OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2621/1

Decision and Discrete Mathematics 2

Thursday **20 JUNE 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- An **insert** is provided for use in Question **3**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages and an insert.

[Turn over

1 (i) Draw the switching circuit representing $(a \wedge (b \vee c)) \vee ({\sim}a \wedge b \wedge c)$. [4]

Fig. 1 shows a circuit for a voting machine for 3 people, A, B and C. Person A voting for a proposal is represented by a. Person A voting against the proposal is represented by $\sim a$.

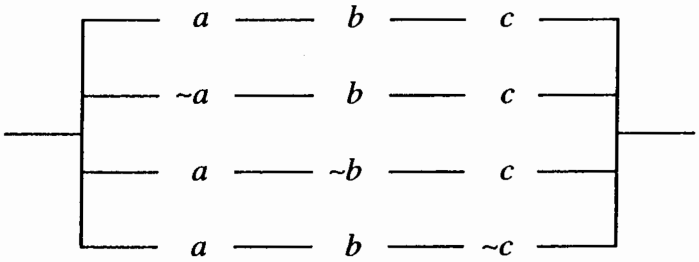

Fig. 1

(ii) Show that the expression in part **(i)** is equivalent to the voting machine in Fig. 1. [3]

(iii) Draw an equivalent circuit in which the symbols a, b and c are used, twice each, and in which the symbol ~ is not used. [2]

(iv) Draw a circuit for a voting machine in which each of A, B and C has a veto. [1]

[Total 10]

2 White, yellow, blue, green and red dyes are to be used separately in a dyeing vat (a container in which materials are dyed). Each colour is to be used once during each day. The vat has to be cleaned between colours, and the cost of this depends on which colour was previously used, and on which colour is going to be used. For example, if the blue dye was previously used and the yellow dye is going to be used next, the cost is 5. These costs, in suitable units, are shown in the table. The vat must be returned to its original colour at the end of the day.

		To				
		W	Y	B	G	R
From	W	–	0	2	1	2
	Y	4	–	4	3	4
	B	8	5	–	1	2
	G	7	3	1	–	3
	R	7	4	3	3	–

(i) Explain why this problem is similar to the travelling salesperson problem. [2]

(ii) Use the nearest neighbour algorithm five times, starting from each colour in turn, to find a low-cost sequence of colours. [6]

(iii) Give a colour sequence of cost 12, starting and ending with white. [1]

(iv) Because the network is directed, the technique of deleting a vertex and finding a minimum connector for the remainder to produce a lower bound for the cost will not work. Explain why not. [1]

[Total 10]

3 [There is an insert for use in this question.]

One of three similar types of new car, A, B or C, is to be purchased. The decision is to be made on the basis of annual service and repair costs. For each car a warranty can be purchased which insures against unexpected costs. Otherwise a chance can be taken on whether the particular car purchased turns out to be reliable or unreliable. The costs and probabilities are shown in the table below.

	Annual costs (£)			Probabilities	
	with warranty	reliable	unreliable	reliable	unreliable
A	1000	750	2100	$\frac{2}{3}$	$\frac{1}{3}$
B	1100	800	2000	$\frac{3}{4}$	$\frac{1}{4}$
C	1100	810	1800	$\frac{5}{6}$	$\frac{1}{6}$

(i) Complete the decision tree on Fig. 3.1 on the insert, and give the best decision, together with its EMV. [8]

An alternative is to buy a cheaper second-hand car. Annual service and repair costs of second-hand cars are higher, and warranties are more expensive. A free independent inspection of one car can be arranged. Approval by the inspector gives a good indication of the car being reliable. If the report is not favourable then a warranty will be purchased, fixing costs at £1150 per year. The relevant probabilities are summarised on the decision tree in Fig. 3.2 on the insert.

(ii) Complete the EMV calculations on the decision tree in Fig. 3.2 on the insert and give the best course of action and its EMV. [8]

(iii) For each type of car give the value of having an inspection. [2]

(iv) The cost of a warranty increases. To what value would the fixed cost of £1150 per year have to rise to change the decision in part **(ii)**? [2]

[Total 20]

4 A manufacturer of garden furniture produces chairs, round tables and square tables.

There must be at least 4 chairs produced for each table. At least 100 round tables and 80 square tables must be produced.

The costs of manufacture are £4 per chair, £10 per round table and £8 per square table.

(i) Using x, y and z to represent the numbers of chairs, round tables and square tables produced respectively, formulate as a linear program the problem of deciding how many of each item to produce at minimum cost. [5]

(ii) The initial tableau and the final tableau for a two-stage simplex solution to the LP are shown below. Explain the structure of the initial tableau, including the variables and the two objective functions. Interpret the final tableau. [10]

Initial tableau

Q	C	x	y	z	$s1$	$s2$	$s3$	$a2$	$a3$	RHS
1	0	0	1	1	0	–1	–1	0	0	180
0	1	–4	–10	–8	0	0	0	0	0	0
0	0	–1	4	4	1	0	0	0	0	0
0	0	0	1	0	0	–1	0	1	0	100
0	0	0	0	1	0	0	–1	0	1	80

Final tableau

Q	C	x	y	z	$s1$	$s2$	$s3$	$a2$	$a3$	RHS
1	0	0	0	0	0	0	0	–1	–1	0
0	1	0	0	0	–4	–26	–24	26	24	4520
0	0	1	0	0	–1	–4	–4	4	4	720
0	0	0	1	0	0	–1	0	1	0	100
0	0	0	0	1	0	0	–1	0	1	80

(iii) Chairs are sold to retailers at £8 each, round tables at £15 each and square tables at £12 each. Write down an expression in terms of x, y and z for the total profit. [1]

(iv) The manufacturer wishes to maximise the profit, P, while spending no more than £5000 on manufacturing costs. You are given that the tableau shown below takes the solution represented by the final tableau in part **(ii)** as the starting point for this problem. Apply the simplex algorithm to this tableau to find the most profitable production plan, pivoting on the $s1$ column. [4]

P	x	y	z	$s1$	$s2$	$s3$	$s4$	RHS
1	0	0	0	–4	–21	–20	0	3700
0	1	0	0	–1	–4	–4	0	720
0	0	1	0	0	–1	0	0	100
0	0	0	1	0	0	–1	0	80
0	0	0	0	4	26	24	1	480

[Total 20]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS 2621/1

Decision and Discrete Mathematics 2

INSERT

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INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **3** parts **(i)** and **(ii)**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 3 printed pages and 1 blank page.

[Turn over

3 (i)

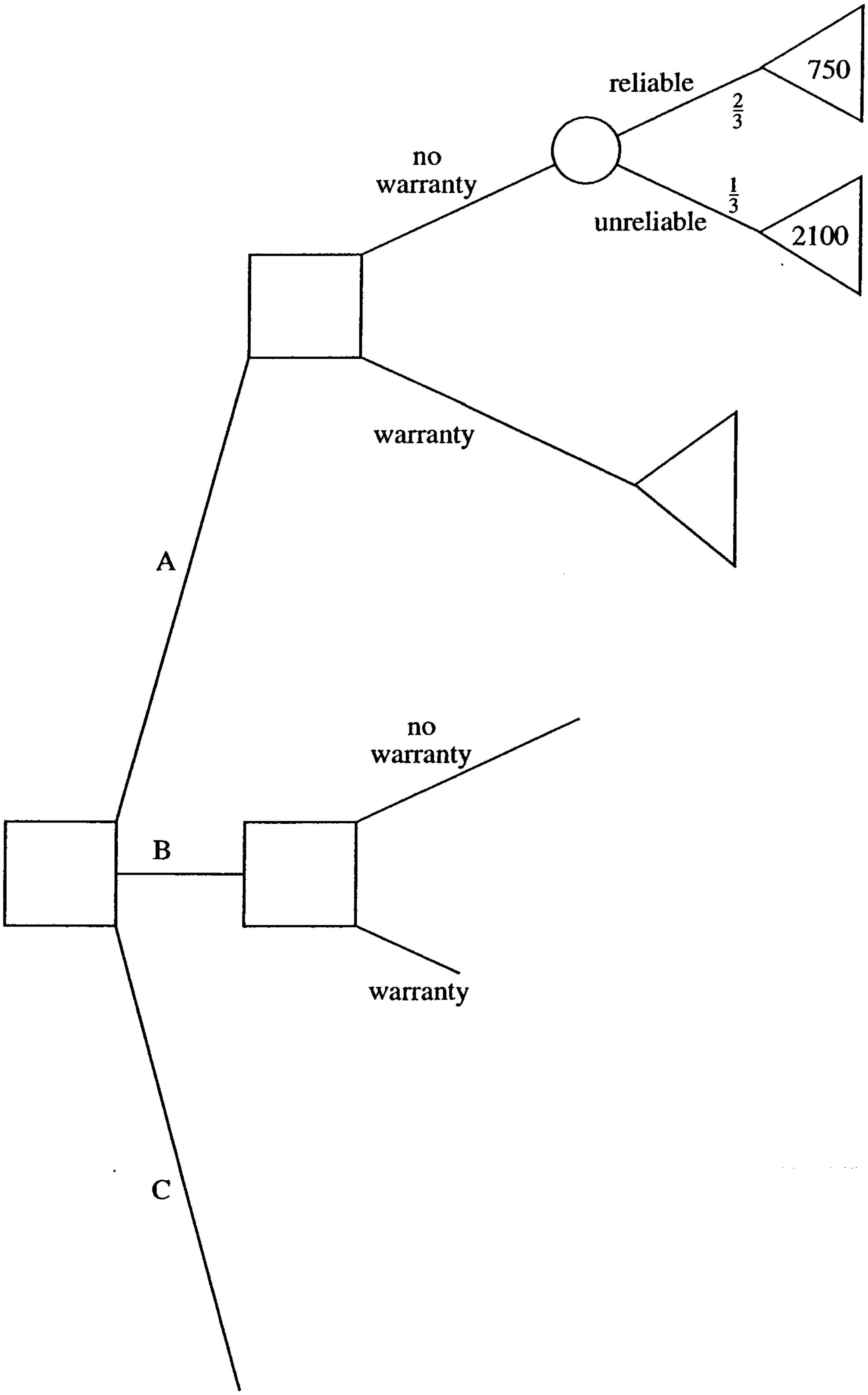
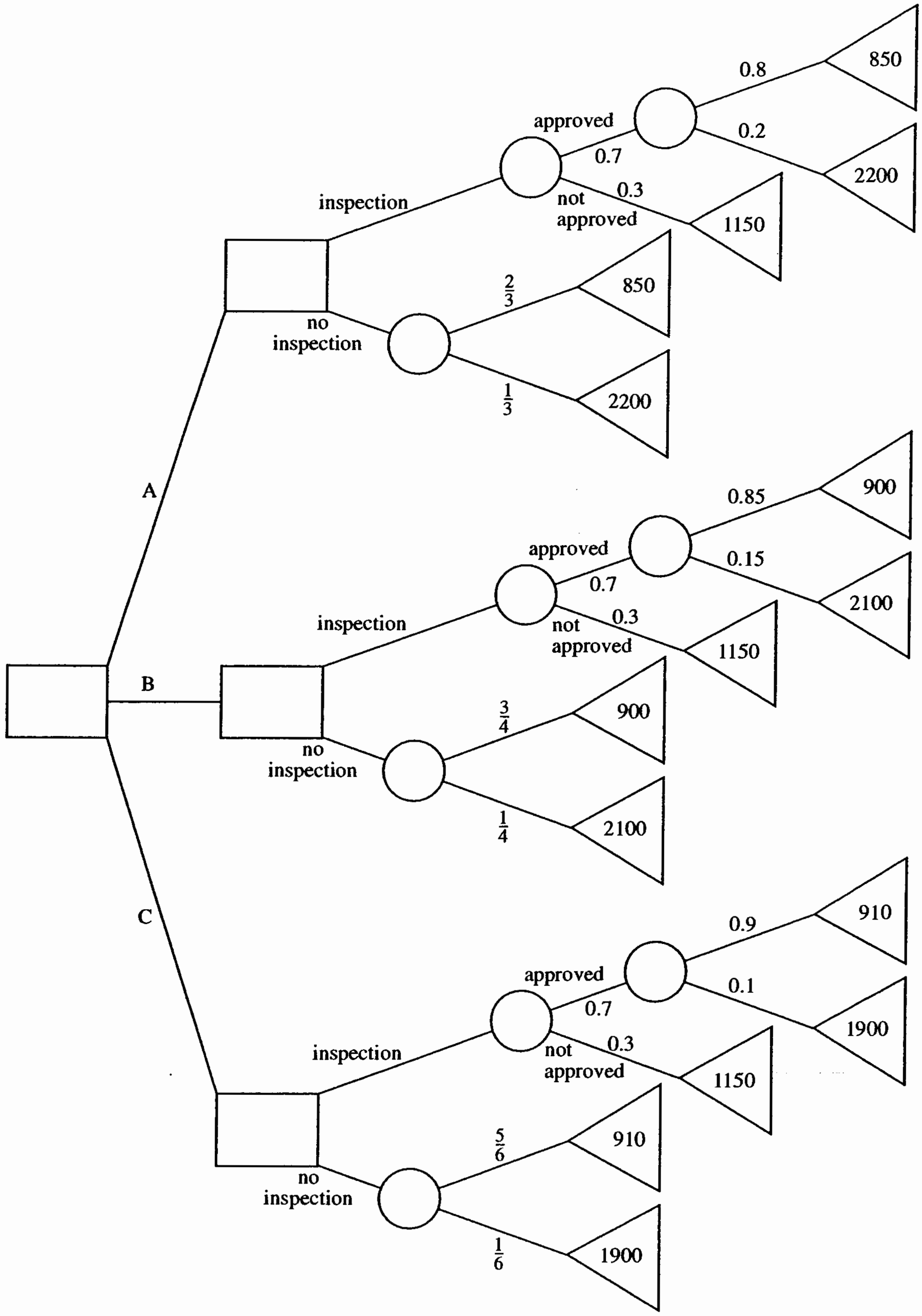

reliable
750
2/3
no
warranty
1/3
unreliable
2100
warranty
A
no
warranty
B
warranty
C

Fig. 3.1

3 (ii)

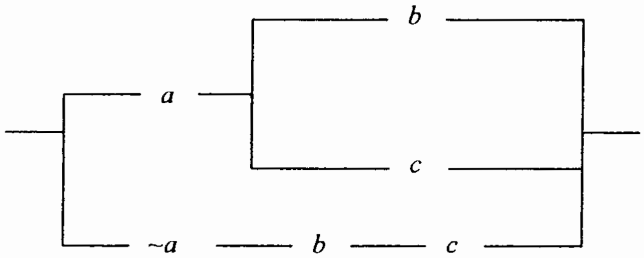
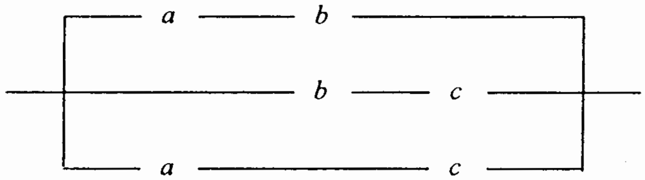
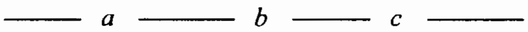


A
B
C
inspection
no inspection
approved
not approved
0.7
0.3
0.8
0.2
850
2200
1150
2/3
1/3
0.85
0.15
900
2100
3/4
1/4
0.9
0.1
910
1900
5/6
1/6

Fig. 3.2

Mark Scheme

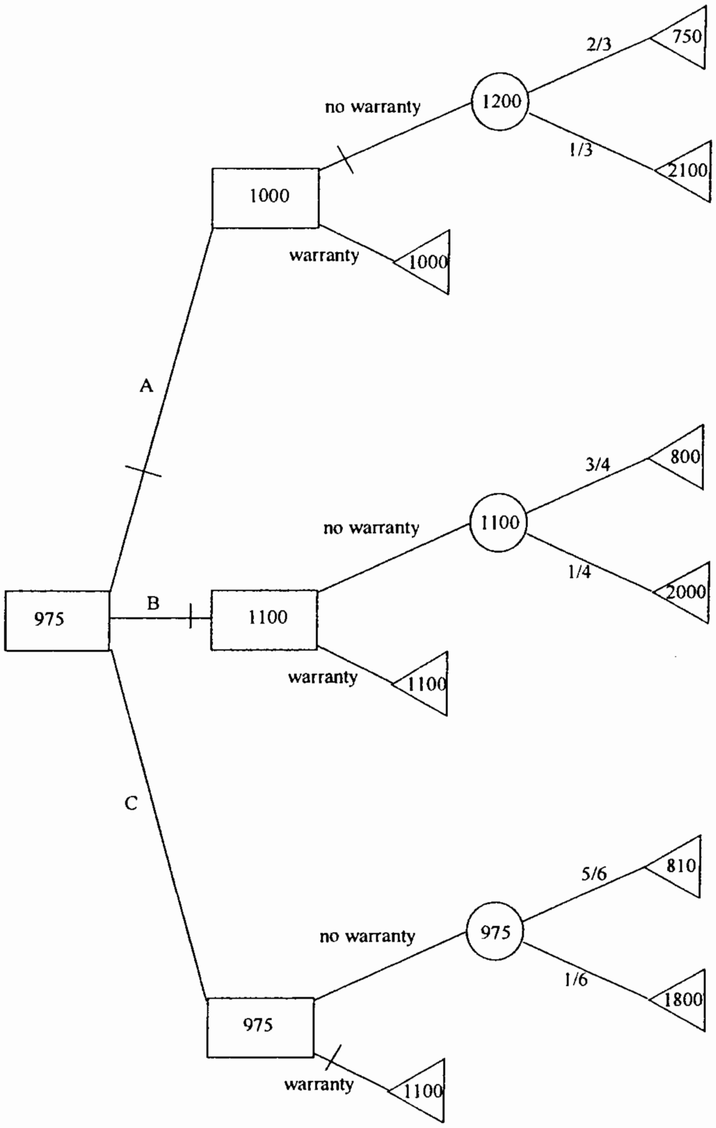
1.

(i) 	M1 switching circuit A2 $a \wedge (b \vee c)$ A1 $\sim a \wedge b \wedge c$
(ii) Argument about majority voting, or tables of outcomes/truth tables, or Boolean algebra	M1 A2 (1 for circuit and 1 for expression)
(iii) 	M1 A1
(iv) 	B1

2.

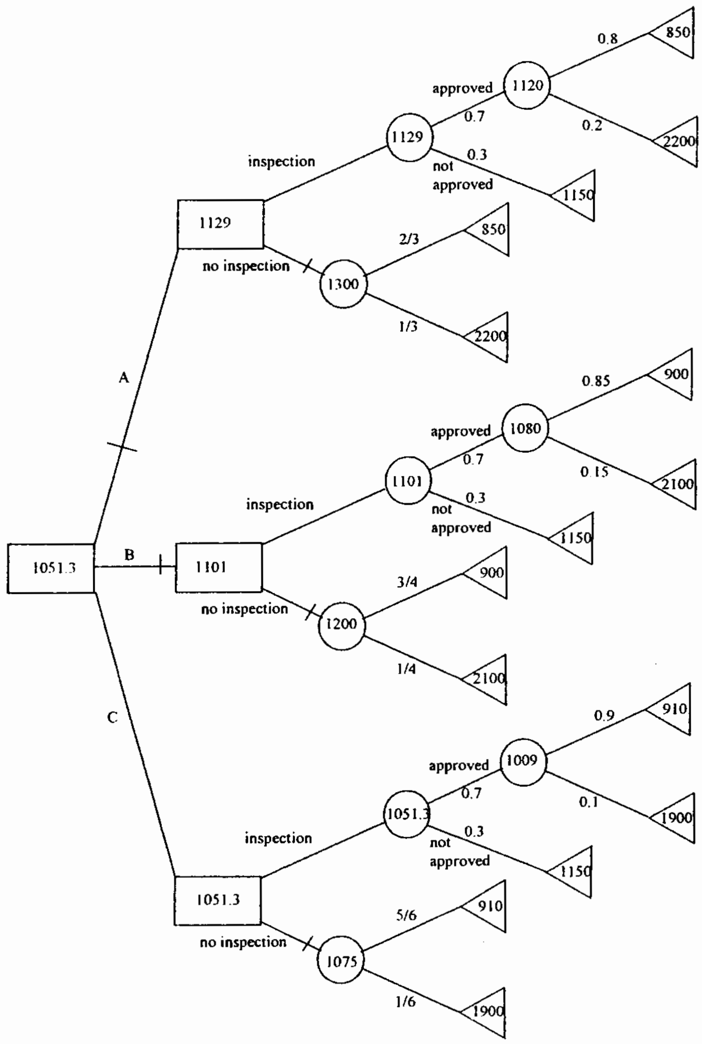
(i) town = colour cost of travel = cost of cleaning visiting town = using colour	B1 B1 B1
(ii) WYGBRW – 13 YGBRWY – 13 BGYRWB – 17 or BGYWRB – 13 or BGRYWB – 14 GBRYWG – 12 RBGYWR – 13 or RGBYWR – 15	B1 B1 B1 B1 B1
(iii) GBRYWG = WGBRYW	B1
(iv) digraph →connectedness ?	B1

3.

(i)	
 	B1 1200 B1 1000 M1 B branch A1 M1 C branch A1
Choose a car of type C and do not buy a warranty. EMV = 975	B1 B1

3 (cont).

(ii)



	M1 handling chance nodes A2 (−1 each error)
	M1 handling decision nodes A2 (−1 each error)
Choose a second-hand car of type C, and have it inspected. EMV = £1051.30	B1 B1
(iii) A: £171 B: £99 C: £23.70	M1 A1
(iv) Require x st 0.7*1009 + 0.3*x = 1075, giving x = 1229	M1 A1

4.

(i) Min $4x + 10y + 8z$ s.t. $x \geq 4(y + z)$ $y \geq 100$ $z \geq 80$	M1 A1 A1 A1 A1
(ii) First constraint has been written as $-x + 4y + 4z \leq 0$ using slack s1.	B1
Second constraint is $y \geq 100$ using surplus s2 and artificial a2.	B1
Third constraint is $z \geq 80$ using surplus s3 and artificial a3.	B1
Q is sum of artificial variables = a2 + a3	B1
Q row has been re-written using $y - s2 + a2 = 100$ $z - s3 + a3 = 80$	B1
C is cost = $4x + 10y + 8z$	B1
Final tableau shows x = 720, y = 100 and z = 80, with cost = 4520. This is feasible since Q = 0. (It is also optimal since there are no non-artificial positive numbers in the C row.)	M1 A1 B1 B1
(iii) $4x + 5y + 4z$	B1
(iv) (tableau below)	M1 A2 (–1 each error)
Produce 840 chairs, 100 round tables and 80 square tables.	B1

P	x	y	z	s1	s2	s3	s4	RHS
1	0	0	0	0	5	4	1	4180
0	1	0	0	0	2.5	2	0.25	840
0	0	1	0	0	–1	0	0	100
0	0	0	1	0	0	–1	0	80
0	0	0	0	1	6.5	6	0.25	120

Examiner’s Report

2621 Decision and Discrete Mathematics 2

General Comments

Most candidates seemed to enjoy this paper. It was perhaps less difficult than its predecessor, largely due to the decision analysis question (question 3) which candidates found to be easy. Against that, question 1 was found to be difficult, and it was perhaps that which led to some candidates running short of time.

There were a few problems relating to the handing-in of insert sheets. Centres should ensure that invigilators are aware of the existence of inserts, and of the importance of collecting them with the scripts.

Comments on Individual Questions

Q.1 Parts (i), (ii) and (iii) were about majority voting, but candidates were not able to step back, identify this, and use it.

Most were comfortable with part (i), although some combinatorial circuits were seen.

Few candidates who attempted to use Boolean algebra in part (ii) were able to develop a purposeful attack. Those fewer candidates who used truth tables tended to fare better. The question said "show", not "prove", and arguments about outcomes (viz. majority voting) would have been acceptable. A few such arguments were seen.

There were some "imaginative" answers to part (iii), and it seems that a few candidates were stymied in part (iv) by not knowing what was meant by a veto.

Q.2 A very large number of candidates failed to score well in part (i). The most common approach was to describe in detail the travelling salesperson problem, without relating it to the problem which was specified in the question (towns↔colours; visiting↔using; distance↔cost of cleaning; etc.)

Parts (ii) and (iii) were done well. Part (iv) was intentionally difficult, and only the best candidates were able to be sufficiently incisive to score the mark.

(ii) WYGBRW; YGBRWY; BGYRWB or BGYWRB or BGRYWB; GBRYWG;
RBGYWR or RGBYWR
(iii) WGBRYW

Q.3 Most candidates found this question to be straightforward, and good marks were scored. There were some centres where candidates did not fill in their EMVs in the chance nodes and/or in the decision nodes, and this often made it difficult to follow through on their solutions.

(i) Choose a type C car and do not buy a warranty; EMV = £975
(ii) Choose a second-hand type C, and have it inspected; EMV = £1051.30
(iii) A: £171; B: £99; C: £23.70
(iv) £1229

Q.4 Not all candidates were able to score all 5 marks in part (i). The $x \geq 4(y+z)$ inequality caused most of the difficulties.

As in Q.2 part (i), some candidates failed to relate their explanations in part (ii) to the question in hand. They described how to do two-phase Simplex, but not how that related to each line of the given tableau. In interpreting the final tableau many failed to note that it represented a solution which was feasible.

Surprisingly few candidates scored the mark for part (iii).

Part (iv) was quite well done, although, as was mentioned in the preamble to this report, some ran out of time.

(iii) $4x + 5y + 4z$