OCR
RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2613

Statistics 1

Wednesday **16 JANUARY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 3 printed pages and 1 blank page.

1 George records the time he spends per day surfing the internet for the first three weeks of May. The times, given to the nearest minute, are as follows.

0	26	13	5	18	12	35
24	61	16	10	26	15	0
0	73	21	17	16	42	32

(i) Illustrate the data using a sorted stem and leaf diagram with eight stems. Comment briefly on the shape of the distribution. [3]

(ii) Find the mode, median and mean, commenting on their relative usefulness as measures of central tendency for this data set. [5]

(iii) Calculate the standard deviation and hence find any outliers. [4]

(iv) George's Dad claims that he is spending too much time on the internet. He tells George to reduce his usage so that the mean daily time for May is 20% less than the current mean.

What is the maximum total time George can spend surfing the internet for the remaining 10 days of May? [3]

2 Dick and Kate are taking part in a *Head to Head* final of a quiz show. Each contestant is asked a question in turn, starting with Dick. One point is awarded for a correct answer.

The probability that Dick gets a question right is 0.6 and the probability that Kate gets a question right is 0.7, both independently of other questions answered.

(i) Find the probability that

(*A*) Dick and Kate both get their first questions correct, [2]

(*B*) both Dick and Kate have no points after two questions each. [2]

(ii) Show that the probability that Dick and Kate have one point each after two questions each is 0.2016. [3]

(iii) Kate and Dick have answered two questions each. By considering the possible scores, find the probability that Kate is now exactly one point ahead of Dick. [4]

(iv) Find the probability that Kate is exactly two points ahead of Dick after they have answered three questions each. [4]

3 During a general election in the United Kingdom, a national TV company wishes to undertake a sample survey of the electorate.

(i) Describe the method of cluster sampling. [3]

(ii) Explain briefly why cluster sampling might be appropriate. [1]

An exit poll is conducted in a constituency. [An exit poll asks a sample of voters, immediately after they have voted, which party they voted for.]

(iii) Explain why an exit poll will not in practice be a simple random sample of all those who voted in this constituency. [2]

In another constituency, the proportions of votes actually cast were as follows.

Conservative	34%
Labour	43%
Liberal Democrat	15%
Green	8%

(iv) Three voters from this constituency are chosen at random and contacted by telephone to ask which party they voted for. Find the probability that

(*A*) all three voted for the same party, [2]

(*B*) each voted for a different party. [4]

(v) How many voters from this constituency would you need to sample to be 95% sure of obtaining at least one Liberal Democrat voter? [3]

4 In a national survey, it was found that 40% of young people failed a standard fitness test.

A random sample of 30 young people is taken.

(i) Explain why, for such random samples, the mean number of those who failed the fitness test is 12. [1]

(ii) Find the probability of the mean number occurring. Explain how you would check whether the mean is also the modal value. *No further calculations are necessary.* [4]

A local health authority wishes to investigate whether or not the proportion of young people in its area who failed the fitness test is 40%. A random sample of 20 young people contains 12 who failed the test.

(iii) Carry out a suitable hypothesis test at the 5% significance level, stating your hypotheses and conclusions carefully. Find the critical region for the test. [10]

Mark Scheme

January 2002 2613 MEI Statistics 1

Question 1

(i)	0 | 0 0 0 5 10 | 0 2 3 5 6 6 7 8 20 | 1 4 6 6 30 | 2 5 40 | 2 50 | 60 | 1 70 | 3 *Key:* 30 | 2 means 32 minutes Distribution has positive skewness	G1 for suitable stem (including 50) G1 for sorted leaves and alignment E1 for comment	**3**
(ii)	Mode = 0 (most frequent value) Median = 17 (middle value) Mean $\bar{x} = \frac{462}{21} = 22$ • Mode certainly not representative • Mean affected by extreme values (61 and 73) • Median best representative measure	B1 for mode B1 for median B1 for mean E1 for realizing mode is unsuitable / not useful E1 for realizing mean / median is useful	**5**
(iii)	s.d. $= \sqrt{\frac{17220}{21} - 22^2} = 18.3$ (3 sf) Mean + 2 s.d. = 22 + 2 x 18.3 = 58.6 Hence 61 and 73 are outliers	M1 for variance A1 (inc. s.d.) **cao** M1 for mean + 2 s.d. A1	**4**
(iv)	Reduction of current mean (22) by 20% gives 17.6 Max. May total = 17.6 × 31 = 545.6 Max. time for rest of May = 545.6 – 462 = 83.6 min *or* 8.36 minutes per day	B1 for sight of 17.6 M1 for May total A1 **cao**	**3**
			15

Question 2

(i)	(*A*) P(both get first questions right) $= 0.6 \times 0.7 = 0.42$ (*B*) P(both have 0 points after 2 questions each) $= 0.4 \times 0.3 \times 0.4 \times 0.3 = 0.0144$	M1 for product A1 **cao** M1 for quadruple prod. A1 **cao**	**4**
(ii)	P(both have one point each after two questions each) $= 2 \times 0.6 \times 0.4 \times 2 \times 0.7 \times 0.3$ $= 0.2016$	M1 for product of probs. M1 for "$2 \times 2 \times \ldots$" A1 **cao** *(answer given)*	**3**
(iii)	Kate is 1 point ahead if she leads (a) 1 – 0 or (b) 2 – 1 (a) P(Kate is 1 – 0 ahead) $= [2 \times 0.7 \times 0.3] \times 0.4^2 = 0.0672$ (b) P(Kate is 2 – 1 ahead) $= 0.7^2 \times [2 \times 0.6 \times 0.4] = 0.2352$ hence P(Kate leads Dick by 1 point) $= 0.0672 + 0.2352$ $= 0.3024$ *or* 0.302 (to 3 s.f.) *or* 0.30 (to 2 s.f.)	 B1 for $0.7 \times 0.3 \times 0.4^2$ B1 for $0.7^2 \times 0.6 \times 0.4$ M1 for both factors of 2 A1 **cao**	**4**
(iv)	Kate is 2 points ahead if she leads (a) 2 – 0 or (b) 3 – 1 (a) P(Kate is 2 – 0 ahead) $= [3 \times 0.7^2 \times 0.3] \times 0.4^3 = 0.028224$ (b) P(Kate is 3 – 1 ahead) $= 0.7^3 \times [3 \times 0.6 \times 0.4^2] = 0.098784$ hence P(Kate leads Dick by 2 points) $= 0.028224 + 0.098784$ $= 0.127$ (to 3 s.f.) *or* 0.13 (to 2 s.f.)	 B1 for $0.7^2 \times 0.3 \times 0.4^3$ B1 for $0.7^3 \times 0.6 \times 0.4^2$ M1 for both factors of 3 A1 **cao**	**4**
			15

Question 3

(i)	Cluster sampling consists of • dividing [the population] into suitable subgroups • selecting some of these subgroups for study • sampling within the selected subgroup(s)	E1 for division into *subgroups* E1 for *selection* E1 for *sampling*	**3**
(ii)	Voting patterns in the selected clusters (hopefully!) reflect the voting patterns of the nation. *Also allow any one of:* representative, ease, comparisons, cannot perform random sample.	E1 for explanation	**1**
(iii)	For a simple random sample, a sampling frame is required of all those who voted. Refused to answer / not telling the truth / reference to time of day or postal vote.	E1 for *sampling frame* E1 for sensible comment	**2**
(iv)	(*A*) P(all voted for same party) $= 0.34^3 + 0.43^3 + 0.15^3 + 0.08^3 = 0.123$ (to 3 s.f.) (*B*) P(each voted for a different party) $= 6 \times 0.34 \times 0.43 \times 0.15$ $+ 6 \times 0.34 \times 0.43 \times 0.08$ $+ 6 \times 0.34 \times 0.15 \times 0.08$ $+ 6 \times 0.43 \times 0.15 \times 0.08$ $= 0.257$ (3 s.f.)	M1 for sum of correct cubes A1 **cao** M1 for product of any one triplet of probabilities M1 for "6 × any triplet" M1 for sum of 4 products A1 **cao**	**2** **4**
(v)	Require n such that: $P(X \geq 1) \geq 0.95$ $\Rightarrow$ $P(X = 0) \leq 0.05$ Using tables *or* trial and improvement: $n = 18$: $P(X = 0) = 0.0536$ $[= 0.85^{18}]$ $n = 19$: $P(X = 0) = 0.0456$ $[= 0.85^{19}]$ $\Rightarrow$ Minimum value of n is 19 Using logarithms: $0.85^n \leq 0.05$ $\Rightarrow$ $n \log 0.85 \geq \log 0.05$ $\Rightarrow$ $n \leq 18.43\ldots$ $\Rightarrow$ Minimum value of n is 19	M1 for 0.85 (SOI) from use of binomial tables M1 for a valid method using 0.85 and either tables, trial and improvement or logarithms A1 for $n = 19$ or $n \geq 19$ **cao**	**3**
			15

Question 4

(i)	Let $X \sim B(30, 0.4)$ Mean = np = 30 × 0.4 *or* 40% of 30 [= 12]	B1 for mean value *[may be seen in part (ii)]*	**1**
(ii)	$P(X = 12) = {}^{30}C_{12} \times 0.4^{12} \times 0.6^{18}$ = 0.147 (to 3 s.f.) *or* 0.15 (to 2 s.f.) Compare $P(X = 12)$ with $P(X = 11)$ and $P(X = 13)$ *or equivalent*	M1 for $0.4^{12} \times 0.6^{18}$ M1 for ${}^{30}C_{12} \times \ldots$ A1 **cao** E1 for convincing explanation	**4**
(iii)	[Let p represent the probability that a young person chosen at random fails the fitness test and Let $X \sim B(20, p)$] H_0: $p = 0.4$ H_1: $p \neq 0.4$ (i.e. two-tail test) $P(X \geq 12) = 1 - P(X < 12)$ $= 1 - 0.9435 = 0.0565$ Since 0.0565 > 0.025, there is not enough evidence to reject H_0, i.e. accept the hypothesis that the proportion of young people who fail the fitness test is 40% The critical region is the set of x-values for which we reject H_0 : *Using tables:* Lower tail: $P(X \leq 3) = 0.0160 < 0.025$ $P(X \leq 4) = 0.0510 > 0.025$ Upper tail: $P(X \geq 12) = 0.0565 > 0.025$ $P(X \geq 13) = 0.0210 < 0.025$ *Critical region:* {0, 1, 2, 3, 13, 14, 15, 16, 17, 18, 19, 20}	B1 for H_0 B1 for H_1 B1 for $P(X \geq 12)$ B1 for 0.0565 **cao** M1 for comparison A1 for conclusion in words in context B1 for 0.0160 & 0.0510 B1 for 0.0565 & 0.0210 B1 for lower tail B1 for upper tail	**2** **4** **4**
			15

Examiner’s Report

Statistics 1 (2613)

General Comments

The overall performance of candidates was better than last summer, but on a par with the last January sitting. There was more evidence of genuine knowledge and understanding. Generally, questions were attempted sensibly with appropriate evidence of working and method. There were fewer scripts than usual scoring 50+ but also less than usual in single figures.

Very pleasing work was seen in questions 1 (Data handling) and 2 (Probability). The main areas of weakness occurred in question 3 (Cluster sampling) and question 4 (Binomial distribution: the two-tailed hypothesis test - particularly with the upper tail). There was widespread misunderstanding of how to find the correct upper tail for the critical or rejection region.

Comments on Individual Questions

Question 1 (Data analysis; stem and leaf diagram, measures of central tendency and standard deviation; times spent on the internet)

A good starting question for most candidates. All parts except the last were well answered. Some candidates did not understand the phrase 'relative usefulness'.

(i) The stem and leaf diagram and the identification of positive skewness were generally well done. Th main source of error was the gross misalignment of the leaves. There were fewer references to 'rigl skew' than in the past.

(ii) Many candidates did well here, with a sizeable number scoring all 5 marks. A common error was tl mode found as '16 and 26', believing that the three zeroes in the times did not count. The on common error on the median was '16.5' due to $\frac{n}{2}$ instead of $\frac{n+1}{2}$ being used. Most candidates wer able to gain credit for their comments about the relative usefulness of the measures of central tenden but there were many unwanted definitions seen or comments made out of context of the data.

(iii) The standard deviation was well attempted by the majority of candidates. Quite a few showed r working at all, relying on their calculator to process the data. Most did well when finding the outlier Common errors included using 1, 1.5, 2.5, 3 standard deviations from the mean in the definition of outlier.

(iv) This part was poorly answered. The most common response was '80% of 22 = 17.6 followed l $17.6 \times 10 = 176$'. Some candidates believed there were 30 or even 29 days in May, which caus further problems.

[(i) sorted stem and leaf diagram; (ii) mode = 0, median = 17, mean = 22; comments; (iii) s.d. = 18.3; outliers: 61 and 73; (iv) 83.6 minutes in total]

Question 2 (Probability; combining probabilities by multiplication and addition; Dick and Kate *Head to Head*)

A full range of marks was seen. Some candidates who struggled at the start seemed to get a helpful bo from part (ii), which enabled them to continue successfully with the rest of the question. For many of t abler candidates this was their best question, with frequent scores of full marks.

(i) Part (*A*) was answered correctly by nearly all candidates. Part (*B*) was not so well answered with 0. + 0.09 = 0.25 being a common error.

(ii) Most candidates found the probability correctly, no doubt helped by the given printed answer. M justified 0.2016 with 0.48×0.42.

(iii) Most candidates scored at least 2 marks; many achieved all the marks. The omission of the factor o was the most common error. Another common error in the weaker scripts was to add all of Kat possible scores first, and the same with Dick's and then multiply at the end i.e. $(0.7^2 + 0.3 \times 0.7)$ $(0.4^2 + 0.4 \times 0.6)$.

(iv) Those who succeeded with part (iii) invariably were correct in part (iv). The most common error v the omission of the factor of 3.

[(i) (*A*) 0.42, (*B*) 0.0144; (ii) 0.2016; (iii) 0.3024; (iv) 0.127 (to 3 s.f.)]

Question 3 (Sampling and probability; analysis of voting patterns)

This was a poorly answered question. Hardly any candidate was able to score full marks.

(i) The definition of cluster sampling was very badly done. Many confused it with stratified sampl Some had clusters based on age, political parties. Many candidates preferred to discuss examples t had met before – especially penguin or puffin colonies, all of which were out of context of question.

(ii) Often a correct answer appeared amongst other incorrect statements. Many gave reasons why o kinds of sampling were inappropriate instead of why cluster sampling was.

(iii) Only a very few candidates mentioned the need for a sampling frame before a random sample could be taken. Many gained partial credit for responses about not telling the truth/refusing to answer/postal votes. A large number seemed confused about the nature and purpose of an exit poll with some believing that it should relate to the whole of the country and not to just a constituency. Perhaps some were unfamiliar with the word constituency.

(iv) Part (*A*) was well answered. In a small, but important, minority 8% was converted to 0.8 instead of 0.08. In part (*B*) the most common error was the omission of the × 6 (i.e. 3! permutations). Most were able to score 2 of the 4 available marks here.

(v) There were very few correct answers. Only a small number of better candidates made significant progress. Those who did usually used trial and improvement or tables to give 19. Most did not appreciate the use of 0.85.

[(i) description of cluster sampling; (ii) comment; (iii) comment;
(iv) (*A*) 0.123 (to 3 s.f.), (*B*) 0.257 (to 3 s.f.); (v) 19]

Question 4 (Binomial distribution and hypothesis testing; fitness testing of young people)

A full spectrum of marks was seen. This remains a challenging topic for many candidates and many misconceptions abound.

(i) The majority of candidates could explain the derivation of the mean value.

(ii) The probability of the mean was well answered by most. Many were unable to give a correct response as to why the mean could be the modal value. Many thought it involved repeat sampling rather than checking the probabilities either side of $P(X = 12)$. Similar questions have been set before but this seemed unfamiliar to many candidates during this session.

(iii) The two-tail test was not well understood. Failure to split the 5% significance level into two lots of 2.5%, one for each tail, was common. The upper tail, more often than not, was 'one out'. Many (even the very ablest) did not put their conclusion in context. The lower critical region was well attempted but the omission of 0 was the most likely reason for losing marks. The upper region was usually wrong at {12, 13, ..., 20} for the same reasons as before.

[(i) 30×0.4; (ii) 0.147, comment; (iii) $H_0: p = 0.4$, $H_1: p \neq 0.4$; $P(X \geq 12) = 0.0565 > 0.025$, hence accept H_0; critical region = {0, 1, 2, 3, 13, 14, 15, 16, 17, 18, 19, 20}]

Statistics 2 (2614)

General Comments

The overall standard was slightly better than in recent examinations, with a larger than usual number of candidates achieving marks above 45 and rather less weak candidates than normal. Most candidates were able to achieve a good degree of success in each question, with better attempts at questions on the Normal distribution and Poisson distribution (Questions 2 and 3) than in recent examinations. As ever it was the discussion and interpretation questions which proved the stumbling block for even the most successful candidates, who frequently were only able to score one or two of the five such marks in Question 1, despite losing only two or three marks in the whole of the rest of the paper. Premature approximation was occasionally seen, leading to unnecessary loss of marks. In particular, candidates should be advised against rounding of z-values to 2 decimal places, prior to looking them up in the tables.