RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2601**

Pure Mathematics 1

Thursday **10 JANUARY 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use only a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

Section A (30 marks)

1 Find the term in x^3 in the binomial expansion of $(1-x)^5$. [2]

2 Find, in the form $y = mx + c$, the equation of the line joining the points (5, 6) and (2, –3). [3]

3 Solve the equation $|2x+1| = 6$. [3]

4 Show that $(x + 3)$ is not a factor of the polynomial $x^3 - 7x^2 + 2x + 40$.

Find one factor of this polynomial. [3]

5 You are given that $\tan\theta = 3$. Find in an *exact* form the possible values of $\cos\theta$. [3]

6 Find the range of values of p for which the quadratic equation $x^2 + 5x + p = 0$ has real roots.

State the value of x when the equation has equal roots. [4]

7

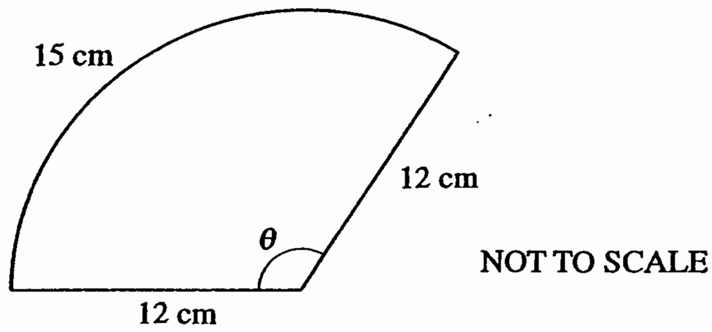

Fig. 7

Fig. 7 shows a sector of a circle. Calculate θ, in radians.

Find the area of the sector. [4]

8 Differentiate $x^3 + 3x$.

Hence show that the curve with equation $y = x^3 + 3x$ has no turning points. [4]

9 Solve the equation $2\sin^2 x = \sin x$ for $0° \leqslant x \leqslant 180°$. [4]

Section B (30 marks)

10

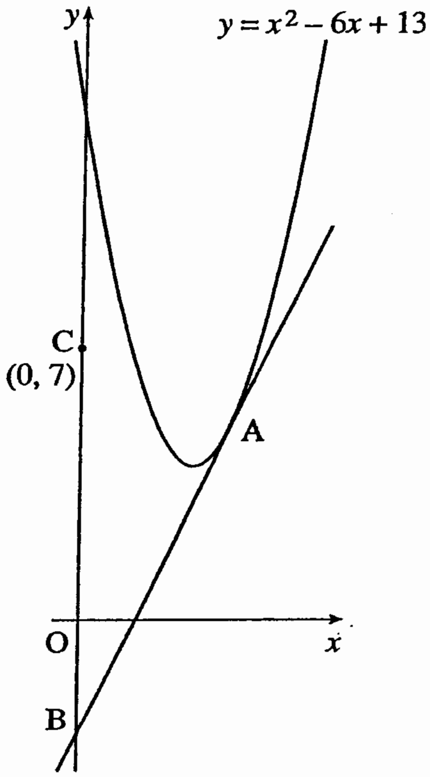

Fig. 10

Fig. 10 shows a sketch of the graph of $y = x^2 - 6x + 13$. The line AB is the tangent to the curve at A and meets the y-axis at B. The point A has coordinates (4, 5).

(i) Show that the equation of AB is $y = 2x - 3$. [4]

(ii) The point C is (0, 7). Show that C lies on the normal to the curve at A. [2]

(iii) Find the equation of the circle with BC as diameter.

Show that this circle passes through A. [4]

(iv) Find the area bounded by the curve, the line AB and the y-axis. [4]

[Total 14]

[Turn over

11 Fig. 11.1 shows a horizontal display shelf in a supermarket. Fig. 11.2 is a plan view of the shelf with measurements in centimetres. Assume that these measurements are exact.

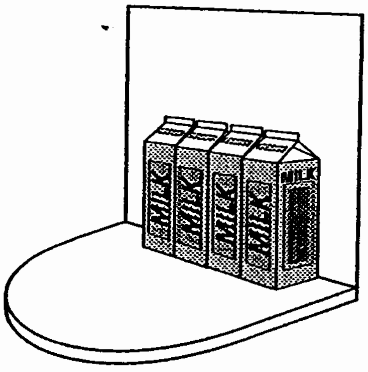

Fig. 11.1

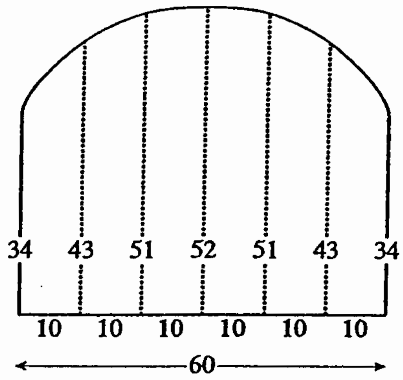

Fig. 11.2

(i) Draw a sketch to show that the area in cm^2 of the shelf is certainly less than

$$2(10 \times 43 + 10 \times 51 + 10 \times 52).$$ [2]

(ii) (*A*) Referring to Fig. 11.2, explain why the area of the shelf is certainly greater than any estimate given by the trapezium rule.

(*B*) Using the 7 ordinates given, calculate the trapezium rule estimate. [4]

A student, Michelle, models the curve of the shelf using the equation $y = 34 + 12x - 2x^2$, where one unit of x represents 10 cm and one unit of y represents 1 cm. This curve is shown in Fig. 11.3.

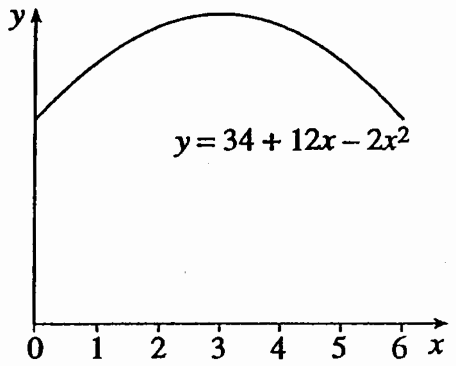

Fig. 11.3

(iii) Calculate the value of y when $x = 1$. Hence find, correct to 3 decimal places, the relative error in the model when $x = 1$. [2]

(iv) Find the area of the shelf given by this model. [3]

(v) Express $34 + 12x - 2x^2$ in the form $a - b(x - c)^2$.

Identify two good features of Michelle's model. [5]

[Total 16]

Mark Scheme

Section A

1	$-10[x^3]$	B2	B1 for 10 seen	2
2.	$y = 3x - 9$	B3	M1 for m. = 3 or $(6-{}^-3)/(5-2)$ M1 for $(y-6) = m(x-5)$ or $(y+3) = m(x-2)$ or $6 = 5m + c$ or $-3 = 2m + c$ or M2 for $\frac{y-6}{-3-6} = \frac{x-5}{2-5}$ o.e.	3
3.	2.5 o.e. −3.5 o.e.	B1 B2	 M1 for $2x + 1 = -6$	3
4.	f(−3) = -27 – 63 – 6 + 40 (at least 2 terms correct) = −56 $(x+2)$ or $(x-4)$ or $(x-5)$ found	M1 A1 *B*1	or M1 for long div as far as $x^2 + kx$ A1 for long div as far as $32x + 96$ or B*2* for '3 is not a factor of 40'	3
5.	$\cos\theta = \pm 1/\sqrt{10}$ or $\pm\sqrt{0.1}$ or $\pm\sqrt{\frac{1}{10}}$	*B3*	B2 for 1 correct <u>or</u> M1 for hyp = $\sqrt{10}$ or $\sqrt{(3^2+1^2)}$ or use of $\sec^2 = 1 + \tan^2$	3
6.	$p \le 6.25$ o.e. −5/2 o.e.	B2 B2	M1 for $5^2 - 4p$ o.e. seen or p (any sign) 6.25 B1 for $(\pm)\sqrt{6.25}$ or $\frac{-5\pm\sqrt{0}}{2}$ or $(x+2.5)^2$ or 2.5	4
7.	θ = 15/12 o.e. i.s.w. Area = 90 [cm^2]	B2 B2	M1 for $15 = 12\,\theta$ o.e. M1 for $0.5 \times 12^2 \times \theta$ or for $\frac{15}{2\pi\times12}\times\pi\times12$ allow B2 for 89.5 – 90.5	4
8.	$3x^2 + 3$ $3x^2 + 3 = 0$ has no real solutions or is always +ve	B2 B2	1 for each term or for showing $x^2 = -1$ at t.p. M1 for using $f'(x) = 0$	4
9.	$x = 0, 180, 30, 150$	B4	1 each; or M1 for sin x = 0, and M1 for sin x = ½, mark to advantage of cand.	4
			Total Section A	30

Section B

10.	(i)	$dy/dx = 2x - 6$ Gradient of tgt at A = 2 $y - 5 = 2(x - 4)$ [or M1 use of y = their $m\,x$ + c, M1 subst of (4,5)]	M1 A1 M2	or M1 for $2x - 3 = x^2 - 6x + 13$ M1 for $x^2 - 8x + 16 = 0$ M1 for $(x - 4)^2$ [=0] M1 for equal roots, so tgt, or showing $b^2 = 4ac$ if 0, then M1 for showing (4, 5) lies on $y = 2x - 3$	4
	(ii)	grad of normal = -1/2 o.e. obtaining normal as $y = -1/2x + 7$ o.e. and testing (0,7)	B1 B1	ft for -1/their grad. in (i) or B1 for gradient of (4,5) to (0,7) = –1/2 or showing line through C with grad –1/2 goes through A or vv	2
	(iii)	$x^2 + (y - 2)^2 = 25$ o.e. subst of (4,5) in circle eqn to show consistent	B3 1	B1 for midpt of BC = (0,2) B1 for radius = 5 M1 for $(x-h)^2 + (y-k)^2 = r^2$ ft their (h,k) and r or use of angle in semicircle = 90° or showing A is 5 from (0,2)	4
	(iv)	$\int([x^2 - 6x + 13] - [2x - 3])dx$ or $\int(x^2 - 8x + 16)dx$ Answer 21(.3..)	M2 A2	M1 for $\int(x^2 - 6x + 13)dx$ if A0, B1 for 25.3(..) or at least 2 terms of $x^3/3 - 4x^2 + 16x$ or $x^3/3 - 3x^2 + 13x$ (B1 implies at least M1 also)	4
11.	(i)	Sketch showing relevant rectangles above curve.	E2	E1 for partially correct sketch eg gap slightly above or below curve or indication of bits missing	2
	(ii)	(a) curve is convex (b) 2740	E1 B3	B2 for 5480 or 1370 or at least 2 of 385, 470 and 515 seen M1 for trap. rule attempted	4
	(iii)	0.023	B2	condone 1/43 or 0.023(25..) or 2.326%; B1 for $y = 44$	2
	(iv)	$\int_0^6 [34 + 12x - 2x^2]dx$ $34x + 6x^2 - \frac{2}{3}x^3$ 276 or 2760	M1 B1 A1	or 0 to 3	3
	(v)	$a = 52$, $b = 2$, $c = 3$ eg fits symmetry $x = 3$ $y = 52$max 2740 < area < 2920	B3 B1+ B1	1 each 2 relevant facts needed from: symmetry about $x = 3$; fits at 0 and/or 6 fits at 3 small rel. error at 2 and/or 1, 4, 5 sensible comment on area	5
				Total Section B	30
				Total for paper	60

Examiner’s Report

Pure Mathematics 1 (2601)

General Comments

Compared with last summer, the examiners were very relieved to find far less of a tail of very weak candidates this session. Although some candidates were still entered who had no clear knowledge of any parts of the content for this unit, the great majority were able to demonstrate mastery of at least a few aspects. Some excellent scripts were seen, but the examiners felt that there were fewer exceptionally good candidates than in January or June last year. The full range of marks was scored. A few candidates showed evidence of rushing on question 11, or did not complete part (v), but lack of time did not seem to be a major problem.

The long questions proved to be accessible at various points, as hoped, and candidates made good use of this, leaving out parts where necessary but attempting later ones.

In section A, the weakest work was seen on the topics of trigonometry and the discriminant of a quadratic equation.

Comments on Individual Questions

Question 1 (Binomial)

Many candidates coped with Pascal's triangle or use of $\binom{5}{3}$ - perhaps more than last year were aware of the relevance of the formula book to this topic. Omission of the negative was frequent, especially by weaker candidates.

[−10]

Question 2 (Equation of a line)

Most candidates gained full marks, with few having an incorrect method for the gradient or for finding the equation.

[$y = 3x - 9$]

Question 3 (Modulus)

Many ignored the modulus and found only the positive solution, or followed this by stating the solution as $x = \pm 2.5$. Treating the equation as an inequality was also fairly common.

[2.5 and −3.5]

Question 4 (Factor Theorem)

Full marks were fairly common here, with many knowing they should substitute a value in $f(x)$, although using 3 instead of −3 was also common. Those who tried long division often made errors.

[e.g. showing $f(-3) = -56$; $(x + 2)$ or $(x - 4)$ or $(x - 5)$]

Question 5 (Trigonometry)

There were very few fully correct answers, with the vast majority not appreciating the meaning of 'exact' and using their calculator to give 0.316. Of those who did give an exact value, few gave the negative one also.

[$\pm\sqrt{\frac{1}{10}}$ or equivalent]

Question 6 (Quadratic)

Many did not appreciate the relevance of the discriminant and, of those who did, handling the inequality correctly was rare. The second part was also poorly done.

[$p \leqslant 6.25$; -2.5]

Question 7 (Sector)

This was done well by many candidates, although some demonstrated their confusion with radians by introducing an additional π factor after finding the angle correctly. Those who worked in degrees often got there in the end, but wasted time.

[$\theta = 1.25$; area = 90 cm^2]

Question 8 (Differentiation)

Almost all candidates could differentiate, and most went on to complete the question successfully.

[$3x^2 + 3$; showing that $3x^2 + 3 = 0$ has no real solutions]

Question 9 (Trig Equation)

Few recognised this as a quadratic in $\sin x$, and finding all four angles was rare. Many candidates cancelled factors (often wrongly) or did not know where to start. However, some of the A grade candidates for whom this question was designed produced elegant or efficient solutions.

[0, 180, 30, 150]

Question 10 (Tangent, normal, area under a graph)

(i) Many candidates gained full marks for this part, although some weaker ones assumed $y = 2x - 3$, found B and used the gradient of AB, instead of differentiating. Some better candidates successfully showed that the intersection of the curve and line has a double root at $x = 4$.

(ii) Most knew the condition for lines to be perpendicular and many had good solutions. A common error was to use the gradient of the normal and C to find the equation of the line, then testing that C was on the line again instead of using A. The most efficient method was to find the gradient of AC and show this is perpendicular to the tangent at A.

(iii) Some candidates seemed unfamiliar with this topic, but many others knew the form of the equation of a circle and found it correctly. A few took B or C as the centre. Some omitted to show that the circle passes through A. Some candidates exhibited efficient solutions in this part, whilst others took a page or more to arrive at correct answers.

(iv) Most realised that integration was required, and many were able to find the area under the line. However, mishandling of the area under the line was common, with attempts to calculate the two triangles often failing, or wrongly added/subtracted. Those who used the efficient method of integrating $x^2 - 8x + 16$

were much more successful. Sadly, some saw the term 'y-axis' and interpreted this as the need to integrate with respect to y, which foundered. Notation in integration was frequently poor.

Full marks were obtained on question 10 by an encouraging number of candidates, who were able to apply a range of techniques accurately.

[(iii) $x^2 + (y-2)^2 = 25$; (iv) $21\frac{1}{3}$]

Question 11 (Area, numerical methods)

(i) Usually fairly well done, although some sketch diagrams did not show the curve and rectangles clearly.

(ii) Explanations helpfully supported by a diagram were common, although some assumed that the trapezium rule always underestimates an area.. More candidates than in the summer used the trapezium rule correctly – perhaps more were aware of its existence in the formula book. Many gained full marks in this part.

(iii) Many candidates stopped short at calculating the y-value. Others were confused between relative and absolute error, or used 44 as the true value. However, correct answers were also seen, usually from the better candidates.

(iv) Most candidates obtained full marks here, integrating correctly and using the correct limits.

(v) Few candidates were able to complete the square and obtain all the coefficients correctly, with the value of a being a particular problem, although weaker candidates who attempted this part were often able to find the value of b and received credit for this. The identification of good features of the model presented problems even for strong candidates, though a few centres produced better responses. Comments such as 'easier to calculate the area' and 'more accurate' were common, the latter usually with no clear reference as to whether area or y-values were being compared. Many assumed that the model was exact and the shelf inaccurate.

Full marks were rare on question 11, mostly due to parts (iii) and (v).

[(i) sketch showing relevant rectangles above curve, (ii)(A) explanation/diagram (B) 2740,
(iii) 0.023, (iv) 2760, (v) $a = 52$, $b = 2$, $c = 3$, two relevant facts from: symmetry about $x = 3$; fitting at $x = 0$ and/or 6; fitting at $x = 3$; small relative error at $x = 1$ and/or 2, 4, 5; sensible comment on area e.g. $2740 < \text{area} < 2920$.]