = 3$ instead of $x = 0$. Some went on to find the equation of the tangent rather than the normal. Some found the gradient of the normal and stopped there.

(iii) The majority knew what they had to do to find the turning points, but the negative coefficients caused problems for some. Some candidates spotted that (1, 0) is a turning point.

(iv) The method for finding the area under the curve was well known, but some made errors in integrating or substitution. A few integrated the differential instead of the original function.

(ii) $y = \frac{1}{7}x + 3$, (iii) (1, 0) or $(\frac{7}{3}, \frac{32}{27})$ (iv) $\frac{4}{3}$

Question 11

The structure of the question enabled candidates to gain marks where appropriate, although few appreciated the relevance of all the parts to each other and completed (iii)(C).

(i) Some were confused as to how to find the gradient of the tangent, for instance finding (2,8) and giving the answer as 4, but most differentiated and found it successfully.

(ii) A common error was to substitute 2.1 and 2.01 into $3x^2$, but there were alo many good solutions, although some premature approximation was seen.

(iii)(*A*) Many candidates expanded the 3 brackets, rather than using the binomial theorem. There were plenty of good answers, but also plenty of slips.

(iii)(*B*) Most gained the method mark but many mis-cancelled the h to obtain their answer.

(iii)(*C*) This approach to differentiation was clearly not familiar to many candidates. There were a few full and clear explanations, but most omitted this part or attempted some algebra with $12 + 6h + h^2 = 12$, for example.

(i) $y = 12x - 16$ (ii) (*A*) 12.61, (*B*) 12.0601, (iii) (*A*) $8 + 12h + 6h^2 + h^3$,
(*B*) $12 + 6h + h^2$, (*C*) This is gradient of chord; as h tends to 0, gradient of tangent = 12