FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper B

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. Find $\int x\,e^{3x}\ dx$. [4]

2. Find the quotient and remainder when $(x^4 + x^3 - 5x^2 - 9)$ is divided by $(x^2 + x - 6)$. [4]

3. Differentiate each of the following with respect to x and simplify your answers.

(i) $\cot x^2$ [2]

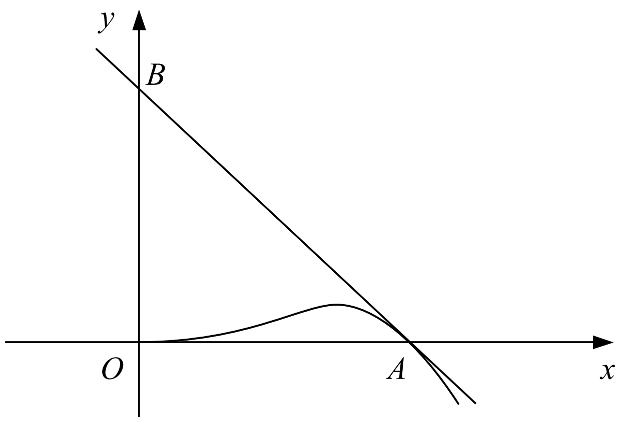
(ii) $\dfrac{\sin x}{3 + 2\cos x}$ [4]

4. *(i)* Expand $(1 - 3x)^{-2}$, $|x| < \frac{1}{3}$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [4]

(ii) Hence, or otherwise, show that for small x,

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3.$$ [3]

5.



The diagram shows the curve with parametric equations

$$x = a\sqrt{t}\,,\quad y = at(1 - t),\quad t \geq 0,$$

where a is a positive constant.

(i) Find $\dfrac{dy}{dx}$ in terms of t. [3]

The curve meets the x-axis at the origin, O, and at the point A. The tangent to the curve at A meets the y-axis at the point B as shown.

(ii) Show that the area of triangle OAB is a^2. [5]

6. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = (7\mathbf{j} - 4\mathbf{k}) + s(4\mathbf{i} - 3\mathbf{j} + \mathbf{k}),$$

and $$\mathbf{r} = (-7\mathbf{i} + \mathbf{j} + 8\mathbf{k}) + t(-3\mathbf{i} + 2\mathbf{k}),$$

where s and t are scalar parameters.

(i) Show that the two lines intersect and find the position vector of the point where they meet. [5]

(ii) Find, in degrees to 1 decimal place, the acute angle between the lines. [4]

7. At time $t = 0$, a tank of height 2 metres is completely filled with water. Water then leaks from a hole in the side of the tank such that the depth of water in the tank, y metres, after t hours satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -k\mathrm{e}^{-0.2t},$$

where k is a positive constant,

(i) Find an expression for y in terms of k and t. [4]

Given that two hours after being filled the depth of water in the tank is 1.6 metres,

(ii) find the value of k to 4 significant figures. [2]

Given also that the hole in the tank is h cm above the base of the tank,

(iii) show that $h = 79$ to 2 significant figures. [3]

Turn over

8. A curve has the equation

$$x^2 - 4xy + 2y^2 = 1.$$

(i) Find an expression for $\frac{dy}{dx}$ in its simplest form in terms of x and y. [4]

(ii) Show that the tangent to the curve at the point P (1, 2) has the equation

$$3x - 2y + 1 = 0.$$ [3]

The tangent to the curve at the point Q is parallel to the tangent at P.

(iii) Find the coordinates of Q. [4]

9. *(i)* Show that the substitution $u = \sin x$ transforms the integral

$$\int \frac{6}{\cos x(2 - \sin x)}\ dx$$

into the integral

$$\int \frac{6}{(1-u^2)(2-u)}\ du.$$ [4]

(ii) Express $\frac{6}{(1-u^2)(2-u)}$ in partial fractions. [4]

(iii) Hence, evaluate

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos x(2 - \sin x)}\ dx,$$

giving your answer in the form $a \ln 2 + b \ln 3$, where a and b are integers. [6]