FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper I

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Solomon Press

Written by Shaun Armstrong

1. A balloon is filled with air at a constant rate of 80 cm^3 per second.

Assuming that the balloon is spherical as it is filled, find to 3 significant figures the rate at which its radius is increasing at the instant when its radius is 6 cm. [5]

2. Solve the equation

$$3 \operatorname{cosec} \theta° + 8 \cos \theta° = 0$$

for θ in the interval $0 \leq \theta \leq 180$, giving your answers to 1 decimal place. [6]

3. *(a)* Given that $y = \ln x$,

(i) find an expression for $\ln \frac{x^2}{e}$ in terms of y, [2]

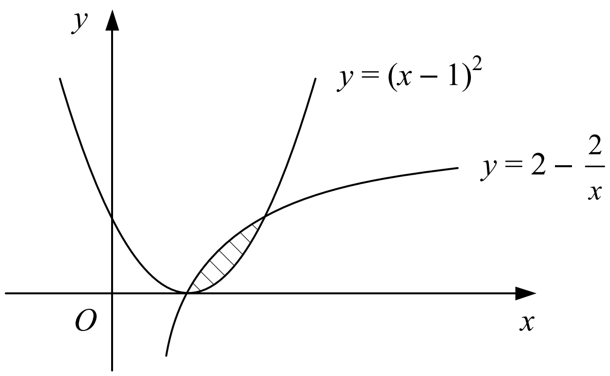
(ii) show that $\log_2 x = \frac{y}{\ln 2}$. [3]

(b) Hence, or otherwise, solve the equation

$$\log_2 x = 4 - \ln \frac{x^2}{e},$$

giving your answer to 2 decimal places. [3]

4.



The diagram shows the curves $y = (x - 1)^2$ and $y = 2 - \frac{2}{x}$, $x > 0$.

(i) Verify that the two curves meet at the points where $x = 1$ and where $x = 2$. [2]

The shaded region bounded by the two curves is rotated completely about the x-axis.

(ii) Find the exact volume of the solid formed. [7]

5. $$f(x) = 5 + e^{2x-3}, \quad x \in \mathbb{R}.$$

(i) State the range of f. [1]

(ii) Find an expression for $f^{-1}(x)$ and state its domain. [3]

(iii) Solve the equation $f(x) = 7$. [2]

(iv) Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. [4]

6. *(i)* Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(ii) State the maximum value of $\sqrt{3}\sin\theta + \cos\theta$ and the smallest positive value of θ for which this maximum value occurs. [3]

(iii) Solve the equation

$$\sqrt{3}\sin\theta + \cos\theta + \sqrt{3} = 0,$$

for θ in the interval $-\pi \le \theta \le \pi$, giving your answers in terms of π. [4]

7. $$f(x) = \frac{x^2 + 3}{4x + 1}, \quad x \in \mathbb{R}, \quad x \ne -\tfrac{1}{4}.$$

(i) Find and simplify an expression for $f'(x)$. [3]

(ii) Find the set of values of x for which $f(x)$ is increasing. [4]

(iii) Use Simpson's rule with six strips to find an approximate value for

$$\int_0^6 f(x)\ dx.$$ [3]

Turn over

8. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow |2x - 5|, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \rightarrow \ln (x + 3), \quad x \in \mathbb{R}, \quad x > -3.$$

(i) State the range of f. [1]

(ii) Evaluate fg(–2). [2]

(iii) Solve the equation

$$\mathrm{fg}(x) = 3,$$

giving your answers in exact form. [5]

(iv) Show that the equation

$$\mathrm{f}(x) = \mathrm{g}(x)$$

has a root, α, in the interval [3, 4]. [2]

(v) Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}[5 + \ln (x_n + 3)],$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures. [2]

(vi) Show that your answer for x_4 is the value of α correct to 4 significant figures. [2]