GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper F

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.

- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.

- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.

- The total number of marks for this paper is 72.

- **You are reminded of the need for clear presentation in your answers.**

1. Evaluate

$$\int_{2}^{6} \sqrt{3x-2} \ dx.$$ [4]

2. Differentiate each of the following with respect to x and simplify your answers.

(i) $\dfrac{6}{\sqrt{2x-7}}$ [2]

(ii) $x^2 e^{-x}$ [3]

3. (i) Prove the identity

$$\sqrt{2} \cos (x + 45)° + 2 \cos (x - 30)° \equiv (1 + \sqrt{3}) \cos x°.$$ [4]

(ii) Hence, find the exact value of $\cos 75°$ in terms of surds. [3]

4. $$f(x) = x^2 + 5x - 2 \sec x, \quad x \in \mathbb{R}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

(i) Show that the equation $f(x) = 0$ has a root, α, such that $1 < \alpha < 1.5$ [2]

(ii) Show that a suitable rearrangement of the equation $f(x) = 0$ leads to the iterative formula

$$x_{n+1} = \cos^{-1} \left(\frac{2}{x_n^2 + 5x_n} \right).$$ [3]

(iii) Use the iterative formula in part (ii) with a starting value of 1.25 to find α correct to 3 decimal places. You should show the result of each iteration. [3]

5. The function f is defined by

$$f(x) \equiv 2 + \ln (3x - 2), \quad x \in \mathbb{R}, \quad x > \tfrac{2}{3}.$$

(i) Find the exact value of $ff(1)$. [2]

(ii) Find an equation for the tangent to the curve $y = f(x)$ at the point where $x = 1$. [4]
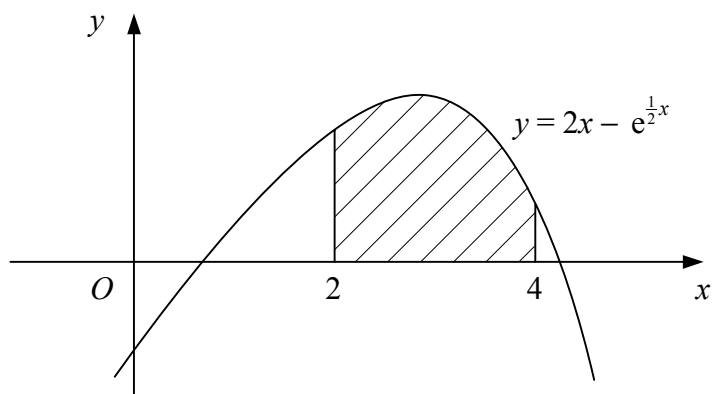
(iii) Find an expression for $f^{-1}(x)$. [2]

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6. *(i)* Sketch on the same diagram the graphs of $y = |x| - a$ and $y = |3x + 5a|$, where a is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes. [5]

(ii) Solve the equation

$$|x| - a = |3x + 5a|.$$ [4]

7.

The diagram shows the curve with equation $y = 2x - e^{\frac{1}{2}x}$.

The shaded region is bounded by the curve, the *x*-axis and the lines $x = 2$ and $x = 4$.

(i) Find the area of the shaded region, giving your answer in terms of e. [4]

The shaded region is rotated through four right angles about the *x*-axis.

(ii) Using Simpson's rule with two strips, estimate the volume of the solid formed. [5]

8. *(i)* Sketch on the same diagram the graphs of

$$y = \sin^{-1} x, \quad -1 \le x \le 1$$

and $\qquad y = \cos^{-1}(2x), \quad -\frac{1}{2} \le x \le \frac{1}{2}.$ [3]

Given that the graphs intersect at the point with coordinates (a, b),

(ii) show that $\tan b = \frac{1}{2}$, [3]

(iii) find the value of a in the form $k\sqrt{5}$. [4]

Turn over

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9. $$f(x) = e^{3x+1} - 2, \quad x \in \mathbb{R}.$$

(i) State the range of f. [1]

The curve $y = f(x)$ meets the y-axis at the point P and the x-axis at the point Q.

(ii) Find the exact coordinates of P and Q. [3]

(iii) Show that the tangent to the curve at P has the equation

$$y = 3ex + e - 2.$$ [4]

(iv) Find to 3 significant figures the x-coordinate of the point where the tangent to the curve at P meets the tangent to the curve at Q. [4]