OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Friday 18 JANUARY 2002 Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 3 printed pages and 1 blank page.

[Turn over

1 **(a)** Find $\int_0^{\frac{1}{2}\pi} x\sin x\,dx$. [4]

(b) Differentiate $\sin^3 2x$ with respect to x. [3]

(c) Express $2\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where α is in degrees. Give the values of R and α correct to 1 decimal place. [3]

(d) Write down small-angle approximations for $\sin h$ and $\cos h$. Hence show that, for small values of h,

$$\frac{\sin(x+h)-\sin x}{h} \approx \cos x - \tfrac{1}{2}h\sin x.$$

What does this result suggest as $h \to 0$? [5]

[Total 15]

2 **(i)** Express $\dfrac{1-x}{(1+x)(1+x^2)}$ in the form $\dfrac{A}{1+x}+\dfrac{Bx+C}{1+x^2}$. [4]

(ii) Hence show that the solution of the differential equation

$$\frac{dy}{dx}=\frac{y(1-x)}{(1+x)(1+x^2)},$$

given that $y = 1$ when $x = 0$, is

$$y=\frac{1+x}{\sqrt{1+x^2}}.$$ [7]

(iii) Find the first three terms of the binomial expansion of $\dfrac{1}{\sqrt{1+x^2}}$. Hence find a polynomial approximation for $y=\dfrac{1+x}{\sqrt{1+x^2}}$. up to the term in x^5. [4]

[Total 15]

3 With respect to coordinate axes Oxyz, A is the point (2, 0, 0), B is (0, 0, 1) and C is (3, 1, 3).

(i) Find the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$. Hence find angle ACB. [4]

(ii) Write down the cartesian equation of the plane p through A with normal vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

Verify that B also lies in this plane. [3]

(iii) Write down the vector equation of the line through C perpendicular to the plane p.

Find the point of intersection of this line with the plane, and the distance from C to the plane. [7]

[Total 14]

4 Fig. 4 shows a sketch of the curve with equation $y^2 = (1 - 2x)^3$. The curve meets the x-axis at A and crosses the y-axis at the points B and C.

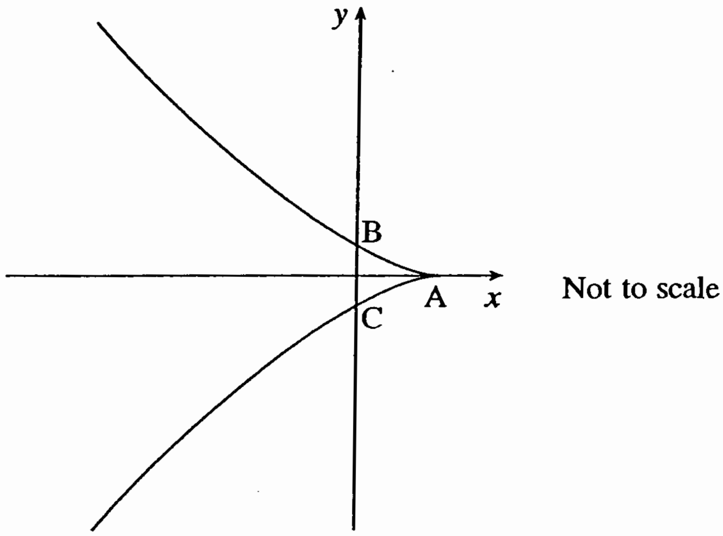

Fig. 4

(i) Find the coordinates of the points A, B and C. [3]

(ii) Show that the gradient of the curve at the point B is –3. [5]

(iii) Verify that

$$x = \tfrac{1}{2}\left(1 - t^2\right), \qquad y = t^3$$

are parametric equations for the curve.

Find $\dfrac{dy}{dx}$ in terms of t, and show that the equation of the tangent to the curve at the point with parameter t is

$$6tx + 2y + t^3 - 3t = 0.$$ [8]

[Total 16]

Mark Scheme

1(a) $\int_0^{\pi/2} x\sin x\,dx = \int_0^{\pi/2} x\frac{d}{dx}(-\cos x)\,dx$ $= [-x\cos x]_0^{\pi/2} + \int_0^{\pi/2}\cos x\,dx$ $= 0 + [\sin x]_0^{\pi/2}$ $= 1$	M1 A1 A1ft A1 [4]	Parts: $u = x$, $\frac{dv}{dx} = \sin x$ +some attempt to integrate sinx $[-x\cos x]$...... ...+$[\sin x]_0^{\pi/2}$ ft their v w.w.w.
(b) $y = \sin^3 2x$ let $u = \sin 2x$, $\Rightarrow y = u^3$ $\Rightarrow \frac{dy}{du} = 3u^2, \frac{du}{dx} = 2\cos 2x$ $\Rightarrow \frac{dy}{dx} = 3u^2.2\cos 2x$ $= 6\sin^2 2x\cos 2x$ Or use of double angle formula and product rule for differentiation	M1 DM1 A1cao M1 A1 A1cao [3]	chain rule. Attempt at $\frac{dy}{du}$ and $\frac{du}{dx}$ 2cos $2x$ or cos $2x$ Special case SC B1 for 6 $\sin^2 2x$ Use of formula for cos $4x$ and product rule Correct application of both Any correct expression
(c) $2\sin x - 3\cos x \equiv R\sin(x - \alpha)$ $\Rightarrow 2\sin x - 3\cos x \equiv R(\sin x\cos\alpha - \cos x\sin\alpha)$ $\Rightarrow R\cos\alpha = 2$, $R\sin\alpha = 3$, $\Rightarrow R^2 = 2^2 + 3^2 = 13$, $R = \sqrt{13} = 3.6$ $\tan\alpha = 3/2$, $\Rightarrow \alpha = 56.3°$	M1 B1 A1 [3]	The identity s.o.i. (not implied by $\tan\alpha = \frac{2}{3}$ or similar.) Accept $\sqrt{13}$ Accept 0.983 radians
(d) $\sin h \approx h$, $\cos h \approx 1 - \frac{1}{2}h^2$ $[\sin(x + h) - \sin x]/h$ $= [\sin x\cos h + \cos x\sin h - \sin x]/h$ $\approx [\sin x(1 - \frac{1}{2}h^2) + \cos x.h - \sin x]/h$ $= [\sin x - \frac{1}{2}h^2\sin x + h\cos x - \sin x]/h$ $= \cos x - \frac{1}{2}h\sin x$ As $h \to 0$, $\cos x - \frac{1}{2}h\sin x \to \cos x$ So the derivative of $\sin x$ is $\cos x$.	B1 M1 M1 E1 B1	Both correct Use of compound angle formula substituting small angle approximations for sin h and cos h www

	[5] [15]	

2 (i) $\frac{1-x}{(1+x)(1+x^2)} \equiv \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$ $\Rightarrow 1-x \equiv A(1+x^2)+(Bx+C)(1+x)$ $x=-1 \Rightarrow 2=2A, A=1$ constants: $1=A+C \Rightarrow C=0$ coeffτ of x^2: $0=A+B \Rightarrow B=-1$ so $\frac{1-x}{(1+x)(1+x^2)} \equiv \frac{1}{1+x} - \frac{x}{1+x^2}$	M1 B1 A1 A1 [4]	Identity s.o.i. plus attempt to equate coeffs or substitute a value of x $A=1$ $C=0$ $B=-1$ i.s.w. after the above results.
(ii) $\frac{dy}{dx} = \frac{y(1-x)}{(1+x)(1+x^2)}$ $\Rightarrow \int \frac{1}{y}dy = \int \frac{1-x}{(1+x)(1+x^2)}dx$ $\Rightarrow \ln y = \int (\frac{1}{1+x} - \frac{x}{1+x^2})dx$ $= \ln(1+x) - \frac{1}{2}\ln(1+x^2)+c$ When $x=0, y=1$ $\Rightarrow \ln 1 = \ln 1 - \frac{1}{2}\ln 1 + c \Rightarrow c=0$ $\Rightarrow \ln y = \ln(1+x) - \frac{1}{2}\ln(1+x^2)$ $= \ln\frac{1+x}{\sqrt{1+x^2}}$ $\Rightarrow y = \frac{1+x}{\sqrt{1+x^2}}$ *	M1 M1 B1 A1ft A1ft M1 E1 [7]	separating variables substituting their partial fractions $\ln y = \ldots$ $\ln(1+x)\ldots$ $\ldots - \frac{1}{2}\ln(1+x^2)+c$ evaluating c. M0 following incorrect use of the rules of logs.
(iii) $\frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-\frac{1}{2}}$ $= 1 + (-\frac{1}{2})(x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x^2)^2 + \ldots$ $= 1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + \ldots$ so $y = (1+x)(1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + \ldots)$ $= 1 + x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \ldots$	M1 A1 M1 A1ft [4] [15]	binomial series with $p=-1/2$ Allow this M1 if $p=½$. M0 if no working is shown and series is wrong expanding brackets. Ft their previous series

3 (i) $\vec{CA} = -\mathbf{i} - \mathbf{j} - 3\mathbf{k}$, $\vec{CB} = -3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ $\cos\theta = \dfrac{\vec{CA}.\vec{CB}}{\lvert\vec{CA}\rvert.\lvert\vec{CB}\rvert}$	B1	Accept row vectors. Accept the vector equations of CA and CB if they contain the correct vectors **CA** and **CB**
$= \dfrac{(-1).(-3)+(-1).(-1)+(-3).(-2)}{\sqrt{11}.\sqrt{14}}$	M1	Ft their vectors
$= \dfrac{10}{\sqrt{11}.\sqrt{14}} = 0.8058..$	A1	Accept ±0.8058
$\Rightarrow\ \theta = 36.31°$	A1 [4]	Must be the acute angle. Accept 0.634 radians
(ii) $\mathbf{r.n} = \mathbf{a.n}$ $\Rightarrow x - y + 2z = 2$ when $x = 0, y = 0, z = 1$: $0 - 0 + 2 = 2$ valid	B1 B1 E1 [3]	$x - y + 2z = \ldots$Condone i,j,k. $\ldots = 2$ verifying (0, 0, 1) in plane
(iii) Perpendicular is $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ Meets plane when $(3+\lambda) - (1-\lambda) + 6 + 4\lambda = 2$ $\Rightarrow 3 + \lambda - 1 + \lambda + 6 + 4\lambda = 2$ $\Rightarrow\ 6\lambda = -6$, $\lambda = -1$ Point of intersection is (2, 2, 1) Distance from (3, 1, 3) to (2, 2, 1) is $\sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$	B1 B1 M1 A1 A1ft M1 A1cao [7] [14]	$3\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \ldots$ (Condone omission $\ldots + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ of $\mathbf{r}=$) substituting into equation of plane $\lambda = -1$ (2, 2, 1) distance formula w.w.w. Accept 2.45

4 (i) When $x = 0, y^2 = 1 \Rightarrow y = 1$ or -1 so B is (0, 1) and C is (0, −1) When $y = 0$, $(1 - 2x)^3 = 0$ $\Rightarrow 1 - 2x = 0, \Rightarrow x = ½$ so A is (1/2 , 0)	B1 B1 B1 [3]	$y = 1$ or -1 $x = ½$
(ii) $y^2 = (1 - 2x)^3$ $\Rightarrow 2y\frac{dy}{dx} = 3.(-2)(1-2x)^2$ or $-6+24x-24x^2$ $\Rightarrow \frac{dy}{dx} = -\frac{3(1-2x)^2}{y}$ At P, $x = 0, y = 1, \Rightarrow \frac{dy}{dx} = -\frac{3.1^2}{1} = -3$ *	M1 M1 A1 M1 E1	$2y\frac{dy}{dx} =$ Chain rule $3.(-2)(1-2x)^2$ or expansion and differentiation Correct result. Unsimplified will do. Substituting $x = 0$, $y=1$ www.
or $y = (1 - 2x)^{3/2}$ $\Rightarrow \frac{dy}{dx} = (3/2)(1-2x)^{1/2}(-2)$ $= -3(1-2x)^{1/2}$ When $x = 0$, $\frac{dy}{dx} = -3$ *	M1 M1 A1 M1 E1 [5]	 Chain rule Correct result, unsimplified will do Substituting $x = 0$ into their dy/dx www
(iii) $y = t^3 \Rightarrow t = y^{1/3}$ $\Rightarrow x = \frac{1}{2}(1-t^2) = \frac{1}{2}(1-y^{2/3})$ $\Rightarrow 2x = 1 - y^{2/3}$ $\Rightarrow y^{2/3} = 1 - 2x$ $\Rightarrow y^2 = (1 - 2x)^3$ * <u>Or</u> $y^2 = (t^3)^3$, $(1-2x)^3 = (1-2(1/2(1-t^2)))^3$ $= t^6$ $= t^6$ $\Rightarrow$ $y^2 = (1-2x)^3$	M1 A1 E1 M1, A1 E1www	Attempt to eliminate t any valid relationship between x and y www Substituting parametric coordinates into the LHS and RHS of the equation, correct expressions unsimplified
$\frac{dy}{dx} = \frac{3t^2}{-\frac{1}{2}.2t} = -3t$ Equation of tangent: $y - y_1 = m(x - x_1)$ $\Rightarrow y - t^3 = -3t\,[x - \frac{1}{2}(1-t^2)\,]$ $= -3tx + \frac{3}{2}t - \frac{3}{2}t^3$ $\Rightarrow 2y - 2t^3 = -6tx + 3t - 3t^3$ $\Rightarrow 6tx + 2y + t^3 - 3t = 0$ *	M1 A1cao M1 M1 E1 (5)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $-3t$ using $y - y_1 = m(x - x_1)$, or finding c in $y=mx+c$ after substituting $x_1 = \frac{1}{2}(1-t^2)$, $y_1 = t^3$ www

	[8] [16]	

Examiner’s Report

Pure Mathematics 3 (2603)

General Comments

Candidates performed remarkably well on this paper with a very high proportion scoring 60 marks or mor out of the 75 marks available. Presentation was generally good, candidates appeared to be confident applying the techniques required and often obtained the solutions to the questions with a minimum working. There was little evidence of candidates being short of time; most completed the four questions Section A, and work crossed out was usually replaced by a correct version. It was, perhaps, significant th there was much less evidence than in recent papers of poor algebra.

Comments on Individual Questions

Question 1 (Various)

(a) Most candidates were familiar with integration by parts and chose the correct order of terms; the on error that was at all common was an error in the sign of $\int \cos x \, dx$.

(b) This was one question which was not well done except by the stronger candidates. Others were unable to apply the chain rule with $u = \sin 2x$ and $y = u^3$, sometimes choosing, instead, to put $u = 2x$. A number of candidates resorted to the product rule with $u = \sin 2x$ and $v = \sin^2 2x$, which, of course, still required the chain rule unless the double angle formula was used. Unfortunately for those attempting this approach, $\sin^2 2x$ was often written as $\frac{1}{2}(1-\cos 2x)$ instead of $\frac{1}{2}(1-\cos 4x)$.

(c) This question was most often done correctly, but some candidates, who did not write down the identity, got $\sin\alpha$ and $\cos\alpha$ the wrong way round. A few candidates wrote $\sin\alpha = -3$.

(d) Apart from some cases of rather doubtful cancelling, this question was well done. Most candidates obtained the correct limit $\cos x$, but very few indeed gave this as the derivative of $\sin x$.

[(a) 1; (b) $6\sin^2 2x\cos 2x$; (c) 3.6, 56.3°; (d) $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$]

Question 2 (Partial fractions, differential equations and the binomial theorem)

(i) There was a very good start to this question with most candidates obtaining the correct partial fractions. A rare careless error was $2 = 2A \Rightarrow A = 2$, and sometimes candidates who obtained A, B and C correctly wrote down the partial fractions as $\frac{1}{1+x} - \frac{1}{1+x^2}$.

(ii) Many candidates were able to separate the variables of the differential equation and perform the correct integrations. Occasionally $\int \frac{x}{1+x^2}dx$ was given as $2\ln(1+x^2)$, and, for those who made the error in (i) above, $\int \frac{1}{1+x^2}dx$ was almost always written as $\frac{1}{2}\ln(1+x^2)$.

The final stages of this question were not generally well done. Candidates who found the constant of integration before any attempt to simplify the logarithms usually did so correctly, but those who chose to simplify their expression, with the constant still present, most often made errors in doing so. Most common was $\ln y = \ln(1+x) - \frac{1}{2}\ln(1+x^2) + c \Rightarrow y = \frac{1+x}{\sqrt{1+x^2}} + c$, or, perhaps, the above line but with e^c or A instead of the final c.

(iii) Apart from a few candidates who expanded $(1+x^2)^{1/2}$ or $(1+x^2)^{-1}$, this part was nearly always answered correctly.

[(i) $A = 1, B = -1, C = 0$; (iii) $1 - \frac{1}{2}x^2 + \frac{3}{8}x^4 + \ldots$, $1 + x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \ldots$]

Question 3 (Vectors)

Most candidates knew how to find the angle between two vectors and full marks were common. Infrequent errors included finding **AC** and **BC** instead of **CA** and **CB**, and using $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta$.

(ii) Again, there was a very pleasing number of completely correct solutions but a small number of candidates, who chose to start with $ax + by + cz + d = 0$ or $ax + by + cz = d$ and then to find d using the scalar product **a.n**, were often confused about the sign of d and clearly had to make changes in order to show that the point B lay in the plane. A significant minority of candidates confused the equations of a line and a plane.

(iii) This question was also well answered and there were very few careless errors. Some candidates obtained the final result by using the formula for the distance of a point from a plane, rather than the method suggested in the question.

[(i) $-\mathbf{i}-\mathbf{j}-3\mathbf{k}$, $-3\mathbf{i}-\mathbf{j}-2\mathbf{k}$, 36.3°; (ii) $x-y+2z=2$;
(iii) $\boldsymbol{r}=3\mathbf{i}+\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})$, (2, 2, 1), $\sqrt{6}$]

Question 4 (Coordinate geometry, parametric equations)

Full marks for this question were common reflecting a pleasing strength in handling the algebra.

(i) This was usually answered correctly although sometimes with forms such as $B = 1$, or $\sqrt{1}$ in the answer.

(ii) Both implicit and explicit differentiation were used correctly with very few errors.

(iii) Generally well answered although in some cases methods were somewhat confused. However the essential steps were usually present. There were occasional sign errors in expanding $(1-(1-t^2))^3$. Also, very occasionally, candidates verified the results for a particular point only, instead of generally.

Candidates usually had no difficulty in finding the value of $\frac{dy}{dx}$ in terms of t, but a few left the result as $\frac{3t^2}{-t}$.

Those candidates who used the equation $y-y'=m(x-x')$ most often obtained the equation of the tangent correctly but those who used the form $y=mx+c$ sometimes failed to find the value of c after substituting the coordinates $(\frac{1}{2}(1-t^2), t^3)$.

[(i) $(\frac{1}{2}, 0)$, (0,1), (0, −1); (iii) $\frac{dy}{dx}=-3t$]

Section B (Comprehension)

Most candidates scored ten marks or more on this section and there was a good spread of marks between 10 and 15.

Question 1 Answered by all candidates but quite a large number gave only a verbal argument comparing a circle to a circle.

Question 2 A small number of candidates placed Neptune in the gap in Table 2 giving it aBode's number of 42. Otherwise candidates usually did this question correctly.

Question 3 A few candidates misunderstood the question and gave the first term of the G.P. as 1, but almost all candidates used their first term correctly, making the correct deduction where possible.

Question 4 Two errors were made in this question. Fairly common was the use of $a = 7.4$ or, less common, the failure to take the square root to find the value of e.

Question 5 This question was the least well done of the five questions, many candidates being rather confused by it. Common errors were :-

$$R_1=\frac{2}{3}\times 5.8, \text{ or } \frac{3}{2}\times 6,$$

$$R_n=\frac{3}{2}\times(6+4.5\times 2^{n-2}), \text{ or } 4+3\times\left(\frac{4}{3}\right)^{n-2}.$$

Some candidates left their expressions unsimplified, e.g. $R_1=\frac{6}{1.5}$.

Question 6 Most candidates were able to make the correct choice of moons, but quite a large number failed to show sufficient method to achieve both marks available.

Question 7 There was a wide variety of answers to this question and, although the majority of candidates included the appropriate facts and achieved the mark, many cast some doubt on their understanding by including irrelevant arguments about the underlying physics or referring to newly formed planets. A few candidates missed the point entirely and referred only to our own solar system.

[**3.** ½, 8.25×10^7; **4.** 0.258; **5.** 4, $4 + 3 \times 2^{n-2}$; **6.** Miranda, Umbriel and Oberon]