OCR
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Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Thursday **14 JUNE 2001** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

NOTE

This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

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1 The curve with parametric equations

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta, \quad 0 \leqslant \theta \leqslant 2\pi,$$

is a cycloid. Its graph is shown in Fig. 1.

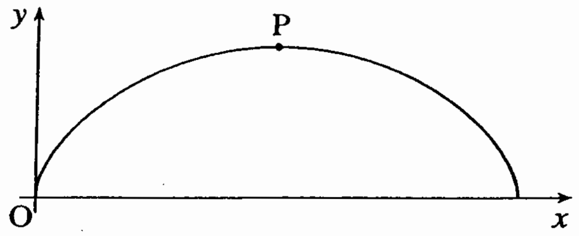

Fig. 1

(i) Find $\frac{dy}{dx}$ in terms of θ. Deduce the coordinates of the stationary point P. [5]

(ii) At the point on the curve with parameter α, the gradient is $\frac{1}{2}$. Show that

$$2\sin\alpha + \cos\alpha = 1.$$

By expressing the left-hand side of the equation in the form $R\cos(\alpha - \beta)$, solve this equation for α, giving your answer in radians correct to 3 decimal places. [5]

(iii) The area of the region enclosed by the curve and the x-axis is A, where

$$A = \int_0^{2\pi} y\frac{dx}{d\theta}\,d\theta.$$

Show that $A = \int_0^{2\pi} (1 - \cos\theta)^2\,d\theta$. Hence find A, giving your answer as a multiple of π. [5]

2 (i) Use the formulae for $\cos(\theta + \phi)$ and $\cos(\theta - \phi)$ to prove that

$$\cos(\theta - \phi) - \cos(\theta + \phi) = 2\sin\theta\sin\phi. \quad (*)$$

Prove also that $\sin(\pi - \theta) = \sin\theta$. [3]

In triangle PQR, angle P $= \frac{1}{6}\pi$ radians, angle Q $= \alpha$ radians, and QR = 1 unit. The point S is at the foot of the perpendicular from R to PQ (see Fig. 2).

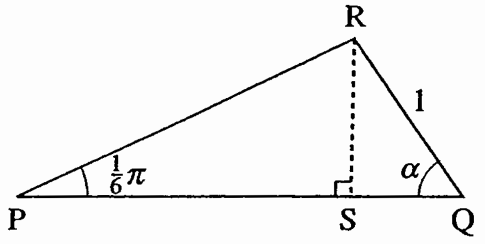

Fig. 2

(ii) Show that PQ $= 2\sin(\alpha + \frac{1}{6}\pi)$. By finding RS in terms of α, deduce that the area A of the triangle is given by

$$A = \sin(\alpha + \tfrac{1}{6}\pi)\sin\alpha.$$

Find the value of α for which the area A is a maximum. [You may find the result (*) helpful.] [7]

(iii) Expand $\sin(\alpha + \frac{1}{6}\pi)$, and hence show that, for small values of α, $A \approx p\alpha + q\alpha^2$, where p and q are constants to be determined.

Find the value of this expression when $\alpha = 0.1$, and find also the corresponding value of A given by the expression in part (ii). [5]

3 (i) Show that $\dfrac{2u^2}{u^2-1} = 2 + \dfrac{2}{u^2-1}$. Hence express $\dfrac{2u^2}{u^2-1}$ in partial fractions. [4]

(ii) Using the substitution $u = \sqrt{x}$, show that $\displaystyle\int_4^9 \frac{\sqrt{x}}{x-1}\,dx = \int_2^3 \frac{2u^2}{u^2-1}\,du$.

Deduce that $\displaystyle\int_4^9 \frac{\sqrt{x}}{x-1}\,dx = \ln 3 - \ln 2 + 2$. [7]

(iii) Use integration by parts, and the result of part (ii), to show that

$$\int_4^9 \frac{\ln(x-1)}{\sqrt{x}}\,dx = 20\ln 2 - 6\ln 3 - 4. \quad [4]$$

4 In Fig. 4, ABCDPQ represents a tent, held up by vertical poles OP and RQ. The axes Ox and Oy are horizontal at ground level, and Oz is vertically upwards. The coordinates of A, B, C, D, P and Q are as shown in the diagram. Lengths are in metres.

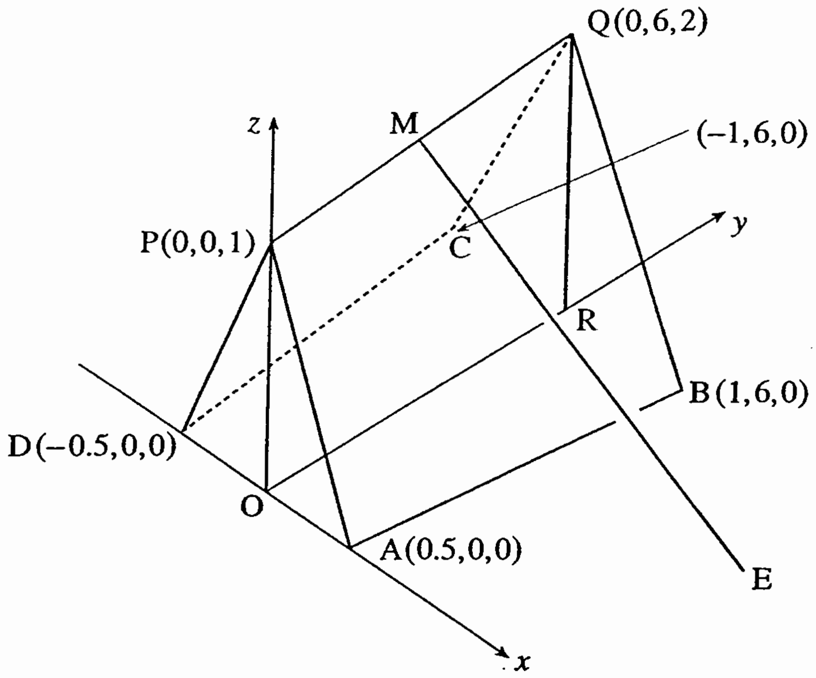

Fig. 4

(i) Find the length of PQ. [2]

(ii) Show that the vector $\mathbf{n}_1 = 12\mathbf{i} - \mathbf{j} + 6\mathbf{k}$ is perpendicular to each of the lines AP and PQ. Hence find the cartesian equation of the plane APQ. Verify that the point B lies in this plane. [5]

(iii) The vector $\mathbf{n}_2 = -12\mathbf{i} - \mathbf{j} + 6\mathbf{k}$ is a normal to the plane DCQP.

Find the angle between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$. Deduce the acute angle in degrees between the planes ABQP and DCQP. [4]

(iv) A rope ME of length 2 metres is stretched from the mid-point M of PQ to the ground. Given that the rope is perpendicular to PQ, find the coordinates of E. [4]

Mark Scheme

1(i) $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\sin\theta}{1-\cos\theta}$	M1 A1	$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$ s.o.i. $\frac{\sin\theta}{1-\cos\theta}$ Answer only B2
$\frac{dy}{dx}=0$ when $\sin\theta=0$, $\theta=\pi$ when $\theta=\pi$, $x=\pi-\sin\pi=\pi$ $y=1-\cos\pi=2$ so P is $(\pi, 2)$	B1 B1 B1 [5]	$\theta=\pi$, and only π, s.o.i. Not 180 $x=\pi$ Accept 3.142 $y=2$ Answers for x and y only without π B3
(ii) $\frac{\sin\alpha}{1-\cos\alpha}=\frac{1}{2}\Rightarrow 2\sin\alpha=1-\cos\alpha$ $\Rightarrow 2\sin\alpha+\cos\alpha=1$*	E1	showing*
$2\sin\alpha+\cos\alpha=R\cos(\alpha-\beta)$ $=R\cos\alpha\cos\beta+R\sin\alpha\sin\beta$ $\Rightarrow R\cos\beta=1$, $R\sin\beta=2$ $\Rightarrow R^2=5$, $R=\sqrt{5}$ $\tan\beta=2$, $\Rightarrow\beta=1.107$	B1 B1	$R=\sqrt{5}$ Condone 2.2 $\beta=1.107$. Accept 1.1, $\tan^{-1}2$, etc., 63.4°. Also accept $\alpha=1.107$ here
so $\sqrt{5}\cos(\alpha-1.107)=1$ $\Rightarrow\cos(\alpha-1.107)=1/\sqrt{5}$ $\Rightarrow\alpha-1.107=1.107$ $\Rightarrow\alpha=2.214$	M1 A1 [5]	α-(their β)$=\cos^{-1}$(1/their R) c.a.o. Accept 0.705π
(iii) $\frac{dx}{d\theta}=1-\cos\theta$ $\Rightarrow A=\int_0^{2\pi}(1-\cos\theta)^2 d\theta$	E1	
$=\int_0^{2\pi}(1-2\cos\theta+\cos^2\theta)d\theta$	M1	Expanding $(1-\cos\theta)^2$. Expansion must have a middle term. Allow this M1, also, for the correct, full first stage of integration by parts, which needs to be applied twice.
$=\int_0^{2\pi}\left[1-2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right]d\theta$	B1	$\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$ correct and used. Or $\sin^2\theta=\frac{1}{2}(1-\cos 2\theta)$ for integration by parts.
$=\left[\theta-2\sin\theta+\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right]_0^{2\pi}$ $=3\pi$	B1 A1cao	$=\left[\theta-2\sin\vartheta+\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right]$

2 (i) $\cos(\theta-\phi)-\cos(\theta+\phi)$ $=\cos\theta\cos\phi+\sin\theta\sin\phi-(\cos\theta\cos\phi-\sin\theta\sin\phi)$ $=2\sin\theta\sin\phi$ *	M1 E1	correct compound angle formulae simplified to show *
$\sin(\pi-\theta)=\sin\pi\cos\theta-\cos\pi\sin\theta$ $=\sin\theta$ *	E1 [3]	or appealing to symmetry of graph, θ and π-θ s.o.i.
(ii) By sine rule: $\frac{PQ}{\sin R}=\frac{1}{\sin\frac{\pi}{6}}=2$ or $PQ=\frac{\sin\alpha}{\tan\frac{\pi}{6}}+\cos\alpha$	M1	sine rule or equivalent alternative or expansion of $2\sin(\alpha+\frac{\pi}{6})$ and use of QS=cosα and PS=sinα/tan$\frac{\pi}{6}$
$\Rightarrow PQ=2\sin R$ $=2\sin[\pi-(\alpha+\pi/6)]$ $=2\sin(\alpha+\pi/6)$ *	E1	must derive $\alpha+\frac{\pi}{6}$
$\Rightarrow A=\frac{1}{2}PQ\times RS$ $=\frac{1}{2}2\sin(\alpha+\pi/6).\sin\alpha$	M1	1/2 their PQ × their RS or 1/2 × 1 × their PQ × sinα
$=\sin(\alpha+\pi/6).\sin\alpha$ *	E1 – [4]	
$A=\sin(\alpha+\pi/6).\sin\alpha$ $=\frac{1}{2}[\cos\pi/6-\cos(2\alpha+\pi/6)]$	B1	$\frac{1}{2}[\cos\pi/6-\cos(2\alpha+\pi/6)]$
A is maximum when $\cos(2\alpha+\pi/6)=-1$ $\Rightarrow 2\alpha+\pi/6=\pi$	M1	$\cos(2\alpha+\pi/6)=-1$ (=0is M0)
$\Rightarrow \alpha=5\pi/12$	A1	$\alpha=5\pi/12$ or 75°
$dA/d\alpha=\sin(\alpha+\pi/6)\cos\alpha+\cos(\alpha+\pi/6)\sin\alpha$	M1	For a reasonable attempt at differentiating any form of A
$=\sin(2\alpha+\pi/6)$	A1	$\sin(2\alpha+\pi/6)$ or their correct dA/dα
$dA/d\alpha=0$ when $2\alpha+\pi/6=\pi\Rightarrow\alpha=5\pi/12$	A1 [7]	$\alpha=5\pi/12$ or 1.309 or 75°
(iii) $\sin(\alpha+\pi/6)=\sin\alpha\cos\pi/6+\cos\alpha\sin\pi/6$ $=\frac{\sqrt{3}}{2}\sin\alpha+\frac{1}{2}\cos\alpha$	B1	
$\Rightarrow A\approx\alpha[\frac{\sqrt{3}}{2}\alpha+\frac{1}{2}(1-\frac{1}{2}\alpha^2)]$ $=\frac{1}{2}\alpha+\frac{\sqrt{3}}{2}\alpha^2+\ldots$	M1	substituting approximations for $\sin\alpha$ and $\cos\alpha$ into A or expansion of $\sin(\alpha+\frac{\pi}{6})$
$\Rightarrow p=\frac{1}{2},q=\frac{\sqrt{3}}{2}$	A1 A1	$\frac{1}{2}$ or 0.5 or $\sin\frac{\pi}{6}$, $q=\frac{\sqrt{3}}{2}$ etc.
when $q=0.1, A\approx0.05866$	B1	0.0587 or better
Using exact result, $A=0.0583$	[5]	0.0583 or better

3 (i) $2+\frac{2}{u^2-1}=\frac{2u^2-2+2}{u^2-1}=\frac{2u^2}{u^2-1}$ *	E1	By any valid method
$\frac{2}{u^2-1}=\frac{A}{u-1}+\frac{B}{u+1}$	M1	$\frac{A}{u-1}+\frac{B}{u+1}$
$\Rightarrow A(u+1)+B(u-1)=2$		
$u=1\Rightarrow 2A=2, A=1$	A1	$A=1$
$u=-1\Rightarrow -2B=2, B=-1$	A1	$B=-1$
so $\frac{2u^2}{u^2-1}=2+\frac{1}{u-1}-\frac{1}{u+1}$	[4]	
(ii) $\int_4^9 \frac{\sqrt{x}}{x-1}\,\mathrm{d}x$ let $u=\sqrt{x}\Rightarrow x=u^2$	M1	For $\frac{dx}{du}=2u$ or $\frac{du}{dx}=\frac{1}{2}x^{-1/2}$ correct
$\mathrm{d}x=2u\mathrm{d}u$		
when $x=4$, $u=2$, when $x=9$, $u=3$		and <u>used</u> in the integral
$=\int_2^3 \frac{u}{u^2-1}2u\,\mathrm{d}u$	M1	Complete substitution of u for x
$=\int_2^3 \frac{2u^2}{u^2-1}\mathrm{d}u$ *	E1	deriving *with correct limits and w.w.w.
$=\int_2^3 [2+\frac{1}{u-1}-\frac{1}{u+1}]\mathrm{d}u$	M1	substituting their partial fractions even if the 2 is missing
$=\left[2u+\ln(u-1)-\ln(u+1)\right]_2^3$		
$=(6+\ln 2-\ln 4)-(4+\ln 1-\ln 3)$	A1ft	$\left[2u+\ln(u-1)-\ln(u+1)\right]_2^3$ ft their partial fractions.
$=2+\ln 2-2\ln 2+\ln 3$	A1f.t.	substituting limits
$=\ln 3-\ln 2+2$ *	E1	correctly simplified
	[7]	
(iii) $\int_4^9 \frac{\ln(x-1)}{\sqrt{x}}\mathrm{d}x$ $u=\ln(x-1)$, $\mathrm{d}v/\mathrm{d}x=1/\sqrt{x}$	M1	'v' $=2\sqrt{x}$
$\Rightarrow v=2\sqrt{x}$		
$=\left[2\sqrt{x}\ln(x-1)\right]_4^9-\int_4^9 \frac{2\sqrt{x}}{x-1}\mathrm{d}x$	A1	$\left[2\sqrt{x}\ln(x-1)\right]_4^9-\int_4^9 \frac{2\sqrt{x}}{x-1}\mathrm{d}x$
$=2.3\ln 8-2.2\ln 3-2\ln 3+2\ln 2-4$	A1	substitutions correct
$=18\ln 2-4\ln 3-2\ln 3+2\ln 2-4$	E1	correctly simplified to *
$=20\ln 2-6\ln 3-4$ *	[4]	

4 (i) PQ = $\sqrt{[(0-0)^2 + (6-0)^2 + (2-1)^2]}$ $= \sqrt{37}$	M1 A1 [2]	use of distance formula $\sqrt{37}$ or 6.08 (m)
(ii) $\mathbf{n}_1 . \vec{AP} = (12\mathbf{i} - \mathbf{j} + 6\mathbf{k}).(-0.5\mathbf{i} + 0\mathbf{j} + 1\mathbf{k})$ $= -6 + 0 + 6 = 0$ $\mathbf{n}_1 . \vec{PQ} = (12\mathbf{i} - \mathbf{j} + 6\mathbf{k}).(0\mathbf{i} + 6\mathbf{j} + 1\mathbf{k})$ $= 0 - 6 + 6 = 0$ So $\mathbf{n}_1$ is perpendicular to plane APQ Equation is $12x - y + 6z = c$ At (0, 0, 1) $12.0 - 0 + 6.1 = c$ $\Rightarrow c = 6$, and equation is $12x - y + 6z = 6$ At B, substituting $x = 1, y = 6, z = 0$: $12 . 1 - 6 + 6 . 0 = 6$ so B lies in the plane.	M1 E1 M1 A1 B1 [5]	use of scalar product verifying $\mathbf{n}_1 . \vec{AP} = 0$, $\mathbf{n}_1 . \vec{PQ} = 0$ $12x - y + 6z = c$, or for a correct form of the vector equation and some attempt at the elimination of the parameters. $12x - y + 6z = 6$ verifying B lies in plane
(iii) $\mathbf{n}_1 . \mathbf{n}_2 = (12\mathbf{i} - \mathbf{j} + 6\mathbf{k}).(-12\mathbf{i} - \mathbf{j} + 6\mathbf{k})$ $= -144 + 1 + 36$ $= -107$ $= \sqrt{181} . \sqrt{181} . \cos\theta$ $\Rightarrow \cos\theta = -107/181, \Rightarrow \theta = 126°$ $\Rightarrow$ angle between planes = 180 − 126 = 54°	 M1 A1 A1 A1ft [4]	 Use of scalar product to find an angle $\cos\theta = -107/181$ $\theta = 126°$ or2.20radians 54°or 0.94 radians
(iv) Midpoint of PQ = (0, 3, 1.5) suppose rope ends at $(a, b, 0)$ $\Rightarrow a\,\mathbf{i} + (b-3)\,\mathbf{j} - 1.5\,\mathbf{k}$ is perpendicular to $6\mathbf{j}+\mathbf{k}$ $\Rightarrow a . 0 + (b-3) . 6 - 1.5 . 1 = 0$ $\Rightarrow b = 19.5 / 6 = 3.25$ length = $\sqrt{(a^2 + 0.25^2 + (-1.5)^2)} = 2$ $\Rightarrow a = 1.3$ so coordinates are (1.3, 3.25, 0)	B1 M1 A1 A1 [4]	midpoint is (0, 3, 1.5) use of scalar product = 0 $b = 3.25$ $a = 1.3$

Examiner’s Report

Pure Mathematics 3 (2603)

General Comments

Although the examiners who marked this paper thought that the level was appropriate there was a very poor response from the candidates. Only a small number of candidates did very well with scores in the range 60 to 75 and there were far more candidates with low marks than might have been expected.

Many problems arose because of poor algebra, the inability of candidates to deal correctly with negative and fractional indices, particularly when dividing, and a tendency to omit brackets in vital places. Poor notation was also a problem, particularly the omission of dx from integrals. On the other hand it was pleasing to note an improvement in the work leading up to results which were given in the question. Simple, but important steps were included in the solution. It was also noted that there was a good response to the vector question.

In Section B, the comprehension, the questions were well answered by the majority of candidates.

Comments on Individual Questions

Section A

Question 1 (Parametric Equations)

There were some errors in differentiating x and y with respect to θ but most candidates were able to find dy/dx with their values of dy/dθ and dx/dθ, and many deduced correctly that $\sin\theta = 0$ gave the stationary point. However, many candidates were unable to write down the appropriate value of θ and deduce the values of x and y at P. A small, but significant number of candidates followed dy/dx = 0 with $\sin\theta = 0$ or $1 - \cos\theta = 0$. Others wrote $\sin\theta = 1 - \cos\theta$.

Those candidates who obtained the correct expression for dy/dx were usually able to obtain the given equation and almost all candidates then expressed this equation in the form $R\cos(\alpha - \beta) = 0$. There was some confusion of α and β, candidates sometimes writing $\alpha = 1.107$ instead of β, which usually meant that those candidates were unable to find the solution for α. Another error which occurred in this part of the question was $\tan\beta = 1/2$ instead of 2.

Many candidates obtained the correct integral to start part (iii) and most of those who did went on to expand $(1 - \cos\theta)^2$, although this was occasionally given as $1 - \cos^2\theta$. About half of the candidates obtaining the correct expansion then failed to express $\cos^2\theta$ in terms of 2θ, possibly integrating it as $\frac{1}{3}\cos^3\theta$. Those candidates who knew how to integrate $\cos^2\theta$ often went on to complete the question correctly. Of the four questions on the paper this question was the one most likely to attract full marks from an able candidate.

(i) $\frac{dy}{dx} = \frac{\sin\theta}{1 - \cos\theta}$, $\theta = \pi$, P is $(\pi, 2)$; (ii) $R = \sqrt{5}$, $\beta = 1.107$, $\alpha = 2.214$; (iii) 3π.

Question 2 (Trigonometry and integration)

The first part of this question was generally well answered, the appropriate formulae were quoted correctly and the given result obtained by subtraction. Just a few solutions, in which the necessary brackets were omitted, left the proof in some doubt. Only a small number of candidates made the error
$\sin(\pi - \theta) = \sin\pi - \sin\theta$, but, again, the omission of a step in the working, or of brackets sometimes left $-\cos\pi\sin\theta = -(-1)(\sin\theta) = \sin\theta$ somewhat doubtful.

Very many candidates were unable to obtain the given expression for the area of the triangle PQR. Few took the most direct root using the sine rule and those who did, often failed to see the significance of the first part of the question to $\sin(\pi - (\alpha + \pi/6))$. Instead, many candidates used PQ = PS + SR, often obtaining the correct expression $\frac{\sin\alpha}{\tan\pi/6} + \cos\alpha$, but being unable to transpose this into the required form. However most candidates obtained the required expression for the area from the given form of PQ.

Few candidates were able to find the value of α for which the area of triangle PQR is a maximum. Some thought that this occurred when $\sin(\alpha + \pi/6)$ is a maximum; others, who followed the recommendation in the question correctly, put $\cos(2\alpha + \pi/6) = 1$, or 0, only very rarely -1. Yet others attempted to differentiate either the given expression, or an expansion of it, or the derived expression, but the result was rarely successful and very few candidates succeeded in obtaining the correct value for α.

In part (iii) most candidates expanded $\sin(\alpha + \pi/6)$ correctly, and then used the correct approximations for $\sin\alpha$ and $\cos\alpha$. The common error, however, was to use this result to find the constants p and q instead of first multiplying by α for the area A. This meant, of course that their numerical value for A was wrong. Many candidates failed to calculate the value of A from the original expression.

(ii) $\alpha = 5\pi/12$; (iii) $p = 1/2, q = \sqrt{3}/2$, $A \approx 0.0587$, $A = 0.0583$.

Question 3 (Partial fractions and integration by substitution and by parts).

With the exception of a small number of very able candidates who produced excellent solutions, this question was very poorly done. Just a few candidates managed to prove the first result by long division, by adding the two terms on the right hand side, or by letting $\frac{2u^2}{u^2-1} = A + \frac{Bu+C}{u^2-1}$. Unfortunately candidates choosing this last method, having found A, B and C, most often thought that they had now found the partial fractions and proceeded to part (ii). The most common error, however, either at this point, or initially, was to write $\frac{2u^2}{u^2-1} = \frac{A}{u-1} + \frac{B}{u+1}$. This often led, by using $u = 1$ and $u = -1$, to the correct partial fractions for $2/(u^2 - 1)$.

Few candidates could deal straightforwardly with the integration by substitution. Most started correctly by differentiating $u = \sqrt{x}$, but having obtained du/dx failed to use it correctly to replace dx in the integral. Many, using poor notation, omitted the dx from their integral, others experienced difficulty writing dx in terms of x and du, or in terms of u and du. Many candidates, having worked at the side of the page, simply asserted the equality of the two integrals.

Those candidates who went on to use their partial fractions, obtained correctly, or in follow through work, very often integrated correctly and made the correct substitutions but perhaps omitted an explanation of $\ln 2 - \ln 4 = -\ln 2$.

Although some candidates made no attempt at part (iii), the integration by parts was, perhaps, better done than the integration by substitution. Common errors were dv/dx = $\sqrt{x}$ instead of $1/\sqrt{x}$. and, if this was correct, $v = \frac{1}{2}x^{1/2}$, instead of $2x^{1/2}$. Some candidates failed to make use of the result in part (ii) and, again, some candidates having made the correct substitutions failed to explain $6\ln 8 + 2\ln 2 = 20\ln 2$.

(i) $2 + \frac{1}{u-1} - \frac{1}{u+1}$.

Question 4 (Vectors)

This was the best answered question in Section A, although the question where fewest able candidates scored full marks; there were very few correct solutions to part (iv). Almost all candidates were able to obtain the length of the vector $\overrightarrow{PQ}$. Part (ii) was also very well done by the great majority of candidates. In finding the Cartesian equation of the plane APQ there was some confusion over the sign of 'd' but this was often corrected by candidates when they came to verify that B lay in the plane. Some candidates overlooked this step. Many candidates still use the vector equation in these circumstances but most of those who did got the form of the equation correct and many went on to eliminate the parameters successfully. Part (iii) was also well done, although there were some arithmetic errors and some candidates failed to find the angle between the planes. A small number of candidates thought that this angle was the angle between AP and PD.

The final part of the question defeated almost all candidates. Most started correctly by finding the coordinates of M, the midpoint of PQ, although a common error was to give the z-coordinate as 0.5 instead of 1.5. Many candidates thought that the direction of the vector $\overrightarrow{ME}$ was the same as the direction of the normal to the plane APQ, or the normal to the plane PQC. Another common error was to assume the y-coordinate of E to be 3, or to use a right angled triangle MSE, where S is the foot of the perpendicular from M to Oy, and to assume that SE is the x-coordinate of E. Those candidates assigning coordinates to E, say (a, b ,c) rarely put $c = 0$ and often put the scalar product of $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and $6\mathbf{j} + \mathbf{k}$ equal to 0.

(i) PQ = $\sqrt{37}$; (ii) $12x - y + 6z = 6$; (iii) 126°, 54°; (iv) (1.3, 3.25, 0).

Section B (Comprehension)

Question 1

Very well answered by most candidates.

Question 2

Also well answered although some candidates transposed the formula to make h the subject but then failed to substitute the values of t, E and r.

Question 3

There was some doubtful algebra in the solutions to this question involving invalid factorising, cancelling, or transposing.

Question 4 & 6

Many candidates achieved the correct values of c in question 4, and p in question 6 although in the latter case some candidates had clearly read the value from the graph in the text.

Question 5

This question was done correctly by many candidates. The most common error was to calculate the denominator as $(1 - p)(c + r - h)$.

Question 7

There was a wide variety of answers to this question but many candidates observed correctly that a formula for domestic dogs might not be appropriate to African Wild Dogs. In part (ii), too many candidates referred to ways in which their errors might have been avoided rather than to how to assess their effect on p.

2. $h = 3.1243$; **5.** 8.2hr; **6.** $p = 0.345$.