GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper C

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.

- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.

- The total number of marks for this paper is 72.

- **You are reminded of the need for clear presentation in your answers.**
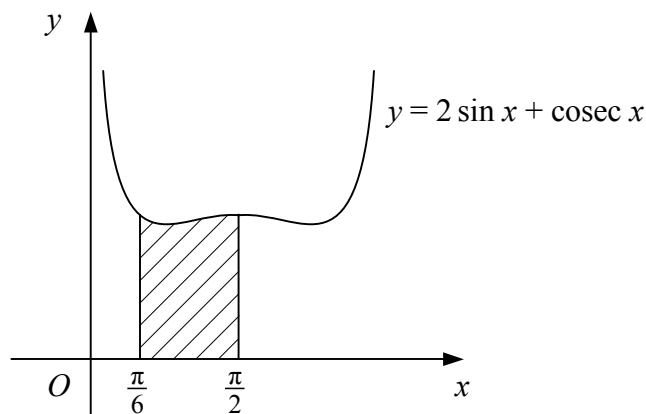
1. A curve has the equation

$$x^2(2 + y) - y^2 = 0.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of x and y. [5]

2. Show that

$$\int_1^2 x \ln x \; dx = 2 \ln 2 - \tfrac{3}{4}.$$ [5]

3.

The diagram shows the curve with equation $y = 2 \sin x + \operatorname{cosec} x, \; 0 < x < \pi.$

The shaded region bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ is rotated through four right angles about the x-axis.

Show that the volume of the solid formed is $\frac{1}{2}\pi(4\pi + 3\sqrt{3}\,)$. [7]

4. *(i)* Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form. [3]

(ii) Simplify

$$\frac{x^3 - 8}{3x^2 - 8x + 4}.$$ [5]

5. A bath is filled with hot water which is allowed to cool. The temperature of the water is $\theta°C$ after cooling for t minutes and the temperature of the room is assumed to remain constant at 20°C.

Given that the rate at which the temperature of the water decreases is proportional to the difference in temperature between the water and the room,

(i) write down a differential equation connecting θ and t. [1]

Given also that the temperature of the water is initially 37°C and that it is 36°C after cooling for four minutes,

(ii) find, to 3 significant figures, the temperature of the water after ten minutes. [8]

Advice suggests that the temperature of the water should be allowed to cool to 33°C before a child gets in.

(iii) Find, to the nearest second, how long a child should wait before getting into the bath. [2]

6. A curve has parametric equations

$$x = 3 \cos^2 t, \quad y = \sin 2t, \quad 0 \le t < \pi.$$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\tfrac{2}{3} \cot 2t.$ [3]

(ii) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. [3]

(iii) Show that the tangent to the curve at the point where $t = \tfrac{\pi}{6}$ has the equation

$$2x + 3\sqrt{3}\,y = 9.$$ [3]

(iv) Find a cartesian equation for the curve in the form $y^2 = \mathrm{f}(x)$. [3]

Turn over

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7. Relative to a fixed origin, the points A and B have position vectors $\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$

respectively.

(i)   Find a vector equation for the line l_1 which passes through A and B.   [2]

The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}.$$

(ii)   Show that lines l_1 and l_2 do not intersect.   [4]

(iii)   Find the position vector of the point C on l_2 such that $\angle ABC = 90°$.   [6]

8. $$\mathrm{f}(x) = \frac{5 - 8x}{(1 + 2x)(1 - x)^2}.$$

(i)   Express f(x) in partial fractions.   [5]

(ii)   Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, simplifying each coefficient.   [6]

(iii)   State the set of values of x for which your expansion is valid.   [1]