OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2607

Mechanics 1

Thursday **23 MAY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1 A particle travels in a straight line.

The motion is initially modelled by the v-t diagram in Fig. 1.

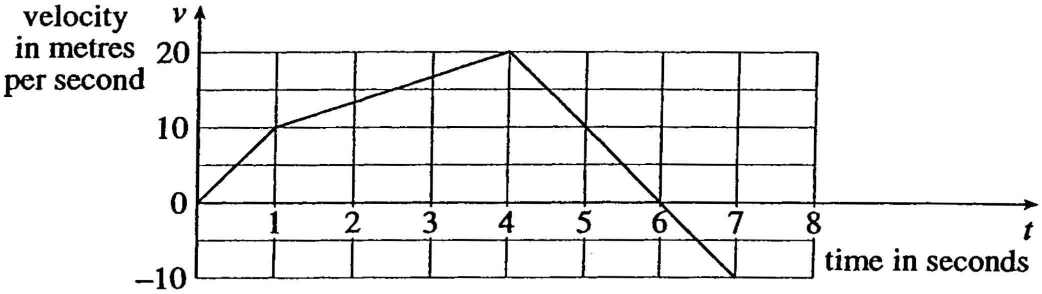

Fig. 1

(i) Calculate the acceleration of the particle in the part of the motion from $t = 1$ to $t = 4$. [1]

(ii) Calculate the displacement of the particle from its position when $t = 0$ to its position when $t = 6$. [4]

(iii) Calculate the displacement of the particle from its position when $t = 0$ to its position when $t = 7$. [2]

(iv) Describe the motion of the particle during the interval $4 \leqslant t \leqslant 7$. [2]

Although the values of v are known accurately at $t = 0$, $t = 1$ and $t = 6$, the value at $t = 4$ is not so certain.

It is now suggested that the relationship between v and t may be modelled, for $0 \leqslant t \leqslant 6$, as

$$v = 12t - 2t^2.$$

(v) Verify that the values for v in this expression agree with those given on the v-t diagram for $t = 0$, $t = 1$ and $t = 6$.

Calculate the displacement of the particle from its position when $t = 0$ to its position when $t = 6$ predicted by this new model. [6]

[Total 15]

2

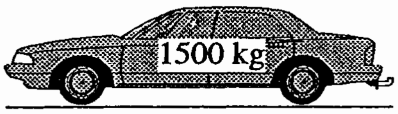

Fig. 2.1

(a) The following two questions are about the motion of a car of mass 1500 kg, shown in Fig. 2.1, travelling along a straight, horizontal road.

(i) The car has negligible resistance to its forward motion and is accelerating at $0.5\ \mathrm{m\ s^{-2}}$. Calculate the driving force. [2]

(ii) The car has a driving force of 2500 N and total resistance to its forward motion of 250 N. Calculate its acceleration. [2]

(b) A car of mass 1500 kg is pulling a trailer of mass 900 kg along a straight, horizontal road, as shown in Fig. 2.2. The coupling between the car and the trailer is light, rigid and horizontal.

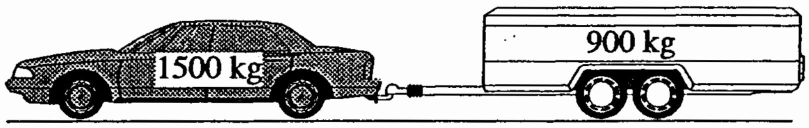

Fig. 2.2

The motion of the car and trailer is first modelled assuming that the resistances to motion are negligible. There is a driving force of 600 N acting on the car.

(i) Draw separate diagrams showing the horizontal force(s) acting on

(A) the car.

(B) the trailer. [2]

(ii) Calculate the acceleration of the car and trailer. Calculate also the force in the coupling, stating whether this is a tension or a thrust. [6]

In a new situation, the trailer has a resistance to motion of 300 N and the car has a resistance R N. There is no driving force and there is zero tension in the coupling.

(iii) Calculate R. [4]

[Total 16]

[Turn over

3 A heavy packing case is on a rough, uniform and horizontal floor. Alf tries to move it by pulling with a force of 600 N in the **i** direction, as shown in Fig. 3.1 and Fig. 3.2. The packing case does not move.

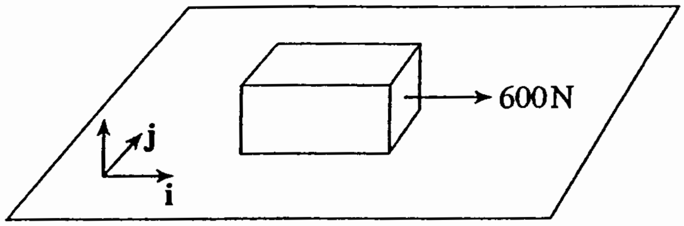

Fig. 3.1

Fig. 3.2

(i) Write down the magnitude and the direction of the frictional force opposing Alf's pull. [1]

Alf is now joined by Bert and Chas who pull with the forces shown in Fig. 3.3. The packing case still does not move.

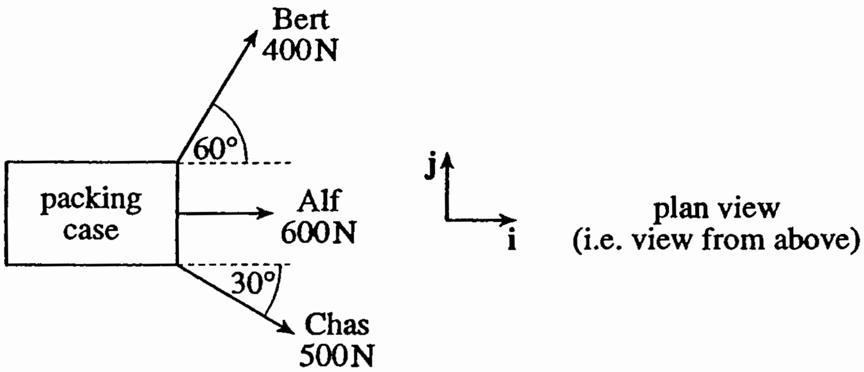

Fig. 3.3

(ii) Calculate the components of the *combined* pulling force on the packing case in

(*A*) the **i** direction,

(*B*) the **j** direction. [5]

(iii) Calculate the magnitude of the *combined* pulling force on the packing case. Calculate also the angle between the direction of this force and the **i** direction. [4]

Dave joins the other three and pulls with a force of $(100\mathbf{i} + p\mathbf{j})$ N. With all four pulling, the box moves at a steady speed in the direction of the vector $(12\mathbf{i} + \mathbf{j})$.

(iv) Calculate the value of p. [4]

[Total 14]

4 In this question you should take $g = 10\,\mathrm{m\,s^{-2}}$. The effects of air resistance should be neglected.

A small stone is fired from a catapult 1 m above horizontal ground at a speed of $30\,\mathrm{m\,s^{-1}}$. The angle of projection with the horizontal is α, where $\cos\alpha = 0.6$ and $\sin\alpha = 0.8$. The stone hits a vertical wall that is a horizontal distance of 27 m from the point of projection. This information is shown in Fig. 4 together with x- and y-axes and the origin O on the ground; the units of the axes are metres.

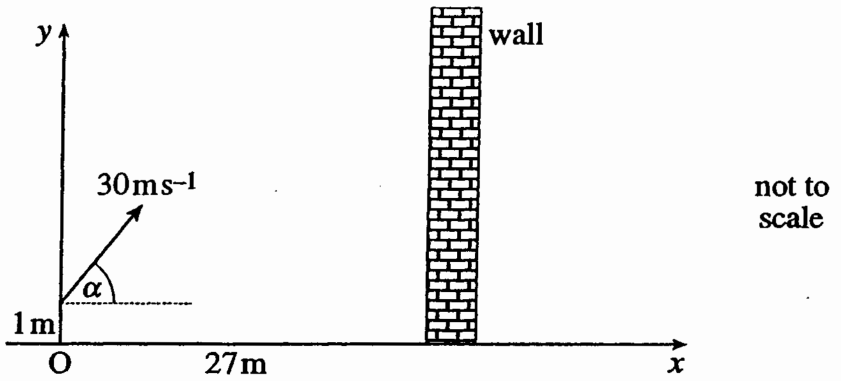

Fig. 4

(i) Show that, after t seconds, the horizontal displacement of the stone from O, x m, and the vertical displacement, y m, are given by

$$x = 18t \qquad \text{and} \qquad y = 1 + 24t - 5t^2.$$ [5]

(ii) What is the value of t when the stone hits the wall?

How high is the stone above the ground when it hits the wall? [3]

(iii) Show that the stone is rising when it hits the wall. [2]

(iv) Find the horizontal displacement of the stone when it is at a height of 17m above the ground. [5]

[Total 15]

Mark Scheme

Paper 2607	Name Mechanics 1	Session June	Year 2002	Final

Solutions and mark scheme

Q 1		Mark		
(i)	$\frac{20-10}{3}=\frac{10}{3}\text{ m s}^{-2}$	B1	Accept 3.33 or better	1
(ii)	Area under the graph $0.5\times10+0.5\times(10+20)\times3+0.5\times20\times2$ $=70\text{ m}$	B1 B1 M1 A1	For any one correct area/ *uvast* application 5, 45 and 20 all correct, addition not necesary Addition of at least 2 attempted areas/ *uvast* calcs cao [SC 70 m seen is awarded SC3]	4
(iii)	From $t=6$ to $t=7$ negative displacement so $70-0.5\times10=65\text{ m}$	M1 A1	Recognition of need to subtract. Accept double -ve FT from **their** (ii) - 5	2
(iv)	Uniform deceleration (or decel of 10 m s^{-2}) from 20 m s^{-1} to - 10 m s^{-1} (in 3 seconds) Changes direction (after 6 s). (Must be clear Not backwards through whole interval)	E1 E1	1 relevant comment another relevant comment	2
(v)	$v(0)=0\times6=0$; $v(1)=2\times5=10$; $v(6)=12\times0=0$, so all agree.	E1	All working must be shown for $t=1$ and $t=6$.	
	$s=\int_0^6(12t-2t^2)dt$	M1	Use of integration. Neglect limits	
	$=\left[6t^2-\frac{2t^3}{3}\right]_0^6$	A1	Either term correct. Neglect limits	
	$=(216-144)-0$ or $=216-144$	M1 A1	Subst of both limits or evaluation of arb const attempted Correct substitutions or correct arb constant	
	$=72\text{ m}$	A1	cao	6
				Tot 15

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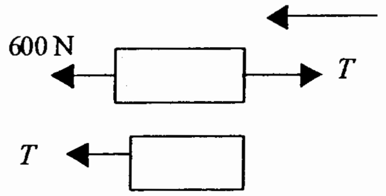
Paper 2607	Name Mechanics 1	Session June	Year 2002	Final

Solutions and mark scheme

Q 2		Mark		
(a) (i)	$F = 1500 \times 0.5$	M1	Use of N2L. Allow use of *mga*. No 'extra' forces.	
	$= 750$ so 750 N	A1		2
(ii)	$(2500 - 250) = 1500a$	M1	Use of N2L, $(2500 - 250)$ seen Allow use of *mga*	
	$a = 1.5$ so 1.5 m s^{-2}	A1		2
(b) (i)		B1	600 N and 'T' present. No other forces. Ignore any vertical forces on either diag	
		B1	Labelling consistent. Condone single diagram if all correct, including T	2
(ii)	**Either** ← N2L Overall $600 = 2400a$	M1	Use of N2L. All correct. Do not accept *mga*	
	$a = 0.25$ so 0.25 m s^{-2}	A1		
	← N2L Trailer $T = 900 \times 0.25$	M1	Use of N2L on trailer (or car). All relevant forces. No extras. Do not accept *mga*	
		A1	Signs correct. Need to see **their** 0.25 used.	
	$= 225$ so 225 N	A1	[If N2L used inappropriately in finding tension, M1,A0,A0]	
	Tension	E1	Award if seen	6
	or For pair of simultaneous equns	M1	Use of N2L on trailer (or car). All relevant forces. No extras. Do not accept *mga*	
	First equation	A1	Equation correct	
	Second equation	M1	Correct + attempt to solve simultaneously. Do not accept *mga*.	
	$a = 0.25$	A1		
	$T = 225$	A1		
	Tension	E1	Award if seen	

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Q 2		Mark		
(iii)	← direction of motion			
	For trailer $-300 = 900a$	M1	Condone only sign errors. Do not accept *mga*	
	Either $-300 - R = 2400 \times a$ **or** $-R = 1500 \times a$	M1	Allow only sign errors. Do not accept $a = 0.25$ or 0.	
	so $a = -\frac{1}{3}$	A1		
	$R = 500$ so 500 N	A1	If a not calculated award A2 for $R = 500$. Do not accept – 500N [SC $R/1500 = 300/900$ M2 $R = 500$ so 500 N A2]	4
				Tot 16

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Solutions and mark scheme

Q3		Mark		
(i)	600 N in $-\mathbf{i}$ direction	B1	Both required. Allow $-600\mathbf{i}$, 600 at 270°, 600 to the left, 600 N due west	1
(ii) (A)	$400\cos 60 + 500\cos 30 + 600$	M1	Resolving with at least one resolution attempted. All forces present. No extras. Allow sin/cos interchange	
	= 1233.01... so 1230 N (3 s. f.)	A1	Evaluation necessary. Allow surds.	2
(B)	$400\sin 60 - 500\sin 30$	M1	Resolving both forces in correct direction with at least 1 resolution correct. Both forces present. No extras.	
		A1	Signs correct	
	96.4101... so 96.4 N (3 s. f.)	A1	Evaluation necessary. Allow surds. [The two A marks in (A) and (B) may be awarded retrospectively from (iii)]	3
(iii)	magnitude is $\sqrt{1233.01...^2 + 96.4101...^2}$	M1	Use of Pythagoras provided each cpt is non-zero	
	= 1236.77... so 1240 N (3 s. f.)	A1	FT (ii) (A) + (B)	
	$\arctan\left(\dfrac{96.4101...}{1233.01..}\right)$ or equivalent	M1	Use of arctan, angle w.r.t. $\pm\mathbf{i}$ or $\pm\mathbf{j}$ provided each cpt is non-zero	
	= 4.4709..° so 4.47° (3 s. f.)	A1	FT (ii) (A) + (B). Angle w.r.t. $\pm\mathbf{i}$ only	4
(iv)	Total force is now $1333.01..\mathbf{i} + (96.4101.. + p)\mathbf{j}$	M1	Expression for **total** force e.g. [(ii)(A) + 100]**i** + {(i)(B) + *p*]**j**	
	We require $\dfrac{1333.01...}{96.4101..+p} = \dfrac{12}{1}$ or equivalent	M1	Attempt to form valid equation(s)	
		B1	Correct equation not involving an arbitrary constant	
	$p = 14.674..$ so 14.7 (3 s. f.)	A1	cao	4
				Tot 14

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Solutions and mark scheme

Q 4		Mark		
			Condone $\alpha = 53.1°$ used (or more accurate)	
(i)	$x = (30\cos\alpha t) = 30 \times 0.6t = 18t$	E1	Use of horizontal component. Award if $30 \times 0.6t$ or $30\cos 53.1t$ seen	
	$s = 30\sin\alpha t - 5t^2$ above pt of projection	M1 A1	Use of correct vertical cpt with $\pm g$ All correct	
	so $y = 30\sin\alpha t - 5t^2(+1)$	A1	Subst of $\sin\alpha = 0.8$ or $\alpha = 53.1°$. Need not have + 1.	
	$= 30 \times 0.8t - 5t^2 + 1 = 24t - 5t^2 + 1$	E1	The '+ 1' made clear. Dependent on M1	5
(ii)	Taking $x = 27$, $t = \frac{27}{18} = 1.5$	B1		
	$y = 36 - 11.25 + 1 = 25.75$	M1	Subst **their** t in the **correct** expression for y	
	so 25.8 m (3 s. f.)	A1	cao. Accept any reasonable accuracy	3
	or Obtain correct cartesian equation	B1		
	Subst of $x = 27$ into their quadratic cartesian equation	M1		
	so 25.8 m (3 s. f.)	A1	cao. Accept any reasonable accuracy	
(iii)	Investigate vertical velocity cpt provided $1.5 \le t \le 2.4$	M1	Vertical component must be correct	
	+ ve so rising	E1	FT provided $t < 2.4$ and t chosen between **their** t and 2.4	2
	or			
	Evaluate y in $1.5 < t \le 3.3$	M1		
	comparison with their $y(1.5)$	E1	FT y in **'their** t**'** $< t < 4.8 -$ **'their** t**'**	
	or			
	Find time to greatest height	M1		
	Compare with $t = 1.5$	E1	FT compare with **their** t	

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Solutions and mark scheme

Q 4		Mark		
(iv)	**Either**			
	When stone is 17 m above the ground	M1	Equating **their** quadratic y to 17	
	$24t - 5t^2 + 1 = 17$	A1	Obtaining a correct quadratic equation, not necessary to re-arrange	
	so $5t^2 - 24t + 16 = 0$			
	thus $(5t-4)(t-4) = 0$	M1	Solving a quadratic in three terms	
	and $t = \frac{4}{5}$ or 4	A1	Obtaining the smaller root (condone only this root found)	
	As stone is still rising at the wall			
	$t = \frac{4}{5}$ and horizontal distance is $18 \times \frac{4}{5} = 14.4\text{ m}$	F1	FT **their** time dependent on 2nd M1 awarded [FT only if two positive roots found]	5
	or			Tot 15
	$y = 1 + \frac{4}{3}x - \frac{5x^2}{324}$ or equivalent	M2	Attempt to find cartesian eqn: $t = \frac{x}{18}$ and substitution to obtain a quadratic cartesian equation.	
		A1		
	$16 = \frac{4}{3}x - \frac{5x^2}{324}$ or equivalent	M1	(Dep on 1st M1) for subst $y = 17$ + attempt to solve quadratic with 3 terms	
	14.4 m	F1	**Their** smaller root [FT only if two positive roots found]	
	or			
	Find vertical velocity when 16 m above start point			
	$v^2 = 24^2 - 20 \times 16$	M1	Allow $\pm g$ or use of 17 as only errors	
	$v = +16$	A1		
	$16 = \frac{(16+24)t}{2}$	M1	Use of their v to find t. Correct signs. Vertical displacement must be 16 or 17.	
	$t = 0.8$	A1		
	distance is 14.4 m	F1		

Examiner’s Report

MEI 2607/1 (Mechanics 1) Summer 2002

General

This paper was found to be very accessible by a large number of candidates; the majority of these were able to demonstrate a good knowledge of the basic principles involved in the unit and many worked accurately through most of the paper. However, poor organisation and many algebraic and arithmetic errors spoiled the attempts of a substantial number of candidates. There was no evidence that candidates were under any serious time pressure.

Q1 (Velocity-time graph & non-uniform acceleration)

In part (i) a common error was to give the acceleration as $\frac{10}{4}$ m s^{-2}.

The calculation of the displacement in part (ii) caused some candidates problems simply because they failed to organize their work properly. All but a handful were aware of the need to find the area beneath the graph but elementary errors and double counting were quite commonly seen.

Candidates usually followed through correctly into part (iii) although some weaker candidates added the moduli of the two displacements.

Many candidates failed to be sufficiently precise in their answers to part (iv). They realised that the particle decelerated but did not state or even imply that this deceleration was constant. A large number of candidates recognised that the direction of motion of the particle changed when $t = 6$ although some thought this happened when $t = 4$ (presumably they thought this was so because the slope of the graph became negative at this time).

Part (v) was usually done very well. The majority of the candidates knew that integration was needed and performed this accurately. However, some candidates ignored the constant of integration and only substituted $t = 6$ into their expression for displacement. On this occasion they happened upon the correct answer simply because the initial displacement was zero. Weaker candidates either used the trapezium rule (an inefficient method, especially as the integral is so easily obtained) or assumed that the constant acceleration formulae remained valid.

[(i) $\frac{10}{3}$ m s^{-2}; (ii) 70 m; (iii) 65 m; (iv) 72 m]

Q2 (Newton's Second Law applied to a car & trailer)

Part (a) (with only a car) was done very well by the majority of candidates. Only a handful of them used the weight of the car in their equations and the only other common error was to omit the resistive force in part (ii).

In part (b)(i) the diagrams were very often poor. There were inconsistencies in the tension marked on the two bodies, some marked the 600N driving force as acting on the trailer as well as the car and others added extra forces to their diagrams. It was interesting to note that these wrong diagrams often did not affect some of the following work – it seems that some candidates have just learned a method for dealing with connected particles by heart!

In part (b)(ii) the majority of candidates were able to apply Newton's second law to the system and obtain the correct common acceleration. Calculation of the force in the coupling caused problems, however, either due to carelessness (the wrong masses used) or sign errors. Some candidates thought the weight of the car, trailer or both were involved and others wrote ' $F = 1500 \times 0.25$ ' and ' $F = 900 \times 0.25$ ' and thought the tension was the difference of these two forces. Despite such errors, it was quite common to see candidates score full marks.

The final part of the question was often poorly done. Many candidates failed to realise that there was a change in the acceleration (0.25 m s^{-2} was still used) and others thought the system must be moving with uniform speed. Some candidates obtained the correct answer by using a ratio method; this was acceptable provided the reasoning was sound. Some of those who correctly attempted to find R by first finding the new acceleration made sign errors, obtained $R = -500$ and then simply dropped the negative sign (and lost a mark).

[(a)(i) 750 N; (ii) 1.5 m s^{-2}; (b)(ii) 0.25 m s^{-2}, 225 N, tension; (iii) 500 ;]

Q3 (Analysis in vector form of a system of forces acting on a packing case)

Part (i) was done well although some candidates failed to make the direction of the line of action clear. One common error was to give the answer as –600N in the **i** direction, another was to state that the frictional force is *greater than* 600 N.

Part (ii)(A) was extremely well done with candidates showing a good understanding of resolution but in part (ii)(B), although many scored full marks, many others incorrectly added the two non-zero vertical components as if they were in the same direction.

The majority of candidates correctly calculated the magnitude and direction of their resultant force for part (iii).

Many candidates found part (iv) hard. Although they were able to write an expression for the total force using their answers from part (ii), some then wrote nonsense or equated the total force to 12 **i** + **j**, failing to realize one vector was a force and the other only a direction. There were various correct strategies employed and good candidates obtained the correct answer very quickly.

[(i) 600 N in the – **i** direction; (ii) 1230 N (3 s. f.), 96.4 N (3 s. f.); (iii) 1240 N (3 s. f.), 4.47° (3 s. f.); (iv) 14.7 (3 s. f.)]

Q 4 (Motion of a projectile fired from above ground level)

In part (i), those who wrote a vector equation of the form $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}t^2\mathbf{a}$, quickly and efficiently scored full marks. Some candidates, despite being given the values of cosα and sinα, found and used a value of α occasionally to an unsatisfactory degree of accuracy. Many candidates were able to show that $x = 18t$. Often marks were lost when attempting to show that $y = 1 + 24t - 5t^2$ because the last term was written without any indication of its derivation from the acceleration of -10 m s^{-2}. Some candidates also failed to justify why the first term was needed. However the majority of candidates who did well overall scored full or high marks here.

In part (ii), apart from those who took $y = 27$ or formed the equation for the vertical motion again but this time omitted the initial height, the vast majority of candidates scored full marks.

In part (iii) there were many correct solutions but also some attempts showed flawed logic. Some candidates considered the displacement of the stone at a time *before* it hit the wall, others found the greatest height without stating the time at which this occurred, others applied $v^2 = u^2 + 2as$ in an attempt to find the vertical component of velocity took the positive root and then claimed the stone must be rising!

Part (iv) was often done well with many candidates scoring full marks. One common error was to equate 17 to an incorrect expression for the vertical displacement (again this was often due to candidates forming the equation for the vertical motion yet again but without the initial height, ignoring the printed expression they had already correctly shown in the first part!). A worrying number of candidates, some of whom scored highly in the earlier questions, were unable to solve the quadratic equation correctly and many seemed not to know how to set about doing so. Without such basic techniques, candidates will find this unit hard. Some candidates spoiled an otherwise correct solution by giving *two* horizontal displacements corresponding to the two times found from their quadratic equation. These candidates disappointingly failed to appreciate that the later time was after the wall had been hit.

[(ii) 1.5, 25.8 m (3 s. f.); (iv) 14.4 m;]