OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2603(A)**

Pure Mathematics 3

Section A

Tuesday **11 JUNE 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 3 printed pages and 1 blank page.

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1 **(a)** Split $\dfrac{x+4}{(x+1)(x-2)}$ into partial fractions. [3]

(b) Find the first four terms of the binomial expansion of $(1+2x)^{\frac{1}{2}}$. [3]

(c) Solve the equation $4\sin x\cos x = 1$, where $0 \leqslant x \leqslant \pi$. [4]

(d) Show that $\sin^3 x = \sin x - \sin x\cos^2 x$.

Differentiate $\cos^3 x$ with respect to x.

Hence find $\int \sin^3 x \, dx$. [5]

2 In Fig. 2, OAB is a bent rod, with OA = 1 metre, AB = 2 metres and angle OAB = 120°. The bent rod is in a vertical plane. It is free to rotate in this plane about the point O.

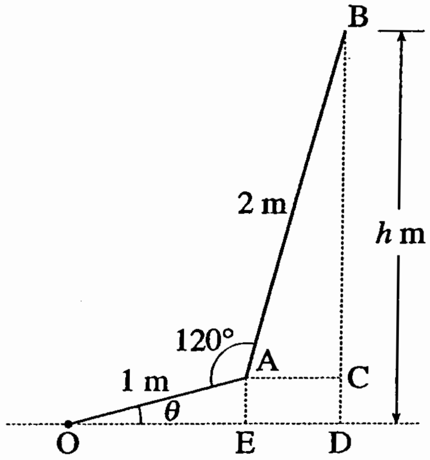

Fig. 2

OA makes an angle θ with the horizontal, where $-90° < \theta < 90°$. The vertical height BD of B above the level of O is h metres. The horizontal through A meets BD at C.

(i) Show that angle BAC = $\theta + 60°$, and show that $h = \sin\theta + 2\sin(\theta + 60°)$. [3]

(ii) Hence show that $h = 2\sin\theta + \sqrt{3}\cos\theta$, and find the angle θ for which $h = 0$. [6]

(iii) Express $2\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta + \alpha)$. Hence or otherwise find the maximum value of h, and find an angle θ for which $h = 2.5$. [6]

3 **(a)** Find $\int xe^x \,dx$. Hence show that the solution of the differential equation

$$e^y \frac{dy}{dx} = -xe^x,$$

for which $y = 0$ when $x = 0$, is

$$y = x + \ln(1 - x).$$ [5]

(b) A curve is defined by the equation $e^x + e^y = 2$.

(i) By differentiating implicitly, or otherwise, show that $\frac{dy}{dx} = -e^{x-y}$. [2]

(ii) Verify that

$$x = \ln(1 + t), \qquad y = \ln(1 - t)$$

are parametric equations for the curve. [2]

(iii) Find $\frac{dy}{dx}$ in terms of t, and hence or otherwise find the exact coordinates (in terms of logarithms) of the point on the curve where the gradient is -2. [6]

4 With respect to coordinate axes Oxyz, A is the point (3, 0, 1), B is (1, 0, 3), C is (3, 2, 3) and D is (2, −1, 1).

(i) Show that triangle ABC is equilateral. [3]

(ii) Show that the vector $\overrightarrow{AD}$ can be expressed as $\lambda\overrightarrow{AB} + \mu\overrightarrow{AC}$, where λ and μ are constants to be determined. What can you deduce about the points A, B, C and D? [5]

(iii) Verify that the vector $\mathbf{n} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ is perpendicular to the plane ABC. Hence or otherwise find the cartesian equation of the plane ABC. [4]

(iv) Find the angle between the lines AB and DC. [3]

Mark Scheme

Section A

1 (a) $\dfrac{x+4}{(x+1)(x-2)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-2}$ $\Rightarrow x+4 \equiv A(x-2)+B(x+1)$ $x=-1 \Rightarrow 3=-3A \Rightarrow A=-1$ $x=2 \Rightarrow 6=3B \Rightarrow B=2.$	M1 A1 A1 [3]	Correct partial fractions **s.o.i.** $A=-1$ $B=2$
(b) $(1+2x)^{1/2} =$ $1+\frac{1}{2}(2x)+\dfrac{\frac{1}{2}.(-\frac{1}{2})}{2!}(2x)^2+\dfrac{\frac{1}{2}.(-\frac{1}{2})(-\frac{3}{2})}{3!}(2x)^3+\ldots$ $= 1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+\ldots$	M1 A1 A1cao. [3]	Any three of the binomial coefficients $1+\dfrac{1}{2}(\ )+\dfrac{(1/2)(1/2-1)}{1.2}(\ )^2+\dfrac{(1/2)(1/2-1)(1/2-2)}{1.2.3}(\ )^3\ldots$ Correct expansion unsimplified Correct series
(c) $4\sin x\cos x = 2\sin 2x = 1$ $\Rightarrow \sin 2x = \dfrac{1}{2}$ $\Rightarrow 2x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ $\Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ Accept 0.26, 1.31. Allow unsupported answers B2 ,B2 or B1,B1 in degrees.	M1 M1 A1A1 cao [4]	use of double angle formula solving for $2x$ $\dfrac{\pi}{6}$ or 0.524 For 15°, 75° give A1, for both.
(d) $\sin^3 x = \sin x \sin^2 x$ $= \sin x(1-\cos^2 x)$ $= \sin x - \sin x\cos^2 x$ * $\dfrac{d}{dx}(\cos^3 x) = -3\sin x\cos^2 x$ $\int \sin^3 x\,dx = \int(\sin x - \sin x\cos^2 x)dx$ $= \int \sin x\,dx - \int \sin x\cos^2 x\,dx$	E1 M1 A1 B1 B1 [5]	Generously applied Chain rule, $3\cos^2 x$ or $3u^2 \times$ an attempt to differentiate $\cos x$., or any other correct method. A1 for any correct result $-\cos x$, $1/3\cos^3 x$, condone omission of c

$= -\cos x + \frac{1}{3}\cos^3 x + c$		

2 (i) $B\hat{A}C = 360 - 90 - 120 - (90 - \theta)$ $= \theta + 60°$ * AE = 1. sin θ = sin θ BC = 2 sin(θ + 60) and h = sin θ + 2sin(θ + 60) *	E1 M1 E1 [3]	 AE = sin θ or BC = 2 sin(θ + 60) **soi** AE + BC . Some justification must be seen.
(ii) h = sin θ + 2sin(θ + 60) = sin θ + 2(sinθcos60 + cosθsin60) $= \sin\theta + 2(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta)$ = 2 sin θ +√3 cosθ * $h = 0 \Rightarrow$ 2 sin θ +√3 cosθ = 0 ⇒ 2 sin θ = −√3 cosθ ⇒ tan θ = −√3/2 ⇒ θ = − 41° or better – not in radians.	 M1 A1 E1 M1 A1 A1 [6]	 Expanding sin(θ + 60) correctly $\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$ Must be exact form Accept working backwards using Rcos (θ + α) method For 2 tan θ = −√3 or equivalent. **(Or** after the first part of (iii) h=0 when √7sin(θ +40.89°) = 0 M1 θ + 40.89° = 0 A1 θ = −41° A1**)**
(iii) 2 sin θ +√3 cosθ = R sin(θ + α) = R(sin θ cos α + cos θ sin α) ⇒ Rcos α = 2 , Rsin α = √3 ⇒ $R^2 = 2^2 + (\sqrt{3})^2 = 7, R = \sqrt{7}$ tan α = √3/2, α = 40.89° so h = √7 sin (θ + 40.89°) $h_{max} = \sqrt{7}$ √7 sin (θ + 40.89°) = 2.5 ⇒ sin (θ + 40.89°) = 2.5/√7 ⇒ θ + 40.89 = 70.89 (or 109.11) ⇒ θ = 30° (or 68°) Allow unsupported answer B2	B1 M1 B1 B1ft M1 A1 cao [6]	R = √7 For the correct identity- implied by Rcos α = 2 , Rsin α = √3 or tan α = √3/2 Accept 40.9° or better f.t. their R Allow this M1 for √7 sin(θ+ 49.11°) = 2.5 after error in α above. One angle only required

3(a) $\int xe^x\,dx = xe^x - e^x$ $e^y \frac{dy}{dx} = -xe^x \Rightarrow \int e^y dy = -\int xe^x dx$ $\Rightarrow e^y = -\int x \frac{d}{dx}(e^x)dx$ $= -x e^x + \int e^x dx$ $= -x e^x + e^x + c$ when $x = 0, y = 0, \Rightarrow 1 = 1 + c, c = 0$ $\Rightarrow e^y = -x e^x + e^x = e^x(1-x)$ $\Rightarrow y = \ln(e^x(1-x))$ $= \ln(e^x) + \ln(1-x)$ This step must $= x + \ln(1-x)$ be seen for E1	M1 M1 A1 M1 M1 E1 [6]	Integrating by parts $u = x$, $v = e^x$. Separating variables $= -x e^x + e^x + c$ condone omission of c evaluating c Factorising their solution to the DE and taking lns. If the M1 for finding c comes after this M1 it becomes DM1
(b)(i) $e^x + e^y = 2$ $\Rightarrow e^x + e^y \frac{dy}{dx} = 0$ $\Rightarrow e^y \frac{dy}{dx} = -e^x$ $\Rightarrow \frac{dy}{dx} = -e^x / e^y = -e^{x-y}$ *	M1 E1 (2)	Differentiating implicitly $e^x + e^y \frac{dy}{dx}$ (**Or** $e^y = 2 - e^x \Rightarrow y = \ln(2-e^x)$ $\Rightarrow \frac{dy}{dx} = \frac{-e^x}{2-e^x}$ M1 chain rule $= \frac{1}{e^y}(-e^x)$ (must be seen) $= -e^{x-y}$ E1)
(ii) $x = \ln(1+t)$, $y = \ln(1-t)$ $\Rightarrow e^x = 1 + t$, $e^y = 1 - t$ $\Rightarrow e^x + e^y = 1 + t + 1 - t = 2$ *	M1 E1 (2)	Anti-logging, either result seen. (**Or** after above $t = e^x - 1 \Rightarrow$ $y = \ln(2 - e^x)$ $\Rightarrow e^y = 2 - e^x \Rightarrow e^x + e^y = 2$)
(iii) $\frac{dy}{dx} = \frac{-\frac{1}{1-t}}{\frac{1}{1+t}} = -\frac{1+t}{1-t}$ $\frac{dy}{dx} = -2$ when $-\frac{1+t}{1-t} = -2$ $\Rightarrow 1 + t = 2 - 2t$ $\Rightarrow 3t = 1, t = 1/3$ $x = \ln(4/3)$, $y = \ln(2/3)$	M1 A1 M1 A1ft A1cao (5) [Total 9]	$\frac{dy/dt}{dx/dt}$ or substitution in their $\frac{dy}{dx}$ from **(i)**. A1 for any correct expression for $\frac{dy}{dx}$ (in terms of t) Equating their dy/dx to -2 and attempt to solve. $t = 1/3$ allow f.t. for $t = -1/3$, 3, or -3 $x = \ln(4/3)$ and $y = \ln(2/3)$

4 (i) $AC^2 = 0^2 + (0-2)^2 + (1-3)^2 = 8$ or $\cos ABC = \frac{4+0+0}{\sqrt{8}\sqrt{8}} \Rightarrow ABC = 60°$	M1 A1 E1 [3]	Method of finding the (length)^{2}of a side or finding an angle of the triangle One angle or (side)^2 correct A sufficient number of results to prove the result
(ii) $\vec{AD} = -\mathbf{i} - \mathbf{j}, \vec{AB} = -2\mathbf{i} + 2\mathbf{k},\ \vec{AC} = 2\mathbf{j} + 2\mathbf{k}$ $-\mathbf{i} - \mathbf{j} = \lambda(-2\mathbf{i} + 2\mathbf{k}) + \mu(2\mathbf{j} + 2\mathbf{k})$ $\Rightarrow -1 = -2\lambda \Rightarrow \lambda = 1/2$ $-1 = 2\mu \Rightarrow \mu = -1/2$ $0 = 2\lambda + 2\mu = 2.\frac{1}{2} + 2.(-\frac{1}{2})$ correct A, B, C and D are coplanar	M1 A1 A1 E1 B1 [5]	Substituting components into $\vec{AD} = \lambda\ \vec{AB} + \mu\ \vec{AC}$ $\lambda = 1/2$ $\mu = -1/2$ **(Or** showing that any of **AB,AC,AD** is dependent on the other two. M1 E1)
(iii) $\mathbf{n} . \vec{AB} = (\mathbf{i} - \mathbf{j} + \mathbf{k}).(-2\mathbf{i} + 2\mathbf{k})$ $= -2 + 0 + 2 = 0$ $\mathbf{n} . \vec{AC} = (\mathbf{i} - \mathbf{j} + \mathbf{k}).(2\mathbf{j} + 2\mathbf{k})$ $= 0 - 2 + 2 = 0$ so **n** is perpendicular to the plane Equation of the plane is **n.r** = **n.a** $\Rightarrow$ **(i–j+k).(** *x***i**+*y***j**+*z***k)** = **(i–j+k).(3i+k)** $\Rightarrow x - y + z = 3 + 1 = 4$	B1 B1 B1 B1 [4]	Both scalar products = 0 At least one evaluation seen $x - y + z = \ldots$**(Or** M1 A1 for other $\ldots = 4$ acceptable methods eg forming a vector equation **and** eliminating the parameters M! Result AI)
(iv) $\vec{AB} = -2\mathbf{i} + 2\mathbf{k}$, $\vec{DC} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ $\cos\theta = \frac{\vec{AB}.\vec{DC}}{\lvert\vec{AB}\rvert.\lvert\vec{DC}\rvert} = \frac{-2+0+4}{\sqrt{8}.\sqrt{14}} = 0.189$ $\Rightarrow \theta = 79°.$	M1 A1ft A1cao [3]	Correct use of the formula. Correct substitution of their vectors. Accept 79° or 101°or better. Accept 1.38 or 1.76 radians or better.

Examiner’s Report

2603 Pure Mathematics 3

General Comments

There was a very good response to this paper with some 15 to 20 per cent of candidates scoring marks in the range 60 to 75 and only about 6 or 7 per cent scoring 15 or fewer marks. Between these extremes there was a very good spread of marks.

The new format of question 1 proved to be a help to all candidates giving even the weaker candidates an encouraging start; the majority of candidates were able to achieve full marks on the partial fractions question and the binomial series.

There were parts of other questions which most candidates found accessible: expanding $\sin(\theta + 60)$, expressing $2\sin\theta + \sqrt{3}\cos\theta$ in the form $R\sin(\theta + \alpha)$, integration by parts, finding a gradient in parametric form, finding the lengths of vectors, the angle between two directions, all attracted full marks for a very large number of candidates.

There were also parts of questions which differentiated well between candidates: $\frac{d}{dx}\cos^3 x$, $\int \sin^3 x\,dx$, the particular integral of the differential equation, and the complete proof that the vector **AD** is linearly dependent on the vectors **AB** and **AC** in the vector question, were generally only answered correctly by the more able candidates.

Section B also attracted a full range of marks with some parts answered well by almost all candidates and others which only a minority of candidates were able to complete correctly. There appeared to be little correlation between the marks in the two sections.

There were, perhaps, more presentable scripts than in previous years but the work of some candidates was very untidy, poorly written and generally difficult to interpret.

Comments on Individual Questions

Q.1 (a) The correct partial fractions were found by almost all candidates.

(b) Most candidates obtained the correct binomial expansion. Occasional errors included $2x^2$ instead of $(2x)^2$ and an error in the sign of the term $\frac{1}{2}x^2$.

(c) A pleasing number of candidates changed $4\sin x\cos x$ to $2\sin 2x$ and proceeded to the first solution, but $4\sin x\cos x = \sin 4x$ was not uncommon. Most candidates using the correct method failed to find the second solution for $2x$ and gave $\frac{11\pi}{12}$ instead of $\frac{5\pi}{12}$ as the second solution for x.

(d) Too many candidates used the formulae $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ and $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ in this question leading to longer solutions than necessary and a greater possibility of errors. Perhaps the first part prompted some candidates to express $\cos^3 x$ as $\cos x - \cos x\sin^2 x$ and then to differentiate using the product formula. These candidates seemed prepared to use the chain rule to differentiate $\sin^2 x$ having avoided using it to differentiate $\cos^3 x$ directly.

(a) $\frac{-1}{x+1} + \frac{2}{x-2}$; (b) $1 + x - \frac{x^2}{2} + \frac{x^3}{2} + \ldots$; (c) $\frac{\pi}{12}, \frac{5\pi}{12}$; (d) $-3\cos^2 x\sin x$, $-\cos x + \frac{1}{3}\cos^3 x + c$.

Q.2 (i) and (ii) The first three results here were given, and so full justification was required to obtain full marks. A very clear diagram was acceptable for angle BAC = $\theta + 60°$, but some justification of the expressions for AE and BC was required for h. Direct solutions of the equation $2\sin\theta + \sqrt{3}\cos\theta = 0$ were reasonably common, although $\tan\theta = \frac{\sqrt{3}}{2}$ was a frequent error. Other candidates anticipated part (iii) to obtain $\sqrt{7}\sin(\theta + 40.89°) = 0$ and hence the solution.

(iii) Not all candidates wrote down the identity $2\sin\theta + \sqrt{3}\cos\theta = R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$ but attempted the equation $R\cos\alpha = 2$, or etc., from memory. The occasional error $R\cos\alpha = \sqrt{3}$ was, therefore, almost inevitable. Otherwise this part was very well done and a good number of candidates went on to solve the equation $h = 2.5$

correctly. Not so many obtained the maximum value of h, and some gave the angle for which the maximum occurred but not its value. Candidates were not always aware of the range given for θ and solutions to $h = 0$ and $h = 2.5$ were sometimes outside this range, or given in radians.

(ii) –41°, (iii) $\sqrt{7}\sin(\theta + 40.89°)$, $\sqrt{7}$, 30° or 68°.

Q.3 This was, perhaps, the question where the weaker candidates had the most trouble, but even they were usually confident in integrating by parts and if they were also able to separate the variables correctly, the first marks were usually secured. Those candidates who also remembered to include a constant of integration in their solution, and find its value at this stage, most often did so correctly. Many candidates, however, were unable to carry out the next stage of taking logarithms, the rules of logs are still not fully understood by many candidates and $e^y = -xe^x + e^x + c$ often became $y = \ln(-xe^x) + x + \ln c$., or similarly without the constants. Other candidates who wrote, correctly, $y = \ln(x - xe^x)$, attempted to introduce the constant at this stage. Because the result was given it was essential that the steps $e^y = e^x(1-x)$ and $y = \ln e^x + \ln(1-x)$ were shown, in order to obtain full marks. Many candidates failed to do this.

(b)(i) Notation caused problems for some candidates in this question, $\frac{dy}{dx} = e^x + e^y\frac{dy}{dx}$ was sometimes followed by the correct result, but in other cases the surplus $\frac{dy}{dx}$ at the front was taken into the equation. In many such solutions it was not clear whether the candidate had purposefully differentiated the 2 to 0 or not. If = 0 appeared at the end of the above line the first $\frac{dy}{dx}$ was condoned. Candidates needed to show the intermediate line $\frac{dy}{dx} = -\frac{e^x}{e^y}$ to obtain full marks. An occasional error was $\frac{dy}{dx} = -e^x - e^y = -e^{x-y}$.

(ii) The better candidates had little difficulty in deriving e^x and e^y in terms of t but weaker ones often did not know how to proceed. $\ln(1 + t) = \ln 1 + \ln t$ was not uncommon.

(iii) This part was better done than the previous two parts, most candidates knowing how to obtain $\frac{dy}{dx}$ in terms of the parameter t. Unfortunately a very common error was to omit the –ve sign when differentiating $\ln(1 - t)$. Another error was to misread the given gradient as +2. A small number of candidates obtained the expression for $\frac{dy}{dx}$ very neatly by using the result in part (ii). Very few candidates who made sign errors in this question seemed aware that the logarithm of a negative number is not defined.

(i) $xe^x - e^x$; (iii) $\frac{dy}{dx} = \frac{-(1+t)}{1-t}$, $x = \ln\left(\frac{4}{3}\right)$, $y = \ln\left(\frac{2}{3}\right)$.

Q.4 Parts (i) and (iv) were well done by very many candidates who were able to apply the formulae for the length of a vector and the angle between two vectors. There were sometimes errors in the senses of vectors and occasionally candidates failed to give sufficient proof that the triangle ABC was equilateral.

The more able candidates had little difficulty with part (ii), substituting the appropriate components and finding the values of λ and μ. However some of these candidates omitted to check that their values also satisfied the equation $0 = 2\lambda + 2\mu$. Not all the candidates who proved that **AD** = λ**AB** + μ**AC** were able to state that this implied that A,B,C and D were coplanar but some candidates who failed to prove the result, did state the conclusion. Answers such as ABCD is a parallelogram, a square, a pyramid or a kite, were not uncommon.

A very common error in part (iii) was to show only one of the required **n.AB**=0, **n.AC**=0 or **n.AD**=0 instead of two, but many candidates then obtained the correct Cartesian equation of the plane. Those candidates who preferred to use the vector equation of the plane and eliminate the parameters to find the Cartesian equation, were not disadvantaged in this question; the elimination was quite simple and having obtained the equation in this way candidates were able to show that the normal to the plane was the vector **n** = **i** –**j** + **k**, thus answering the first part of the question.

(ii) $\lambda = 1/2$, $\mu = -1/2$, A,B,C,D are coplanar; (iii) $x - y + z = 4$; (iv) 79°.

Section B Comprehension

1. Very many candidates answered this question by quoting from page 3 of the text, "Regular polygons with more than six sides have internal angles greater than 120° and so when three or more of them meet at a point there is an overlap rather than a gap." Rather more justification than this was required for the second of two marks available, namely the actual value of the internal angle.

2. Again some explanation was needed rather than just 12 = 8 + 6 – 2, e.g. V = 6, F = 8, and E = 12. Many candidates did not identify these numbers.

3. Very often correct.

4. Solutions to this question were often lengthy and confused. Many candidates made matters worse by attempting to work back from the answer at some stage in their solution. A common error was to take the sides of the isosceles right-angled triangle to be 1, 1 and $\sqrt{2}$, instead of lengths in the ratio 1: 1: $\sqrt{2}$. Only a few able candidates were able to rationalise the result $x = \frac{a}{(\sqrt{2}+1)}$.

5. Not many candidates were able to visualise the truncated tetrahedron and the correct combination of triangles and hexagons was not stated very often

6. Many candidates were able to state the correct number of faces but the numbers of vertices and edges were obtained correctly by relatively few. Very few candidates thought to use Euler's Law to obtain one result from the other two, or to check their results found in other ways.

(3) 12; (5) 4 triangles and 4 hexagons; (6) F=80, E=120, V=42.