OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Monday **20 JANUARY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 A light spring AB has natural length 0.8 m and modulus of elasticity 200 N. The end A is fixed and a small ball is attached at B. The ball hangs in equilibrium below A.

(i) Given that the length of the spring is 1 metre, calculate the mass of the ball. [2]

(ii) If instead the mass of the ball is 2 kg, calculate the length of the spring. [3]

The end A of the spring is now fixed to a vertical wall and the end B is fixed to the mid-point of a light rod CD of length 1.2 m, as shown in Fig. 1. The end C of the rod is freely hinged to the wall vertically below A and a small ball of mass m kg is attached to the end D of the rod. The rod rests in equilibrium perpendicular to the spring and at an angle of 60° to the upward vertical.

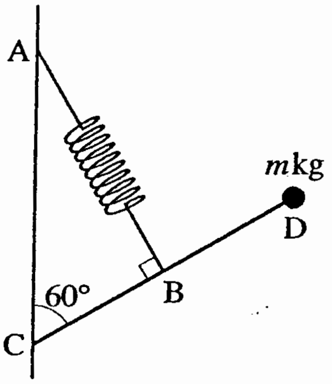


Fig. 1

(iii) Show that the extension of the spring is 0.239 m (correct to 3 significant figures) and hence calculate the tension in the spring. [3]

(iv) Calculate m. [3]

The end A of the spring is now attached to a different point on the wall. In this position the rod rests horizontally in equilibrium and angle ABC = θ.

(v) Show that $3\tan\theta - 4\sin\theta = 1.38$, correct to 3 significant figures. [4]

2

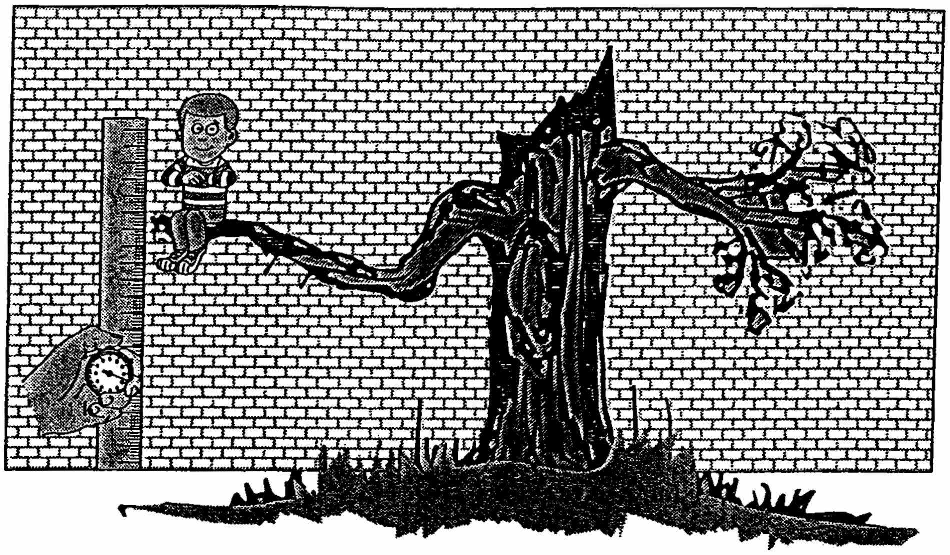


Fig. 2

Peter is bouncing on the end of a branch of a tree. Assume that his motion is vertical. His brother David is recording the height, y metres, of the end of the branch above the ground at time t seconds by means of a stopwatch and a scale which he has marked on an adjacent wall.

David models the motion as simple harmonic using the equation

$$y = A\cos \omega t + B\sin \omega t + C. \qquad (*)$$

At $t = 0$, Peter is at his highest point of 1.5 m above the ground. His lowest point is 1.1 m above the ground.

(i) Sketch a graph of y against t for one oscillation. Write down the values of A, B and C. [4]

At $t = 0.4$, Peter first reaches the point where $y = 1.4$.

(ii) Show that $\omega = \frac{5}{6}\pi$. [2]

(iii) Calculate Peter's maximum speed. [2]

(iv) David records that Peter takes 23.7 s to complete 10 bounces. Verify that this is close to the value predicted by the model. [1]

(v) Show that the equation (*) for y is not a solution of the standard simple harmonic differential equation $\ddot{y} = -\omega^2 y$. State a differential equation for which it is the solution. [2]

(vi) Calculate the maximum force that Peter exerts on the tree, given that his mass is 20 kg. [4]

[Turn over

3 A small ball-bearing of mass m moves in a horizontal circle of radius r on the inside of a hollow cone with its surface inclined at an angle α to the vertical. The inside of the cone is smooth. This situation is shown in Fig. 3.1.

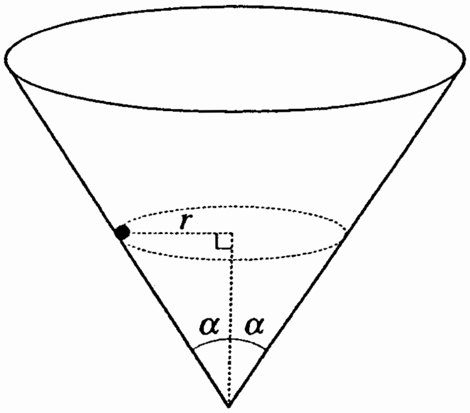


Fig. 3.1

The speed of the ball-bearing is v.

(i) Write down the radial equation of motion and the vertical equilibrium equation for the ball-bearing. Show that

$$v^2 = \frac{rg}{\tan\alpha}.$$

Show that this equation is dimensionally consistent. [7]

(ii) Hence find an expression for the kinetic energy of the ball-bearing in terms of m, g and h, where h is the height of the circle above the vertex. [3]

The ball-bearing is now subjected to a force F. The force F acts throughout the motion and is always in a direction away from the vertex along the surface of the cone, as shown in Fig. 3.2. The ball-bearing moves with speed v_1 in a horizontal circle of radius r_1.

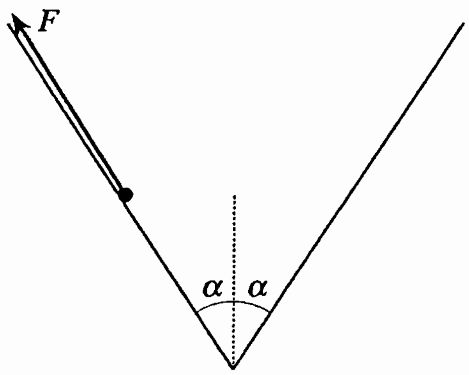


Fig. 3.2

(iii) Express F in terms of m, v_1, α, g and r_1. [5]

4 A uniform lamina is made in the shape of the region between the curve $y = \frac{1}{2}k(1 - x^2)$ and the x-axis, as shown in Fig. 4.1.

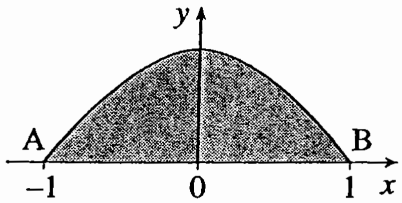


Fig. 4.1

(i) Show that the centre of mass of the lamina is at $(0, \frac{1}{5}k)$. [7]

A uniform lamina is made for an exhibition sign in the form of the shape in part (i) attached to a rectangle ABCD, where AB = 2 and BC = 1. The sign is suspended by two vertical wires attached at A and D as shown in Fig. 4.2.

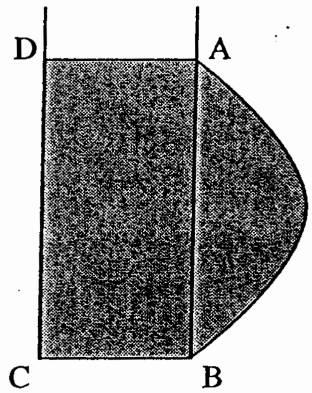


Fig. 4.2

(ii) Show that the centre of mass of the sign is at a distance

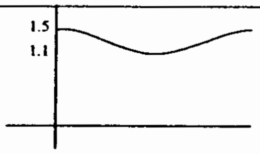
$$\frac{2k^2 + 10k + 15}{10k + 30}$$

from the mid-point of CD. [4]

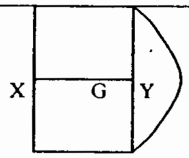
(iii) The tension in the wire at A is twice the tension in the wire at D. Calculate k. [4]

Mark Scheme

1(i)	$mg = \frac{200\times0.2}{0.8}$	M1	use of Hooke's law	
	$m = 5.1$ kg	A1		2
(ii)	$2g = \frac{200x}{0.8}$	M1	use of Hooke's law	
	$x = 0.0784$	M1	solving	
	$l = 0.8784$ m	A1		3
(iii)	$x = 0.6\tan 60° - 0.8 \approx 0.239\,\text{m}$	E1		
	$T = \frac{200\times0.239}{0.8}$	M1	use of Hooke's law	
	$= 59.8$ N	A1		3
(iv)	ΠC: $T\cdot0.6 = mg\cdot1.2\sin 60°$	M1	use of moments	
		A1	correct equation	
	$m = 3.52$	A1	cao	3
(v)	$T = \frac{200}{0.8}(\frac{0.6}{\cos\theta} - 0.8)$	M1	use correct extension	
	ΠC: $T\cdot0.6\sin\theta = mg\cdot1.2$	M1	use of moments	
	$250(\frac{0.6}{\cos\theta} - 0.8)\sin\theta\cdot0.6 = 1.2mg$	M1	eliminate T	
	$\Rightarrow 3\tan\theta - 4\sin\theta = 0.04mg \approx 1.38$	E1		4

2(i)		B1	sketch	
	$A = 0.2$	B1		
	$B = 0$	B1		
	$C = 1.3$	B1		4
(ii)	$1.4 = 1.3 + 0.2\cos(0.4\omega) \Rightarrow 0.4\omega = \frac{1}{3}\pi$	M1	substitute values and solve for ω	
	$\omega = \frac{5}{6}\pi$	E1		2
(iii)	$v_{max} = a\omega = 0.2\times\frac{5}{6}\pi$	M1	use of formula	
	$= \frac{1}{6}\pi \approx 0.524\,\text{m s}^{-1}$	A1		2
(iv)	$10\times\frac{2\pi}{(5\pi/6)} = 24 \approx 23.7$	B1		1
(v)	$\ddot{y} = -\omega^2 A\cos\omega t - \omega^2 B\sin\omega t \neq -\omega^2 y$	E1	clearly shown	
	$\ddot{y} = -\omega^2(y - C)$	B1		2
(vi)	force on tree = force on boy	B1	must consider N3L explicitly in some way	
	$R_{max} - mg = ma\omega^2$	M1	N2L with a correct acceleration expression	
		B1	recognise R maximised at lowest point	
	$R_{max} = 223\,\text{N}$	A1	cao	4

3(i)	$R\sin\alpha = mg$	B1		
	$R\cos\alpha = \dfrac{mv^2}{r}$	M1 A1	N2L with correct acceleration all correct	
	$v^2 = \dfrac{rg}{\tan\alpha}$	M1 E1	eliminate R clearly shown	
	$[v^2] = (\mathrm{LT}^{-1})^2 = \mathrm{L}^2\mathrm{T}^{-2}$	B1		
	$\left[\dfrac{rg}{\tan\alpha}\right] = \dfrac{\mathrm{L}\cdot\mathrm{LT}^{-2}}{1} = \mathrm{L}^2\mathrm{T}^{-2}$	B1	$\tan\alpha$ must not be ignored	7
(ii)	$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\dfrac{rg}{\tan\alpha}$	M1	use of formula with given v^2	
	$r = h\tan\alpha \Rightarrow$	M1	eliminate r	
	$\text{KE} = \frac{1}{2}mgh$	A1		3
(iii)	$R\cos\alpha - F\sin\alpha = \dfrac{mv_1^{\,2}}{r_1}$	M1 A1	N2L with both forces and correct acceleration	
	$R\sin\alpha + F\cos\alpha = mg$	B1	resolve vertically (including all forces)	
	$F = m(g\cos\alpha - \dfrac{v_1^{\,2}}{r_1}\sin\alpha)$	M1 A1	eliminate R	
	or resolve parallel to surface	M2	must be clear intention	
	$mg\cos\alpha - F = \dfrac{mv_1^{\,2}}{r_1}\sin\alpha$	A1 A1	correct component of acceleration all correct	
	$F = m(g\cos\alpha - \dfrac{v_1^{\,2}}{r_1}\sin\alpha)$	A1		5

4(i)	$\bar{x} = 0$ by symmetry	B1		
	$\bar{y}\int_{-1}^{1} y\,dx = \int_{-1}^{1}\frac{1}{2}y^2\,dx$	B1	formula	
	$\bar{y}\int_{-1}^{1}\frac{1}{2}k(1-x^2)\,dx = \int_{-1}^{1}\frac{1}{8}k^2(1-2x^2+x^4)\,dx$	M1	substitute and expand in reasonable attempt at formula	
	$\bar{y}[\frac{1}{2}k(x-\frac{1}{3}x^3)]_{-1}^{1} = [\frac{1}{8}k^2(x-\frac{2}{3}x^3+\frac{1}{5}x^5)]_{-1}^{1}$	M1 A1 M1	integrate $(x-\frac{1}{3}x^3)$ and $(x-\frac{2}{3}x^3+\frac{1}{5}x^5)$ evaluate limits	
	$\frac{2}{3}k\bar{y} = \frac{2}{15}k^2 \Rightarrow \bar{y} = \frac{1}{5}k$	E1		7
(ii)	 $\text{XG} = \dfrac{2(\frac{1}{2}) + \frac{2}{3}k(1+\frac{1}{5}k)}{2+\frac{2}{3}k}$	M1 A1 A1	use of $\Sigma mx/\Sigma m$ or moments numerator denominator	
	$= \dfrac{2k^2+10k+15}{10k+30}$	E1		4
(iii)	$T_D : T_A = 1:2 \Rightarrow \text{XG}:\text{GY} = 2:1$	M1	use of ratio or moments	
	$\Rightarrow \text{XG} = \frac{2}{3}$	A1		
	$\dfrac{2k^2+10k+15}{10k+30} = \frac{2}{3} \Rightarrow k = \frac{1}{6}(-5+\sqrt{115}) \approx 0.954$	M1 A1	solving 3-term quadratic (dependent on first M1)	4

Examiner’s Report

2609 Mechanics 3

General Comments

The general standard of work was high with many very good scripts and few very weak scripts. Candidates generally set their solutions out clearly, although some could have benefitted from signposting their work more clearly (e.g. by stating the direction of resolving or the point about which moments are being taken).

Comments on Individual Questions

Question 1 (Hooke's Law and Equilibrium)

The first two parts of this question were usually done very well. In the third part, some candidates were not able to deduce the given extension, and some candidates omitted to calculate the tension, but again there were many good solutions. In the remaining parts of the question, many candidates did not realise that moments were needed and tried to resolve instead. There were many good solutions although many candidates were unable to accurately find an expression for the extension in terms of θ.

(i) 5.1 kg; (ii) 0.8784 m; (iii) 59.8 N; (iv) 3.52.

Question 2 (Simple Harmonic Motion)

This question was often done well. However, some candidates were unable to find the correct values for the constants and often gave up after the first two parts, even though the remaining parts of the question could be done with their values. The sketch was usually broadly correct, although often included a rather misleading t-axis in the centre of the motion. Some candidates were able to write down the values of A, B and C, but many resorted to (sometimes lengthy) calculations. When considering the differential equation, many did not make clear conclusions, a number of candidates thought that an alternative solution, rather than an alternative differential equation was required. Candidates are expected to be familiar with the alternative form of the simple harmonic motion equation $\ddot{y} = -\omega^2(y - C)$. In the final part of the question, most candidates did not calculate the required force. Many calculated the maximum resultant force on Peter by merely multiplying the mass by the maximum acceleration. Some did better by calculating the maximum reaction force that the tree exerts on Peter, but only a handful of candidates equated this to the force that Peter exerts on the tree.

(i) 0.2, 0, 1.3; (iii) $\frac{1}{6}\pi \approx 0.524\,\text{m s}^{-1}$; (v) $\ddot{y} = -\omega^2(y - C)$; (vi) 223 N

Question 3 (Circular Motion)

The first part was often done very well, although some candidates produced (with no explanation) equations such as $mg\tan\alpha = mv^2/r$ which they then manipulated to the required result. In the final part of the question, similar errors were even more evident (although there were many good solutions also). Candidates are strongly advised to draw a diagram to indicate the forces being considered and to always indicate the direction in which they are applying Newton's second law. This is especially important if the law is applied in a direction which is not parallel to the acceleration. Even candidates who did these things often ran into problems when labelling a force mv^2/r on their diagram and then using it in an equation. Attempts of this type were often unclear and prone to inconsistent signs. However, there were also many good, clear solutions and some candidates produced a neat solution to the problem by applying Newton's second law down the slope clearly and successfully.

(ii) $\frac{1}{2}mgh$; (iii) $F = m(g\cos\alpha - \frac{v_1^2}{r_1}\sin\alpha)$

Question 4 (Centres of Mass)

The calculation of the centre of mass by integration was often done well, but a number of candidates were hampered by not being able to recall the correct formula. Some candidates did not explain why the x-coordinate was zero. When showing the result for the composite body, many candidates made errors in the position of the two centres of mass, but there were many good solutions. Most candidates made a reasonable attempt at the value of k required for the given ratio of tensions, but only a minority produced fully accurate solutions. Only a few candidates used a simple ratio argument to establish the position of the centre of mass, most preferring to take moments and in some cases creating a lot of unnecessary algebra.

(iii) 0.954.