OCR
RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2621/1

Decision and Discrete Mathematics 2

Tuesday **19 JUNE 2001** Morning 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

An **insert** is provided for use in Question **2** part **(ii)** and Question **3** part **(ii)**.

This question paper consists of 4 printed pages and an insert.

1 Let s represent the proposition "There is snow".
Let n represent the proposition "There is a north wind".

You are given that if there is no snow then there is no north wind, and that there is a north wind.

(i) Express what you are given in terms of s, n and logical symbols. [4]

(ii) Use a truth table to prove, from what you are given, that there is snow. [4]

(iii) You are also given that if it snows then the robin hides its head under its wing. What can you deduce about weather and wind if the robin does not have its head hidden under its wing? [2]

[Total: 10]

2 [There is an insert for use in part (ii) of this question.]

A decision has to be made regarding a project. It can be allowed to proceed (A), or it can be cancelled (~A). Outcomes, payoffs and probabilities are summarised in the decision tree in Fig. 2.1.

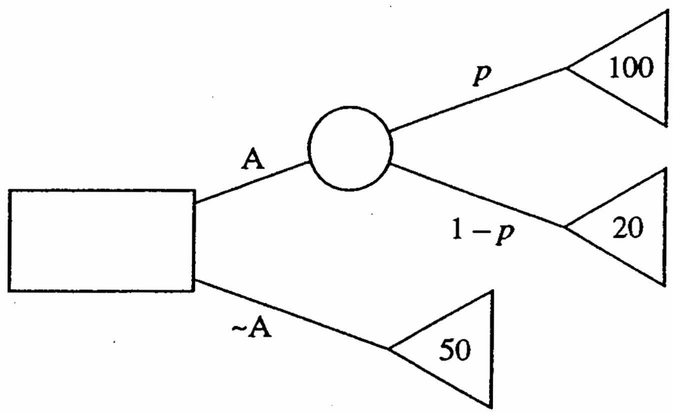

Fig. 2.1

(i) If the EMV of proceeding is equal to the EMV of cancelling, show that $p = 0.375$. [2]

(ii) Advice can be sought on whether or not to proceed. If advice is sought then there is a probability of 0.2 that the advice will be to proceed, in which case the resultant probabilities will be more favourable. If the advice is to cancel, then the probabilities are less favourable. The values are summarised in the decision tree in Fig. 2.2 on the insert.

(*A*) Complete the copy of the tree on the insert, showing the EMVs. [5]

(*B*) What is the value of the advice? [1]

(*C*) To what value would the cancellation payoff of 50 have to increase to make it not worth seeking advice? [2]

[Total: 10]

3 [There is an insert for use in part **(ii)** of this question.]

The weights on the network in Fig. 3 represent distances.

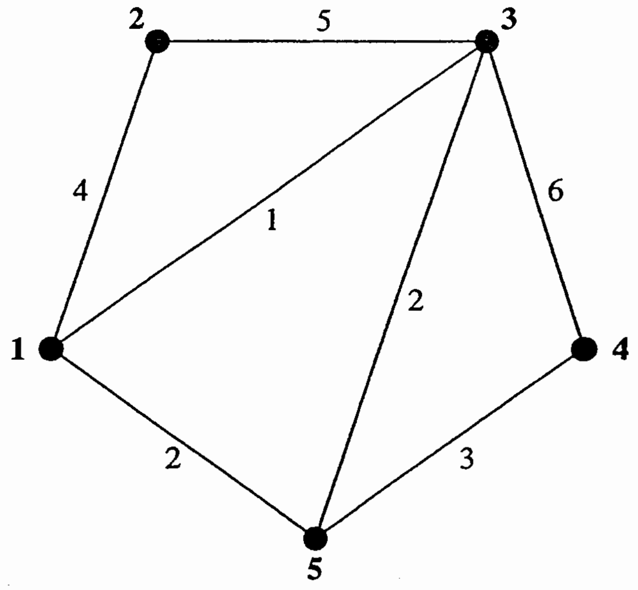

Fig. 3

(i) Give a walk of minimum length which traverses every arc at least once, and which returns to the start. Give the length of your walk. [3]

(ii) The insert shows the initial tables and the results of iterations 1, 2, 4 and 5 when Floyd's algorithm is applied to the network.

(*A*) Complete the two tables for iteration 3. [6]

(*B*) Use the final route table to give the shortest route from vertex **4** to vertex **2**. [1]

(*C*) Use the final distance table to draw a complete network with weights representing the shortest distances between vertices. [2]

(iii) Using the complete network of shortest distances, find a lower bound for the solution to the travelling salesperson problem by deleting vertex **1** and its arcs, and by finding the length of a minimum connector for the remainder.
(You may find the minimum connector by inspection.) [3]

(iv) Use the nearest neighbour algorithm, starting at vertex **1**, to produce a Hamilton cycle in the complete network. Give the length of your cycle. [3]

(v) Interpret your Hamilton cycle in part **(iv)** in terms of the original network. [2]

[Total: 20]

4 Three products, X, Y and Z, are to be manufactured. They all require resources A, B, C and D which are in limited supply. Table 4 summarises these requirements in suitable units per item produced.

	A	B	C	D
X	2	0	2	4
Y	5	2	4	3
Z	4	1	2	2
Availability	60	10	70	180

Table 4

Profits are £3 per item of X produced, £2 per item of Y and £5 per item of Z.

(i) Formulate a linear programming problem to maximise profit within the constraints imposed by resource availabilities. [5]

(ii) Use the simplex algorithm to solve the problem. [9]

(iii) An extra constraint is imposed by a contract to supply at least 5 items of Y. Show how to incorporate this constraint into an initial tableau using a surplus and an additional variable. [2]

(iv) Explain how to use either two-stage simplex or the big-M method to move to a feasible solution to the modified problem. You should show the initial tableau, including the objective function(s), and explain briefly how to proceed. You are not required to do the iterations. [4]

[Total: 20]

Candidate Name	Centre Number	Candidate Number

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MEI STRUCTURED MATHEMATICS **2621/1**

Decision and Discrete Mathematics 2

INSERT

Tuesday **19 JUNE 2001** Morning 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

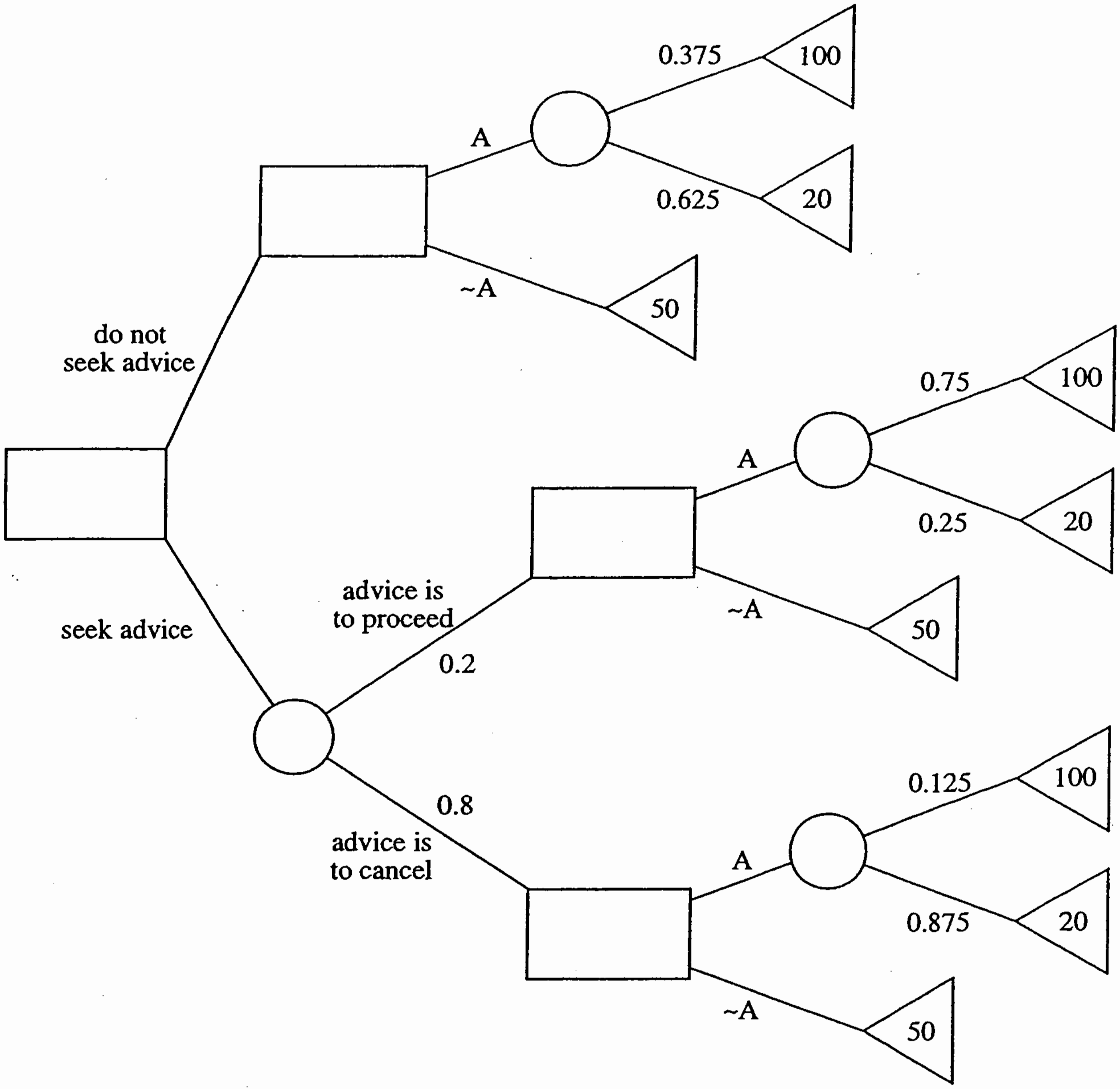
This insert should be used in Question **2** part **(ii)** and Question **3** part **(ii)**.

Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.

This insert should be attached securely to your answer booklet.

This insert consists of 3 printed pages and 1 blank page.

2 (ii)



0.375
100
A
0.625
20
~A
50
do not
seek advice
0.75
100
A
0.25
20
~A
50
advice is
to proceed
0.2
seek advice
0.125
100
0.8
advice is
to cancel
A
0.875
20
~A
50

Fig. 2.2

3 (ii)

	1	2	3	4	5
1	∞	4	1	∞	2
2	4	∞	5	∞	∞
3	1	5	∞	6	2
4	∞	∞	6	∞	3
5	2	∞	2	3	∞

	1	2	3	4	5
1	1	2	3	4	5
2	1	2	3	4	5
3	1	2	3	4	5
4	1	2	3	4	5
5	1	2	3	4	5

	1	2	3	4	5
1	∞	4	1	∞	2
2	4	8	5	∞	6
3	1	5	2	6	2
4	∞	∞	6	∞	3
5	2	6	2	3	4

	1	2	3	4	5
1	1	2	3	4	5
2	1	1	3	4	1
3	1	2	1	4	5
4	1	2	3	4	5
5	1	1	3	4	1

	1	2	3	4	5
1	8	4	1	∞	2
2	4	8	5	∞	6
3	1	5	2	6	2
4	∞	∞	6	∞	3
5	2	6	2	3	4

	1	2	3	4	5
1	2	2	3	4	5
2	1	1	3	4	1
3	1	2	1	4	5
4	1	2	3	4	5
5	1	1	3	4	1

	1	2	3	4	5
1					
2					
3					
4					
5					

	1	2	3	4	5
1					
2					
3					
4					
5					

	1	2	3	4	5
1	2	4	1	7	2
2	4	8	5	11	6
3	1	5	2	6	2
4	7	11	6	12	3
5	2	6	2	3	4

	1	2	3	4	5
1	3	2	3	3	5
2	1	1	3	3	1
3	1	2	1	4	5
4	3	3	3	3	5
5	1	1	3	4	1

	1	2	3	4	5
1	2	4	1	5	2
2	4	8	5	9	6
3	1	5	2	5	2
4	5	9	5	6	3
5	2	6	2	3	4

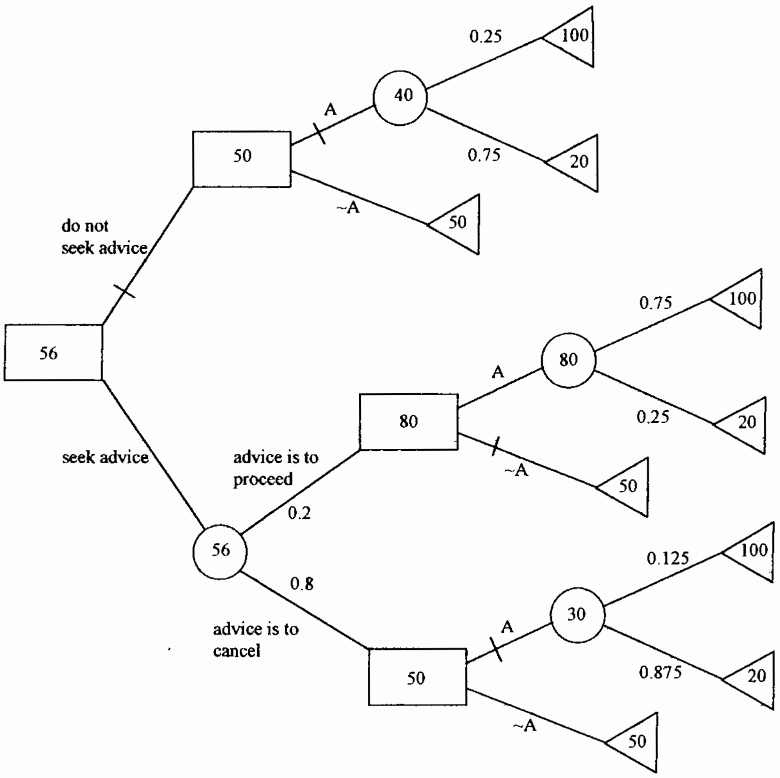
	1	2	3	4	5
1	3	2	3	5	5
2	1	1	3	1	1
3	1	2	1	5	5
4	5	5	5	5	5
5	1	1	3	4	1

Mark Scheme

1.

(i) $(\sim s \Rightarrow \sim n) \wedge n$	M1 $\Rightarrow$ A1 2 x ~ A1 B1 $\wedge n$
(ii) $(\sim s \Rightarrow \sim n) \wedge n \Rightarrow s$ 10 1 10 0 0 1 0 10 0 01 0 1 1 0 01 1 10 0 0 1 1 01 1 01 1 1 1 1	M1 4 rows A3 (–1 each error)
(iii) $s \Rightarrow r$ so $\sim r \Rightarrow \sim s$ But $\sim s \Rightarrow \sim n$ So can deduce no snow and no north wind.	B1 B1

2.

(i) Require 100p + 20(1–p) = 50, giving p = 3/8	M1 A1
(ii) (A) 	M1 handling chance nodes A2 (–1 each error) M1 handling choice nodes A1
(B) Value of advice = 6	B1
(C) Assuming it's not > 80, require X = 16 + 0.8X, giving X = 80.	M1 A1

3.

(i) eg. **151354321** 23+2 = 25

M1 **15** repeated
A1 route
A1 25

(ii)(A)

	1	2	3	4	5
1	2	4	1	7	2
2	4	8	5	11	6
3	1	5	2	6	2
4	7	11	6	12	3
5	2	6	2	3	4

M1
A2 (Ĕ1 each error)

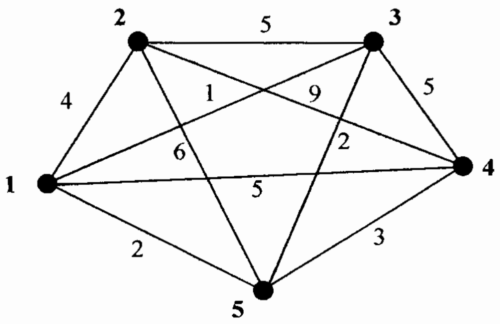
	1	2	3	4	5
1	3	2	3	3	5
2	1	1	3	3	1
3	1	2	1	4	5
4	3	3	3	3	5
5	1	1	3	4	1

M1
A2 (Ĕ1 each error)

(B) 2→1→5→4

B1

(C)



M1 complete
A1

(iii) (5+2+3) + (1+2) = 13

M1
A1 (5+2+3)
A1 (1+2)

(iv) 1→3→5→4→2→1 length = 19

M1
A1 A1

(v) 1→3→5→4→5→1→2→1

M1 A1

4.

(i) Let x be number of units of X, ...

Max $3x + 2y + 5z$
s.t. $2x + 5y + 4z \le 60$
$2y + z \le 10$
$2x + 4y + 2z \le 70$
$4x + 3y + 2z \le 180$

B1 explicit identification
M1
A3 –1 each error

(ii)

P	x	y	z	s1	s2	s3	s4	RHS
1	–3	–2	–5	0	0	0	0	0
0	2	5	4	1	0	0	0	60
0	0	2	1	0	1	0	0	10
0	2	4	2	0	0	1	0	70
0	4	3	2	0	0	0	1	180
1	–3	8	0	0	5	0	0	50
0	2	–3	0	1	–4	0	0	20
0	0	2	1	0	1	0	0	10
0	2	0	0	0	–2	1	0	50
0	4	–1	0	0	–2	0	1	160
1	0	3.5	0	1.5	–1	0	0	80
0	1	–1.5	0	0.5	–2	0	0	10
0	0	2	1	0	1	0	0	10
0	0	3	0	–1	2	1	0	30
0	0	5	0	–2	6	0	1	120
1	0	5.5	1	1.5	0	0	0	90
0	1	2.5	2	0.5	0	0	0	30
0	0	2	1	0	1	0	0	10
0	0	–1	–2	–1	0	1	0	10
0	0	–7	–6	–2	0	0	1	60

x = 30, y = 0, z = 0, giving a profit of 90

M1 initial tableau
A1

M1 1st pivot
A1

M1 2nd pivot
A1

M1 3rd pivot
A1

B1 answer

(iii)

P	x	y	z	s1	s2	s3	s4	s5	a5	RHS
0	0	1	0	0	0	0	0	–1	1	5

M1 A1

(iv)

Q	P	x	y	z	s1	s2	s3	s4	s5	a5	RHS
1	0	0	1	0	0	0	0	0	–1	0	5
0	1	–3	–2	–5	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	–1	1	5
0	0	2	5	4	1	0	0	0	0	0	60
0	0	0	2	1	0	1	0	0	0	0	10
0	0	2	4	2	0	0	1	0	0	0	70
0	0	4	3	2	0	0	0	1	0	0	180

M1 A1 tableau

Start by minimising Q, pivoting on y column. Drop Q row and a5 column when Q attains 0, and proceed as usual.

M1 A1

Or use the single objective

P	x	y	z	s1	s2	s3	s4	s5	a5	RHS
1	–3	–2–M	–5	0	0	0	0	M		–5M

Examiner’s Report

Decision and Discrete Mathematics 2 (2621)

General Comments

This was the first sitting of this examination, and there was only a small number of candidates. Performances were generally good.

Comments on Individual Questions

Question 1 (logic)

This question hinged on understanding that "if ... then ..." is expressed as $\Rightarrow$. Follow through was applied as far as possible to the large numbers of candidates who did not make that step, but subsequent work was not always of much value.

It would help the markers, and some candidates, if truth tables were organised with rows in binary order. They were often not, and lines were sometimes omitted as a consequence.

(i) $(\sim s \Rightarrow \sim n) \wedge n$ (iii) no snow and no north wind

Question 2 (decision analysis)

Most candidates were successful with part (i), weaker candidates using a verification approach.

Part (ii) was also answered well, although the expected number of errors were seen in the handling of chance and choice nodes. Few managed the final calculation.

(ii) value of advice = 6; 50 would have to increase to 80

Question 3 (networks)

Part (i) was found to be easy, although mistakes were made.

Part (ii) was the first time in which Floyd's algorithm has been tested, and candidates were very well prepared for it. Understanding was good.

Parts (iii), (iv) and (v) tested candidates on the TSP, and they responded well.

(i) length = 25 (ii)(B) 4-5-1-2 (iii) 13 (iv) 1-3-5-4-2-1; 19 (v) 1-3-5-4-5-1-2-1

Question 4 (simplex)

Candidates had been well prepared for this question.

In part (i) they often omitted to define their variables. This is an essential step, even if for this problem it was a trivial one.

Candidates scored well in part (ii). The occasional mistake did not lead to any serious problems.

Part (iii) was well answered, but not all were able to produce the two extra rows required in a two-stage simplex tableau. One or two candidates tried the big-M approach, usually successfully.

(ii) $x = 30$, $y = 0$, $z = 0$, giving a profit of 90