OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2620/1

Decision and Discrete Mathematics 1

Wednesday **15 JANUARY 2003** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 A simple graph is one in which there are no loops, and in which there is no more than one edge connecting any pair of vertices. A simple **connected** graph is drawn. It has 5 vertices and 7 edges, and the order of each vertex is either 2, 3 or 4.

(i) Explain why the sum of the orders of the vertices is 14. [2]

(ii) Copy and complete Table 1 to show two of the possibilities for the numbers of vertices of each order.

Number of vertices	Number of order 2	Number of order 3	Number of order 4	Sum of orders
5				14
5				14

[2]

Table 1

(iii) Draw a diagram for each of your possibilities from part **(ii)** [2]

[Total 6]

2 The following algorithm counts the number of complete days from the beginning of the year 2000 to a date given in the form $d/m/y$, where d is the day number, m is the month number and y is the year number, $y \geqslant 2000$.

- start with $d - 1$
- add on $365 \times (y - 2000)$
- if $m > 2$ add on the integer part of $\frac{1}{4}(y - 1996)$, otherwise add on the integer part of $\frac{1}{4}(y - 1997)$
- if $m = 2$ then add on 31
- if $m = 3$ then add on 59
- if $m = 4$ then add on 90
- if $m = 5$ then add on 120
- if $m = 6$ then add on 151
- if $m = 7$ then add on 181
- if $m = 8$ then add on 212
- if $m = 9$ then add on 243
- if $m = 10$ then add on 273
- if $m = 11$ then add on 304
- if $m = 12$ then add on 334

Apply the algorithm to 27/11/2010. [4]

[Total 4]

3 Use the matrix form of Prim's algorithm, starting at A, to find a minimum connector for the network defined by the arc weights given in Table 3.

	A	B	C	D	E
A	–	12	8	7	9
B	12	–	10	–	9
C	8	10	–	4	5
D	7	–	4	–	3
E	9	9	5	3	–

Table 3

Draw your minimum connector and give its total weight. [5]

[Total 5]

Section B

4 The tasks involved in decorating a room are given in Table 4.1.

	Task	Immediate predecessor(s)
A	strip old paper	–
B	rub down wooden surfaces	–
C	paint ceiling	A
D	apply undercoat	A, B
E	apply gloss paint	D
F	paper walls	C, E

Table 4.1

(i) Draw an activity on arc network to illustrate this information. [3]

The duration of each task is shown in Table 4.2.

Task	A	B	C	D	E	F
Duration (days)	1	0.25	0.75	1	1	1

Table 4.2

(ii) Complete forward and backward passes to find early and late event times. Give the critical activities and the minimum duration of the project. [6]

(iii) Produce a resource histogram, given that each task requires one person.

Fred is decorating the room on his own. How long will it take him? [3]

(iv) Fred asks Alice to help him. Show that together they could decorate three rooms within eight days. [3]

[Total 15]

[Turn over

5 Two products, X and Y, require three ingredients, A, B and C, for their manufacture. Table 5 summarises the amounts required and how much of each is available.

		Resource A	Resource B	Resource C
Amount required per unit of product	Product X	15	10	8
	Product Y	5	7	12
Amount available		600	560	768

Table 5

It is required to maximise the total output of the two products subject to the amounts available.

(i) Identify variables and formulate an appropriate linear programming (LP) problem. [5]

(ii) Solve your LP problem graphically, and interpret the solution. [7]

(iii) The amount of B available is increased by 16. Show that the total output can be increased by 1 unit. [2]

(iv) The amount of B available is increased by a further 16. Show that the total output cannot be increased any further. [1]

[Total 15]

6 In a manufacturing process parts arrive at a machine for polishing. The time intervals between arrivals follow the distribution given in Table 6.1.

Arrival intervals

Interval (minutes)	1	2	3
Probability	0.5	0.3	0.2

Table 6.1

(i) Give a rule for using one-digit random numbers to simulate time intervals between part arrivals. [2]

Polishing times follow the distribution given in Table 6.2.

Polishing times

Time (minutes)	1	2
Probability	$\frac{1}{3}$	$\frac{2}{3}$

Table 6.2

(ii) Give a rule for using one-digit random numbers to simulate polishing times. [3]

(iii) Use the random numbers from Table 6.3 to simulate arrival intervals and polishing times for 5 parts arriving at the machine. [3]

Random numbers for arrival intervals	4	8	1	0	8	4	5	0	3	9
Random numbers for polishing times	8	5	9	0	1	3	5	2	3	0

Table 6.3

When parts arrive at the machine they are placed in a stack until the machine is ready to polish them. Parts arriving later are placed on top of parts arriving earlier. When the machine has finished polishing a part the operator has to take the next part to be polished. If a part is just arriving he takes that part. Otherwise he takes the part from the top of the stack.

A simulation is begun as shown in Table 6.4.

Part	1	2	3	...
Arrival interval	2	1	1	...
Arrival time	2	3	4	...
Polishing time	2	1	1	...
Time when polishing starts	2	...	...	...
Time when polishing ends	4	...	...	...

Table 6.4

(iv) Continue the simulation, using your simulated inter-arrival and polishing times.

Show the times at which parts are taken for polishing and the times when polishing is finished. **Stop your simulation when the part which arrived second is taken to be polished.** [4]

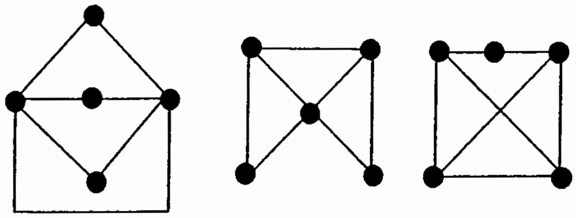
(v) Give the total time for which the second part waited to be polished. [1]

(vi) Your answer to part (v) is an estimate of how long a part has to wait to be polished. Describe two ways to improve the process of obtaining an estimate of this time. [2]

[Total 15]

Mark Scheme

1.

(i) 14 = 2 x 7	M1 A1
(ii) 3, 0, 2 2, 2, 1 1, 4, 0	B1 B1
(iii) e.g.: 	B1 B1

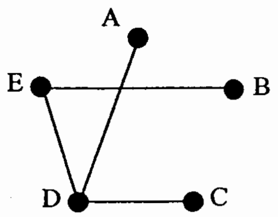
2.

(i) 26	B1
+ 3650	B1
+ 3	B1
+ 304 = 3983	B1

3.

	1	5	4	2	3
	A	B	C	D	E
A	~~–~~	~~12~~	~~8~~	~~7~~	~~9~~
B	12	–	10	–	(9)
C	~~8~~	~~10~~	~~–~~	(4)	~~5~~
D	(7)	~~–~~	~~4~~	~~–~~	~~3~~
E	~~9~~	~~9~~	~~5~~	(3)	~~–~~

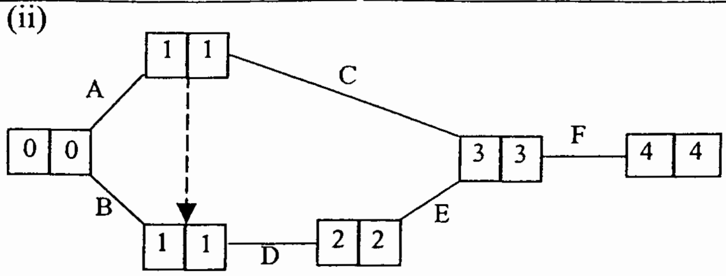
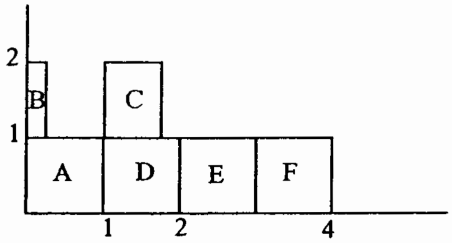
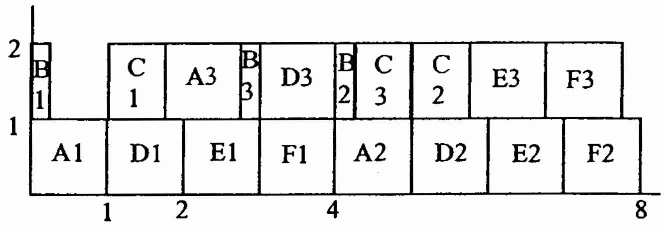
M1
A1 selecting
A1 deleting



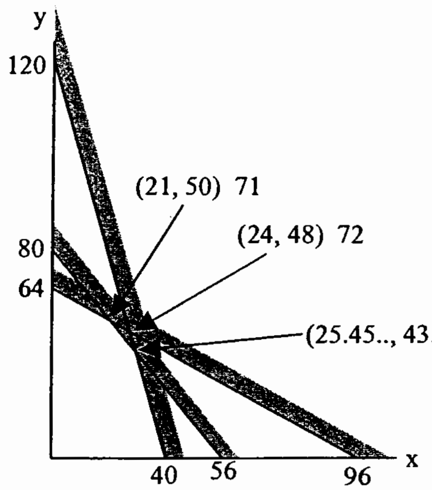
total weight = 23

B1 B1

4.

(i) & (ii) 	M1 A1 dummy A1 rest M1 A1 forward M1 A1 backward
A, D, E and F 4 days	B1 B1
(iii)  5 days	M1 A1 B1
(iv) This is an obvious solution, though it can be done in 7.5 days: 	M1 1&2 side by side A1 3 slotted in A1 B3 correct

5.

(i)	Let x be the number of units of X produced …	B1	
	max x + y	B1	
	st $15x + 5y \leq 600$	B1	
	$10x + 7y \leq 560$	B1	
	$8x + 12y \leq 768$	B1	
(ii)		B1	axes labelled and scaled
		B3	lines
		B1	shading
	21 of X and 50 of Y	M1 A1	
(iii)	Second constraint becomes $10x + 7y \leq 576$, making (24,48) feasible.	M1 A1	
(iv)	Any further increase in availability of B irrelevant since the constraint is now not active.	B1	

6.

(i)	0-4 5-7 8-9 1 2 3	M1 A1
(ii)	0-2 3-8 9 1 2 ignore	M1 missing some A1 1 missed A1 rest OK
(iii)	arrival intervals: 1 3 1 1 3 polishing times: 2 2 1 1 2	M1 A1 arrival intervals A1 polishing times
(iv)	part 1 2 3 4 5 arrival interval 2 1 1 1 3 arrival time 2 3 4 5 8 polishing time 2 1 1 2 2 polishing starts 2 7 4 5 … polishing ends 4 5 7 …	B1 arrival times (parts 4, 5, …) M1 A1 start service A1 end service
(v)	4 minutes	B1
(vi)	Repeated runs + averaging Settling in – looking at a later part (if indeed equilibrium is achieved!)	B1 B1

Examiner’s Report

2620 Decision and Discrete Mathematics I

General Comments

Candidates found one or two of the grade A marks to be challenging, and there were few really high scores. There were occasions when candidates had clearly allowed themselves to lose too much time on these part questions. Conversely, there were many relatively easy marks to be had.

Comments on Individual Questions

Question 1 (Graphs)

This question was disappointingly done. Only a minority of candidates were able to come up with $2 \times$ edges in part (i). More managed correct entries in part (ii) and correct diagrams in part (iii), although many non-simple graphs were seen.

Question 2 (Algorithms)

Relatively few completely correct answers were seen. Commonly, fractional answers were seen following "the integer part of …".
Arithmetic errors were common, particularly $26+365 \times (2010-2000) = 391 \times 10$.

3983 days

Question 3 (Networks)

A very large number of candidates did not know the matrix form of Prim's algorithm. A common score was 2 out of 5, for getting the correct minimum connector and total weight, but not by using the required algorithm.

AD, BE, CD and DE – total weight = 23

Question 4 (CPA)

Candidates did well with the basic CPA work in parts (i) and (ii). They did less well with the resource histogram in part (iii), many producing just a cascade chart. Part (iv) was more difficult.

(ii) Critical activities: A, D, E and F; minimum duration = 4 days
(iii) Fred would take 5 days on his own.

Question 5 (LP)

As ever, candidates were weak in identifying their variables. Surprisingly, many also failed correctly to identify the objective function – many accumulated the left-hand sides of the inequalities to get $33x+24y$. The graphs were generally well done in part (ii), but a substantial majority did not demonstrate any methodology in performing the maximisation. Parts (iii) and (iv) were intended to be more challenging.

Note that accurate answers read from a clear graph are perfectly acceptable. Many candidates solve simultaneous equations to ensure accuracy. This too is perfectly acceptable, but not required, and some lose much time doing it for little reward. This was particularly evident in the final part of this question where all that was required was to note that any further increase in resource availability makes the constraint redundant – the line moves into the infeasible region.

(ii) Max of 71units at (21, 50)
(iii) New max of 72 at (24, 48)

Question 6 (Simulation)

Parts (i), (ii) and (iii) were generally well done. The simulation of a top-down stack was difficult, but was surprisingly well done. Only a very few candidates noted that the waiting time for the second component is not a good indicator of the efficiency of the system.