OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2606

Pure Mathematics 6

Tuesday **11 JUNE 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Option 1: Vectors and Matrices

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 3 \\ 0 & 2 \end{pmatrix}$, where $k \neq 2$.

(i) Find the eigenvalues of **M**, and the corresponding eigenvectors. [7]

(ii) Write down a matrix **P** for which $\mathbf{P}^{-1}\mathbf{MP}$ is a diagonal matrix. [2]

(iii) Hence find the matrix $\mathbf{M}^n$. [7]

(iv) For the case $k = 1$, use the Cayley-Hamilton theorem to find integers p and q such that

$$\mathbf{M}^9 = p\mathbf{M}^8 + q\mathbf{M}^7.$$ [4]

Option 2: Limiting Processes

2 A function $\mathrm{H}(x)$ is defined, for $x \geqslant 0$, by

$$\mathrm{H}(x) = \int_0^x \mathrm{e}^{-t^2}\,\mathrm{d}t.$$

(i) Write down $\mathrm{H}'(x)$. [2]

(ii) Sketch the graphs of $y = \mathrm{e}^{-t^2}$ and $y = \mathrm{e}^{-t}$ (for $t \geqslant 0$) using the same axes, and label them clearly.

Hence show that, for $x > 1$,

$$\int_0^1 \mathrm{e}^{-t}\,\mathrm{d}t < \mathrm{H}(x) < 1 + \int_1^x \mathrm{e}^{-t}\,\mathrm{d}t.$$ [5]

(iii) Deduce that $\mathrm{H}(x)$ tends to a limit L as $x \to \infty$, and show that

$$1 - \frac{1}{\mathrm{e}} < L < 1 + \frac{1}{\mathrm{e}}.$$ [4]

(iv) By first writing $t^2\mathrm{e}^{-t^2}$ as $t(t\mathrm{e}^{-t^2})$, express $\int_0^x t^2\mathrm{e}^{-t^2}\,\mathrm{d}t$ in terms of $\mathrm{H}(x)$, and show that

$$\int_0^\infty t^2\mathrm{e}^{-t^2}\,\mathrm{d}t = \tfrac{1}{2}L.$$ [5]

(v) Use L'Hôpital's rule to find $\lim_{x \to 1} \dfrac{\mathrm{H}(x) - \mathrm{H}(1)}{\mathrm{H}(2x) - \mathrm{H}(2)}$. [4]

Option 3: Multi-Variable Calculus

3 A surface S has equation $g(x, y, z) = 0$, where $g(x, y, z) = (y - x)(x + 2y - z)^2 - 32$.

(i) Show that $\frac{\partial g}{\partial x} = (x + 2y - z)(z - 3x)$, and find $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. [5]

(ii) Verify that $\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + 3\frac{\partial g}{\partial z} = 0$.

Interpret this result in terms of the normal vectors to the surface S. [3]

(iii) Find the equation of the tangent plane to the surface S at the point P(2, 10, 20). [3]

(iv) The point Q$(2 + \delta x, 10 + \delta y, 20 + \delta z)$ is a point on the surface S close to P.

Find an approximate expression for δz in terms of δx and δy. [3]

(v) R$(a, 7, c)$ is a point on the surface S at which $\frac{\partial g}{\partial x} = 0$.

Show that the tangent plane at R has equation $3y - z = 6$. [6]

Option 4: Differential Geometry

4 A curve has parametric equations

$$x = a\left(1 - \cos^3\theta\right), \quad y = a\sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi,$$

where a is a positive constant.

(i) Find the length of this curve. [5]

(ii) Show that, when this curve is rotated through 2π radians about the y-axis, the curved surface area generated is $\frac{9}{5}\pi a^2$. [5]

(iii) Show that the radius of curvature at a general point on the curve is $3a\sin\theta\cos\theta$. [5]

(iv) Find the centre of curvature corresponding to the point on the curve where $\theta = \frac{1}{3}\pi$. [5]

Option 5: Abstract Algebra

5 The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ is a group under the binary operation of multiplication modulo 20.

(i) Give the combination table for G. [4]

(ii) State the inverse of each element of G. [2]

(iii) Find the order of each element of G. [2]

(iv) List all the subgroups of G.

Identify those subgroups which are isomorphic to one another. [7]

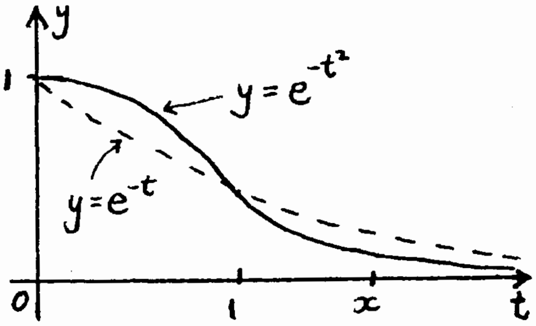
(v) For each of the following, state, giving reasons, whether or not the given set and binary operation is a group. If it is a group, state, giving a reason, whether or not it is isomorphic to G.

(A) $J = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under multiplication modulo 8

(B) $K = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition modulo 8 [5]

Mark Scheme

1 (i)	Characteristic equation is $(k-\lambda)(2-\lambda)-0=0$ Eigenvalues are k and 2 For $\lambda=k$, $\begin{pmatrix}k&3\\0&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=k\begin{pmatrix}x\\y\end{pmatrix}\Rightarrow \begin{matrix}kx+3y=kx\\2y=ky\end{matrix}$ $\Rightarrow y=0$ so an eigenvector is $\begin{pmatrix}1\\0\end{pmatrix}$ For $\lambda=2$, $\begin{pmatrix}k&3\\0&2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=2\begin{pmatrix}x\\y\end{pmatrix}\Rightarrow \begin{matrix}kx+3y=2x\\2y=2y\end{matrix}$ $\Rightarrow y=\frac{1}{3}(2-k)x$ so an eigenvector is $\begin{pmatrix}3\\2-k\end{pmatrix}$	M1 A1A1 M1 A1 M1 A1 7	
(ii)	$\mathbf{P}=\begin{pmatrix}1&3\\0&2-k\end{pmatrix}$	B2 ft 2	
(iii)	$\mathbf{P}^{-1}\mathbf{MP}=\mathbf{D}=\begin{pmatrix}k&0\\0&2\end{pmatrix}$ $\mathbf{D}^n=\begin{pmatrix}k^n&0\\0&2^n\end{pmatrix}$ $\mathbf{P}^{-1}=\frac{1}{2-k}\begin{pmatrix}2-k&-3\\0&1\end{pmatrix}$ $\mathbf{M}^n=\mathbf{PD}^n\mathbf{P}^{-1}$ $=\frac{1}{2-k}\begin{pmatrix}1&3\\0&2-k\end{pmatrix}\begin{pmatrix}k^n&0\\0&2^n\end{pmatrix}\begin{pmatrix}2-k&-3\\0&1\end{pmatrix}$ $=\frac{1}{2-k}\begin{pmatrix}k^n&3(2^n)\\0&(2-k)2^n\end{pmatrix}\begin{pmatrix}2-k&-3\\0&1\end{pmatrix}$ $=\begin{pmatrix}k^n&\frac{3(2^n-k^n)}{2-k}\\0&2^n\end{pmatrix}$	B1 ft B1 ft B1B1 ft M1 A1 cao A1 cao 7	 Give even if order is wrong *Or* $\frac{1}{2-k}\begin{pmatrix}1&3\\0&2-k\end{pmatrix}\begin{pmatrix}(2-k)k^n&-3k^n\\0&2^n\end{pmatrix}$
(iv)	Characteristic equation is $\lambda^2-3\lambda+2=0$ By Cayley-Hamilton theorem, $\mathbf{M}^2-3\mathbf{M}+2\mathbf{I}=\mathbf{O}$ Hence $\mathbf{M}^9-3\mathbf{M}^8+2\mathbf{M}^7=\mathbf{O}$ i.e. $\mathbf{M}^9=3\mathbf{M}^8-2\mathbf{M}^7$ i.e. $p=3,\ q=-2$	B1 ft M1 M1 A1 cao 4	 Applying CH theorem Multiplying by $\mathbf{M}^7$

2 (i)	$H'(x) = e^{-x^2}$	B2	**2**	
(ii)		B2		Both decreasing, and crossing at $t = 0$ and $t = 1$ (Give B1 if $t = 1$ not identified)
	$H(x) > \int_0^1 e^{-t^2}\,dt > \int_0^1 e^{-t}\,dt$	B1 (ag)		
	$H(x) = \int_0^1 e^{-t^2}\,dt + \int_1^x e^{-t^2}\,dt$ $< \int_0^1 1\,dt + \int_1^x e^{-t}\,dt$	M1		
	$= 1 + \int_1^x e^{-t}\,dt$	A1 (ag)	**5**	
(iii)	$\left[-e^{-t}\right]_0^1 < H(x) < 1 + \left[-e^{-t}\right]_1^x$	M1		
	$1 - e^{-1} < H(x) < 1 - e^{-x} + e^{-1} < 1 + e^{-1}$	A1		
	$H(x)$ is an increasing function, and is bounded,	M1		
	so $H(x) \to L$ where $1 - e^{-1} < L < 1 + e^{-1}$	A1 (ag)	**4**	
(iv)	$\int_0^x t(te^{-t^2})\,dt = \left[t(-\tfrac{1}{2}e^{-t^2})\right]_0^x - \int_0^x(-\tfrac{1}{2}e^{-t^2})\,dt$	M1A1		
	$= -\tfrac{1}{2}xe^{-x^2} + \tfrac{1}{2}H(x)$	A1		
	As $x \to \infty$, $xe^{-x^2} \to 0$ and $H(x) \to L$	M1		
	so $\int_0^\infty t^2 e^{-t^2}\,dt = \tfrac{1}{2}L$	A1 (ag)	**5**	
(v)	Limit is $\dfrac{H'(1)}{2H'(2)}$	M1		
	$= \dfrac{e^{-1}}{2e^{-4}}$	A1 A1		or $\dfrac{e^{-x^2}}{2e^{-4x^2}}$
	$= \tfrac{1}{2}e^3$	A1 cao	**4**	

3 (i)	$\frac{\partial g}{\partial x} = -(x+2y-z)^2 + (y-x)2(x+2y-z)$ $= (x+2y-z)(-x-2y+z+2y-2x)$ $= (x+2y-z)(z-3x)$ $\frac{\partial g}{\partial y} = (x+2y-z)^2 + (y-x)2(x+2y-z)(2)$ $= (x+2y-z)(x+2y-z+4y-4x)$ $= (x+2y-z)(-3x+6y-z)$ $\frac{\partial g}{\partial z} = -2(y-x)(x+2y-z)$	M1 A1 (ag) M1 A1 B1 **5**	
(ii)	$\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + 3\frac{\partial g}{\partial z}$ $= (x+2y-z)(z-3x-3x+6y-z-6y+6x)$ $= 0$ All normal vectors are perpendicular to $\begin{pmatrix}1\\1\\3\end{pmatrix}$	 B1 (ag) B2 **3**	 *Or equivalent*
(iii)	At P, $\frac{\partial g}{\partial x} = 28,\ \frac{\partial g}{\partial y} = 68,\ \frac{\partial g}{\partial z} = -32$ Tangent plane is $7x+17y-8z = 14+170-160$ $7x+17y-8z = 24$	M1 M1 A1 cao **3**	 For $7x+17y-8z$
(iv)	$\delta g \approx 28\delta x + 68\delta y - 32\delta z$ (and $\delta g = 0$) so $\delta z \approx \frac{1}{8}(7\delta x + 17\delta y)$	M1A1 ft A1 ft **3**	*Or* $7(2+\delta x)+17(10+\delta y)-8(20+\delta z) \approx 24$
(v)	Since $\frac{\partial g}{\partial x} = 0,\ c = 3a$ Since R lies on S, $(7-a)(a+14-3a)^2 - 32 = 0$ $4(7-a)^3 - 32 = 0$ $7-a = 2$ $a = 5$ $c = 15$ $\frac{\partial g}{\partial y} = 48,\ \frac{\partial g}{\partial z} = -16$ Tangent plane is $3y - z = 21 - 15$ $3y - z = 6$	B1 M1 A1 M1 M1 A1 (ag) **6**	 Either a or c correct Or, using **(ii)**, $\frac{\partial g}{\partial y} = -3\frac{\partial g}{\partial z}$

4 (i)	$\frac{dx}{d\theta} = 3a\cos^2\theta\sin\theta$, $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$	B1	
	$\frac{ds}{d\theta} = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}$		
	$= 3a\sin\theta\cos\theta\sqrt{\cos^2\theta + \sin^2\theta} = 3a\sin\theta\cos\theta$	M1	For finding $\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2$
	Arc length is $\int_0^{\frac{1}{2}\pi} 3a\sin\theta\cos\theta\, d\theta$	A1	Correct integral expression (inc limits)
	$= \left[\frac{3}{2}a\sin^2\theta\right]_0^{\frac{1}{2}\pi}$	B1	$\sin\theta\cos\theta$ correctly integrated
	$= \frac{3}{2}a$	A1 5	
(ii)	Curved surface area is $\int 2\pi x\, ds$	M1	
	$= \int_0^{\frac{1}{2}\pi} 2\pi a(1-\cos^3\theta)(3a\sin\theta\cos\theta)\, d\theta$	A1 ft	Integral expression (limits required)
	$= 6\pi a^2\left[-\frac{1}{2}\cos^2\theta + \frac{1}{5}\cos^5\theta\right]_0^{\frac{1}{2}\pi}$	M1 A1	Method for integrating $\sin\theta\cos^4\theta$ For $-\frac{1}{2}\cos^2\theta + \frac{1}{5}\cos^5\theta$
	$= 6\pi a^2\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{9}{5}\pi a^2$	A1 (ag) 5	
(iii)	$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \tan\theta$	M1A1	
	$\tan\psi = \tan\theta$, so $\psi = \theta$	M1	*Or* $\frac{d^2y}{dx^2} = \frac{\sec^2\theta}{3a\cos^2\theta\sin\theta}$
	$\rho = \frac{ds}{d\psi} = \frac{ds}{d\theta}$	M1	*Or* $\rho = (1+\tan^2\theta)^{\frac{3}{2}} \times \frac{3a\cos^2\theta\sin\theta}{\sec^2\theta}$
	$= 3a\sin\theta\cos\theta$	A1 (ag) 5	
	OR $\dot{x}\ddot{y} - \ddot{x}\dot{y} = \ldots = 9a^2\sin^2\theta\cos^2\theta$ M1A1		
	$\rho = \frac{(3a\sin\theta\cos\theta)^3}{9a^2\sin^2\theta\cos^2\theta}$ M1 M1		
	$= 3a\sin\theta\cos\theta$ A1		
(iv)	When $\theta = \frac{1}{3}\pi$, $\rho = 3a\left(\frac{1}{2}\sqrt{3}\right)\left(\frac{1}{2}\right) = \frac{3}{4}\sqrt{3}a$		
	$\hat{\mathbf{n}} = \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\sqrt{3} \\ \frac{1}{2} \end{pmatrix}$	M1A1	
	$x = \frac{7}{8}a$, $y = \frac{3}{8}\sqrt{3}a$		
	Centre of curvature is		
	$\begin{pmatrix} \frac{7}{8}a \\ \frac{3}{8}\sqrt{3}a \end{pmatrix} + \frac{3}{4}\sqrt{3}a\begin{pmatrix} -\frac{1}{2}\sqrt{3} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4}a \\ \frac{3}{4}\sqrt{3}a \end{pmatrix}$	M1 A1 A1 5	

5 (i)

	1	3	7	9	11	13	17	19
1	1	3	7	9	11	13	17	19
3	3	9	1	7	13	19	11	17
7	7	1	9	3	17	11	19	13
9	9	7	3	1	19	17	13	11
11	11	13	17	19	1	3	7	9
13	13	19	11	17	3	9	1	7
17	17	11	19	13	7	1	9	3
19	19	17	13	11	9	7	3	1

B4 — Give B1 for 16 entries correct; B2 for 32 entries correct; B3 for 48 entries correct — **4**

(ii)

x	1	3	7	9	11	13	17	19
x^{-1}	1	7	3	9	11	17	13	19

B2 — Give B1 for 4 correct — **2**

(iii)

x	1	3	7	9	11	13	17	19
order	1	4	4	2	2	4	4	2

B2 — Give B1 for 4 correct — **2**

(iv)

$\{1\}$, $\{1,9\}$, $\{1,11\}$, $\{1,19\}$	B2	Give B1 for 2 correct
$\{1,3,7,9\}$, $\{1,9,13,17\}$, $\{1,9,11,19\}$, G	B2	Give B1 for 2 correct (G not required)
$\{1,9\}$, $\{1,11\}$, $\{1,19\}$ are isomorphic	B1	For any two subgroups of order 2
$\{1,3,7,9\}$, $\{1,9,13,17\}$ are isomorphic	B1	
	B1	Fully correct, dependent on all subgroups of orders 2 and 4 correctly listed, and no spurious IMs given
	7	

(v)(*A*)

0 has no inverse	B1	For reason
so J is not a group	B1	

(*B*)

K is closed and inverses of 0, 1, 2, 3, 4, 5, 6, 7 are 0, 7, 6, 5, 4, 3, 2, 1	B1	For reason
so K is a group	B1	
Different pattern (2 self-inverse) K is not isomorphic to G	B1	Must include a reason
	5	

Examiner’s Report

2606 Pure Mathematics 6

General Comments

This paper was found slightly harder than recent past papers. There were fewer really excellent scripts than usual, with 10% of candidates scoring more than 50 marks (out of 60), and about a fifth scored less than 20. there are ten possible ways of selecting three questions from five, but these were far from equally likely; more than half the candidates chose Qs.1, 3 and 4.

Comments on Individual Questions

Q.1 This was the best answered question, with half the attempts scoring 15 marks or more (out of 20). All parts were generally well understood, and a good number (about 20% of attempts) scored full marks. In part (i), having obtained the characteristic equation as $(k - \lambda)(2 - \lambda) = 0$, an amazing number of candidates multiplied this out to $\lambda^2 - (k + 2)\lambda + 2k = 0$ then used the quadratic formula to solve it. Many careless errors were made when finding the eigenvectors, such as giving the x and y coordinates the wrong way round. Parts (ii) and (iii) were answered well; candidates could obtain most of the marks even if their eigenvectors were incorrect. The use of the Cayley-Hamilton theorem in part (iv) was very well understood, although some candidates rewrote the question and expressed $\mathbf{M}^9$ in the form $p\mathbf{M} + q\mathbf{I}$.

$$\text{(i) } k, 2, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2-k \end{pmatrix}; \quad \text{(ii) } \begin{pmatrix} 1 & 3 \\ 0 & 2-k \end{pmatrix}, \quad \text{(iii) } \begin{pmatrix} k^n & \dfrac{3(2^n - k^n)}{2-k} \\ 0 & 2^n \end{pmatrix}, \quad \text{(iv) } p = 3, q = -2.$$

Q.2 This was the least popular question, attempted by about 20% of candidates. It was also the worst answered, and half the attempts scored 8 marks or less. Part (i) was usually answered correctly, although H′(x) was sometimes given as a function of t. In part (ii) the graphs were usually drawn correctly, but few candidates earned full marks for deriving the inequalities. There were some excellent explanations, accompanied by *clear* shading on the graphs or starting from $\mathrm{H}(x) = \int_0^1 \mathrm{e}^{-t^2}\,\mathrm{d}t + \int_1^x \mathrm{e}^{-t^2}\,\mathrm{d}t$, but most did little more than copy down the given results. In part (iii) most candidates performed the integrations to obtain $1 - \frac{1}{\mathrm{e}} < \mathrm{H}(x) < 1 + \frac{1}{\mathrm{e}}$ but very few stated that H(x) was an increasing function. In part (iv) about half the candidates realised that integration by parts was needed; these usually obtained the correct expression and deduced the value of the infinite integral. In part (v) L'Hôpital's rule was generally well understood, but correct answers were rare; a very common error was to differentiate H($2x$) to obtain H′($2x$) instead of 2H′($2x$); and some candidates differentiated H(1) (a constant) to obtain H′(1).

$$\text{(i) } \mathrm{e}^{-x^2}, \quad \text{(iv) } -\tfrac{1}{2}x\mathrm{e}^{-x^2} + \tfrac{1}{2}\mathrm{H}(x), \quad \text{(v) } \tfrac{1}{2}\mathrm{e}^3.$$

Q.3 This was the most popular question, but it was found to be more difficult than previous questions on this topic, and most candidates scored between 8 and 13 marks. In part (i) the partial differentiation was usually carried out correctly, although there was often a false start involving multiplying out. In part (ii) the given result was usually obtained (provided part (i) was correct), but few candidates gave a satisfactory interpretation; the expected response was 'all normal vectors are perpendicular to the vector (1 1 3)'. Some said that the vector (1 1 3) lies in every tangent plane; although this is true it could not be given credit as it does not refer to the normal vectors. Part (iii) was well answered, but part (iv) was often omitted or answered incorrectly; common errors were $\delta z \approx \frac{\partial g}{\partial x}\delta x + \frac{\partial g}{\partial y}\delta y$ (which earned no marks) and $\frac{\partial z}{\partial x} = \frac{\partial g}{\partial x} \div \frac{\partial g}{\partial z}$, which resulted in a sign error and scored 1 out of 3. In part (v) most candidates obtained $c = 3a$ and made an attempt to find the ratio $\frac{\partial g}{\partial y} : \frac{\partial g}{\partial z}$ (the easiest method was to use the result in part (ii)). This earned 2 marks, which was the most common mark for this part; few candidates realised that they needed to use the equation of the surface to find a and c.

$$\text{(i) } \frac{\partial g}{\partial y} = (x + 2y - z)(-3x + 6y - z), \quad \frac{\partial g}{\partial z} = -2(y - x)(x + 2y - z), \quad \text{(iii) } 7x + 17y - 8z = 24,$$

$$\text{(iv) } \delta z \approx \tfrac{1}{8}(7\delta x + 17\delta y).$$

Q.4 This question was attempted by about two thirds of the candidates, and the average mark was 11. In part (i) the method was well known and many obtained the correct answer, but some had trouble with the differentiation of x and y, or with the integration of $\sin\theta\cos\theta$. In part (ii) a correct integral expression was often obtained, but many were unable to integrate $\cos^4\theta\sin\theta$. Several candidates rotated the curve about the x-axis; having obtained the wrong answer some, but not all, realised their error and started again. In part (iii) by far the most popular method was to use the parametric formula, but errors in differentiation and manipulation were very common. The method using $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ was sometimes tried, but rarely successfully, as $\frac{dy}{dx} = \tan\theta$ was almost always followed by $\frac{d^2y}{dx^2} = \sec^2\theta$. The quickest method ($\Psi = \theta$ so $\rho = \frac{ds}{d\theta}$ which was found in part (i)) was used by a few candidates. In part (iv) the method was quite well known, but often there were errors in the normal vector, such as signs or reversed coordinates, and sometimes a non-unit vector was used.

(i) $\frac{3}{2}a$, (iv) $\left(-\frac{1}{4}a, \frac{3}{4}\sqrt{3}a\right)$.

Q.5 This question was attempted by about 30% of candidates, which is more than usual for this topic. It was quite well answered, with half the attempts scoring 14 marks or more. All parts were well understood, but most candidates lost marks in part (iv), where usually only one or two of the three subgroups of order 4 were found.

(ii) 1, 7, 3, 9, 11, 17, 13, 19, (iii) 1, 4, 4, 2, 2, 4, 4, 2,
(iv) $\{1\}$, $\{1, 9\} \cong \{1, 11\} \cong \{1, 19\}$, $\{1, 3, 7, 9\} \cong \{1, 9, 13, 17\}$, $\{1, 9, 11, 19\}$, G,
(v) (A) not a group (B) group, not isomorphic to G.