FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C4

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper A – Marking Guide

1.		$= \frac{2x}{2x^2+3x-5} \times \frac{x^2-x}{x^3}$	M1	
		$= \frac{2x}{(2x+5)(x-1)} \times \frac{x(x-1)}{x^3}$	M1	
		$= \frac{2}{x(2x+5)}$	M1 A1	**(4)**
2.		$4x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	M1 A1	
		$\frac{dy}{dx} = 0 \quad \therefore\ 4x + y = 0, \quad y = -4x$	M1 A1	
		sub. $\ 2x^2 - 4x^2 - 16x^2 + 18 = 0$	M1	
		$x^2 = 1,\ x = \pm 1 \ \therefore\ (-1, 4), (1, -4)$	A2	**(7)**
3.	*(i)*	$(1 + ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \ldots$	B1	
		$\therefore\ an = -4, \qquad \frac{a^2n(n-1)}{2} = 24$	B1	
		$\Rightarrow\ a = \frac{-4}{n}$, sub. $\Rightarrow\ \frac{16}{n^2} \times \frac{n(n-1)}{2} = 24$	M1 A1	
		$8(n-1) = 24n, \qquad n = -\frac{1}{2},\ a = 8$	M1 A1	
	(ii)	$(1 + 8x)^{-\frac{1}{2}} = \ldots + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3\times 2}(8x)^3 + \ldots$	M1	
		$\therefore\ k = -\frac{5}{16} \times 512 = -160$	A1	**(8)**
4.	*(i)*	$= \left\| \frac{1\times 6 + 5\times 3 + (-1)\times(-6)}{\sqrt{1+25+1}\times\sqrt{36+9+36}} \right\|$	M1 A1	
		$= \frac{27}{\sqrt{27}\times\sqrt{81}} = \frac{\sqrt{27}}{9} = \frac{3\sqrt{3}}{9} = \frac{1}{3}\sqrt{3}$	M1 A1	
	(ii)	$\sin(\angle AOB) = \sqrt{1 - (\frac{1}{3}\sqrt{3})^2} = \sqrt{\frac{2}{3}}$	M1	
		area $= \frac{1}{2} \times 3\sqrt{3} \times 9 \times \sqrt{\frac{2}{3}} = \frac{27}{2}\sqrt{2}$	M1 A1	
	(iii)	$= OA \times \sin(\angle AOB) = 3\sqrt{3} \times \sqrt{\frac{2}{3}} = 3\sqrt{2}$	M1 A1	**(9)**
5.	*(i)*	$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$		
		$= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$	M1 A1	
		$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$		
		$= \frac{1}{\cos^2 x}$	M1	
		$= \sec^2 x$	A1	
	(ii)	$\frac{dy}{dx} = 2 \times \tan x + 2x \times \sec^2 x = 2\tan x + 2x\sec^2 x$	M1 A1	
		$x = \frac{\pi}{4},\ y = \frac{\pi}{2}$, grad $= 2 + \pi$	B1	
		$\therefore\ y - \frac{\pi}{2} = (2 + \pi)(x - \frac{\pi}{4})$	M1	
		at P, $x = 0$		
		$\therefore\ y = \frac{\pi}{2} - \frac{\pi}{4}(2 + \pi) = -\frac{1}{4}\pi^2$	M1 A1	**(10)**

6.	*(i)*	$= \int (\text{cosec}^2 2x - 1)\ dx$	M1	
		$= -\frac{1}{2}\cot 2x - x + c$	M1 A1	
	(ii)	$u^2 = x + 1 \Rightarrow x = u^2 - 1,\ \frac{dx}{du} = 2u$	M1	
		$x = 0 \Rightarrow u = 1,\ x = 3 \Rightarrow u = 2$	B1	
		$I = \int_1^2 \frac{(u^2-1)^2}{u} \times 2u\ du = \int_1^2 (2u^4 - 4u^2 + 2)\ du$	M1 A1	
		$= [\frac{2}{5}u^5 - \frac{4}{3}u^3 + 2u]_1^2$	M1	
		$= (\frac{64}{5} - \frac{32}{3} + 4) - (\frac{2}{5} - \frac{4}{3} + 2) = 5\frac{1}{15}$	M1 A1	**(10)**
7.	*(i)*	$\int \frac{1}{(x-6)(x-3)}\ dx = \int 2\ dt$	M1	
		$\frac{1}{(x-6)(x-3)} \equiv \frac{A}{x-6} + \frac{B}{x-3},\quad 1 \equiv A(x-3) + B(x-6)$	M1	
		$x = 6 \Rightarrow A = \frac{1}{3},\ x = 3 \Rightarrow B = -\frac{1}{3}$	A2	
		$\frac{1}{3}\int (\frac{1}{x-6} - \frac{1}{x-3})\ dx = \int 2\ dt$		
		$\ln\lvert x-6\rvert - \ln\lvert x-3\rvert = 6t + c$	M1 A1	
		$t = 0,\ x = 0\ \therefore \ln 6 - \ln 3 = c,\quad c = \ln 2$	M1 A1	
		$x = 2 \Rightarrow \ln 4 - 0 = 6t + \ln 2$	M1	
		$t = \frac{1}{6}\ln 2 = 0.1155$ hrs $= 0.1155 \times 60$ mins $= 6.93$ mins ≈ 7 mins	A1	
	(ii)	$\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert = 6t,\quad t = \frac{1}{6}\ln\left\lvert\frac{x-6}{2(x-3)}\right\rvert$		
		as $x \to 3,\ t \to \infty\ \therefore$ cannot make 3 g	B2	**(12)**
8.	*(i)*	$x = 1\ \therefore -1 + 4\cos\theta = 1,\ \cos\theta = \frac{1}{2},\ \theta = \frac{\pi}{3}, \frac{5\pi}{3}$	M1	
		$y > 0\ \therefore \sin\theta > 0\ \therefore \theta = \frac{\pi}{3}$	A1	
	(ii)	$\frac{dx}{d\theta} = -4\sin\theta,\quad \frac{dy}{d\theta} = 2\sqrt{2}\cos\theta$	M1	
		$\therefore \frac{dy}{dx} = \frac{2\sqrt{2}\cos\theta}{-4\sin\theta}$	M1 A1	
		at P, grad $= -\frac{2\sqrt{2}\times\frac{1}{2}}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{2}}{2\sqrt{3}}$	M1	
		grad of normal $= \frac{2\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{6}$	A1	
		$\therefore y - \sqrt{6} = \sqrt{6}(x - 1)$	M1	
		$y = \sqrt{6}x$, when $x = 0, y = 0\ \therefore$ passes through origin	A1	
	(iii)	$\cos\theta = \frac{x+1}{4},\ \sin\theta = \frac{y}{2\sqrt{2}}$	M1	
		$\therefore \frac{(x+1)^2}{16} + \frac{y^2}{8} = 1$	M1 A1	**(12)**
			Total	**(72)**

Performance Record – C4 Paper A

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	rational expressions	differentiation	binomial series	vectors	differentiation	integration	differential equation, partial fractions	parametric equations	
Marks	4	7	8	9	10	10	12	12	72
Student									