FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper K

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper K – Marking Guide

1.		$u = x,\ u' = 1,\ v' = \cos x,\ v = \sin x$	M1	
		$I = [x \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \ \mathrm{d}x$	A1	
		$= [x \sin x + \cos x]_0^{\frac{\pi}{2}}$	A1	
		$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	M1 A1	**(5)**
2.	*(i)*	$= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}$	B1	
		$= \frac{1}{8}[1 + (-3)(-\frac{3}{2}x) + \frac{(-3)(-4)}{2}(-\frac{3}{2}x)^2 + \frac{(-3)(-4)(-5)}{3\times2}(-\frac{3}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \ldots$	A3	
	(ii)	$\lvert x \rvert < \frac{2}{3}$	B1	**(6)**
3.	*(i)*	$\frac{x+11}{(x+4)(x-3)} \equiv \frac{A}{x+4} + \frac{B}{x-3}, \quad x + 11 \equiv A(x-3) + B(x+4)$	M1	
		$x = -4 \Rightarrow 7 = -7A \Rightarrow A = -1$	A1	
		$x = 3 \Rightarrow 14 = 7B \Rightarrow B = 2$	A1	
		$\frac{x+11}{(x+4)(x-3)} \equiv \frac{2}{x-3} - \frac{1}{x+4}$		
	(ii)	$= \int_0^2 (\frac{2}{x-3} - \frac{1}{x+4}) \ \mathrm{d}x$		
		$= [2 \ln\lvert x-3 \rvert - \ln\lvert x+4 \rvert]_0^2$	M1 A1	
		$= (0 - \ln 6) - (2 \ln 3 - \ln 4)$	M1	
		$= \ln \frac{2}{27}$	A1	**(7)**
4.		$8x - 2y - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	
		$(-1, -3) \Rightarrow -8 + 6 + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}$	M1 A1	
		grad of normal $= -4$	M1	
		$\therefore\ y + 3 = -4(x + 1) \qquad [\ y = -4x - 7\ ]$	M1 A1	**(7)**
5.		$u^2 = 1 - x \Rightarrow x = 1 - u^2,\ \frac{\mathrm{d}x}{\mathrm{d}u} = -2u$	M1	
		$x = 0 \Rightarrow u = 1,\ x = 1 \Rightarrow u = 0$	B1	
		area $= \int_0^1 x\sqrt{1-x} \ \mathrm{d}x = \int_1^0 (1 - u^2) \times u \times (-2u) \ \mathrm{d}u$	M1	
		$= \int_0^1 (2u^2 - 2u^4) \ \mathrm{d}u$	A1	
		$= [\frac{2}{3}u^3 - \frac{2}{5}u^5]_0^1$	M1	
		$= (\frac{2}{3} - \frac{2}{5}) - (0) = \frac{4}{15}$	M1 A1	**(7)**
6.	*(i)*	$\frac{\mathrm{d}n}{\mathrm{d}t} = 0 \Rightarrow e^{0.5t} = 5$	M1	
		$t = 2 \ln 5 = 3.219$ mins $=$ 3 mins 13 secs	M1 A1	
	(ii)	$\int \mathrm{d}n = \int (e^{0.5t} - 5) \ \mathrm{d}t$		
		$n = 2e^{0.5t} - 5t + c$	M1 A1	
		$t = 0,\ n = 20 \Rightarrow 20 = 2 + c, \quad c = 18$	M1	
		$n = 2e^{0.5t} - 5t + 18$	A1	
	(iii)	as t increases, n rapidly becomes very large $\therefore$ not realistic	B1	**(8)**

7.	*(i)*	let $\text{f}(x) = 2x^3 - x^2 + 4x + 15$		
		$\text{f}(-\tfrac{3}{2}) = -\tfrac{27}{4} - \tfrac{9}{4} - 6 + 15 = 0 \quad \therefore (2x+3)$ is a factor	B1	
		$$\begin{array}{r\|l} & x^2 - 2x + 5 \\ \hline 2x+3 & 2x^3 - x^2 + 4x + 15 \\ & 2x^3 + 3x^2 \\ \hline & -4x^2 + 4x \\ & -4x^2 - 6x \\ \hline & 10x + 15 \\ & 10x + 15 \end{array}$$	M1 A1	
		$\therefore \text{f}(x) = (2x+3)(x^2 - 2x + 5)$		
		$\therefore \dfrac{2x^2+x-3}{2x^3-x^2+4x+15} = \dfrac{(2x+3)(x-1)}{(2x+3)(x^2-2x+5)} = \dfrac{x-1}{x^2-2x+5}$	M1 A1	
	(ii)	$= \int_2^5 \dfrac{x-1}{x^2-2x+5}\,dx = [\tfrac{1}{2}\ln\|x^2-2x+5\|]_2^5$	M1 A1	
		$= \tfrac{1}{2}(\ln 20 - \ln 5) = \tfrac{1}{2}\ln 4 = \ln 2$	M1 A1	**(9)**
8.	*(i)*	$\overrightarrow{AB} = \begin{pmatrix}10\\-15\\5\end{pmatrix}, \quad \therefore \mathbf{r} = \begin{pmatrix}3\\9\\-7\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$	M1 A1	
	(ii)	$3 + 2\lambda = 9 \quad \therefore \lambda = 3$	M1	
		when $\lambda = 3$, $\mathbf{r} = \begin{pmatrix}3\\9\\-7\end{pmatrix} + 3\begin{pmatrix}2\\-3\\1\end{pmatrix} = \begin{pmatrix}9\\0\\-4\end{pmatrix} \quad \therefore (9, 0, -4)$ lies on l	A1	
	(iii)	$\overrightarrow{OD} = \begin{pmatrix}3+2\lambda\\9-3\lambda\\-7+\lambda\end{pmatrix} \quad \therefore \begin{pmatrix}3+2\lambda\\9-3\lambda\\-7+\lambda\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix} = 0$	M1	
		$6 + 4\lambda - 27 + 9\lambda - 7 + \lambda = 0$		
		$\lambda = 2 \quad \therefore \overrightarrow{OD} = \begin{pmatrix}7\\3\\-5\end{pmatrix}$, $D\,(7, 3, -5)$	M1 A1	
	(iv)	$AB = \sqrt{100+225+25} = \sqrt{350}$, $OD = \sqrt{49+9+25} + \sqrt{83}$	M1	
		area $= \tfrac{1}{2} \times \sqrt{350} \times \sqrt{83} = 85.2$ (3sf)	M1 A1	**(10)**
9.	*(i)*	$x + \dfrac{1}{x} = \sec\theta + \tan\theta + \dfrac{1}{\sec\theta+\tan\theta} = \dfrac{(\sec\theta+\tan\theta)^2+1}{\sec\theta+\tan\theta}$	M1	
		$= \dfrac{\sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta + 1}{\sec\theta+\tan\theta} = \dfrac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta+\tan\theta}$	M1	
		$= \dfrac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}$	M1	
		$= 2\sec\theta$	A1	
	(ii)	$\dfrac{x^2+1}{x} = \dfrac{2}{\cos\theta} \Rightarrow \cos\theta = \dfrac{2x}{x^2+1}$	M1	
		$\dfrac{y^2+1}{y} = \dfrac{2}{\sin\theta} \Rightarrow \sin\theta = \dfrac{2y}{y^2+1}, \quad \therefore \dfrac{4x^2}{(x^2+1)^2} + \dfrac{4y^2}{(y^2+1)^2} = 1$	M1 A1	
	(iii)	$\dfrac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$	M1	
		$= \sec\theta(\tan\theta + \sec\theta) = \dfrac{x^2+1}{2x} \times x = \tfrac{1}{2}(x^2+1)$	M1 A1	
	(iv)	$\dfrac{dy}{d\theta} = -\text{cosec}\,\theta\cot\theta - \text{cosec}^2\,\theta$	M1	
		$= -\text{cosec}\,\theta(\cot\theta + \text{cosec}\,\theta) = -\dfrac{y^2+1}{2y} \times y = -\tfrac{1}{2}(y^2+1)$		
		$\therefore \dfrac{dy}{dx} = -\dfrac{y^2+1}{x^2+1}$	M1 A1	**(13)**
			Total	**(72)**

Performance Record – C4 Paper K

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	integration	binomial series	partial fractions, integration	differentiation	integration	differential equation	rational expressions, integration	vectors	parametric equations	
Marks	5	6	7	7	7	8	9	10	13	72
Student										