

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper G

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

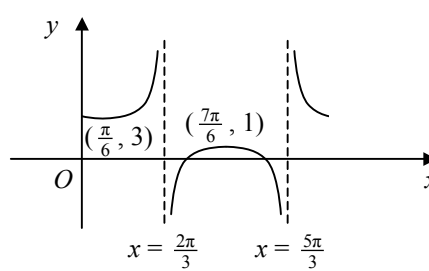


Written by Shaun Armstrong

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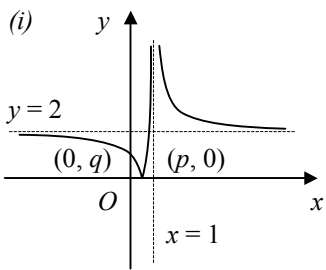
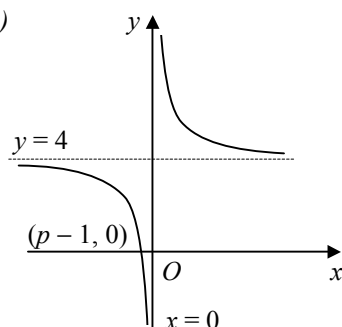
C3 Paper G – Marking Guide

1. (i) $\ln(2 + \cos a) = 0$, $2 + \cos a = 1$, $\cos a = -1$, $a = \pi$ M1 A1
- (ii) x 0 $\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$ π
 y 1.0986 0.9959 0.6931 0.2569 0 M1
 area $\approx \frac{1}{3} \times \frac{\pi}{4} \times [1.0986 + 0 + 4(0.9959 + 0.2569) + 2(0.6931)]$ M1
 $= 1.96$ (3sf) A1 (5)
-
2. (i) $= f(2) = -2$ M1 A1
- (ii) $gf(x) = g(2 - x^2) = \frac{3(2 - x^2)}{2(2 - x^2) - 1} = \frac{6 - 3x^2}{3 - 2x^2}$ M1 A1
- $\therefore \frac{6 - 3x^2}{3 - 2x^2} = \frac{1}{2}$, $2(6 - 3x^2) = 3 - 2x^2$
 $x^2 = \frac{9}{4}$ M1
 $x = \pm \frac{3}{2}$ A1 (6)
-
3. $\frac{dy}{dx} = \frac{1 \times (x^2 - 2x + 5) - (x - 1)(2x - 2)}{(x^2 - 2x + 5)^2} = \frac{-x^2 + 2x + 3}{(x^2 - 2x + 5)^2}$ M1 A2
- SP: $\frac{-x^2 + 2x + 3}{(x^2 - 2x + 5)^2} = 0$, $-x^2 + 2x + 3 = 0$
 $-(x + 1)(x - 3) = 0$ M1
 $x = -1, 3$ A1
 $\therefore (-1, -\frac{1}{4}), (3, \frac{1}{4})$ A1 (6)
-
4. (i)  M2 A2
- (ii) $2 + \sec(x - \frac{\pi}{6}) = 0$
 $\sec(x - \frac{\pi}{6}) = -2$, $\cos(x - \frac{\pi}{6}) = -\frac{1}{2}$ M1
 $x - \frac{\pi}{6} = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}$ M1
 $x = \frac{5\pi}{6}, \frac{3\pi}{2}$ A2 (8)
-
5. (i) $x = 3$, $y = \sqrt{20} = 2\sqrt{5}$ B1
 $\frac{dy}{dx} = \frac{1}{2}(3x + 11)^{-\frac{1}{2}} \times 3 = \frac{3}{2}(3x + 11)^{-\frac{1}{2}}$ M1
 grad $= \frac{3}{4\sqrt{5}}$ A1
 $\therefore y - 2\sqrt{5} = \frac{3}{4\sqrt{5}}(x - 3)$ M1
 $4\sqrt{5}y - 40 = 3x - 9$
 $3x - 4\sqrt{5}y + 31 = 0$ A1
- (ii) normal: $y - 2\sqrt{5} = -\frac{4\sqrt{5}}{3}(x - 3)$ M1
 at Q , $x = 0 \therefore y - 2\sqrt{5} = 4\sqrt{5}$ M1
 $y = 6\sqrt{5}$ A1 (8)

6. (i) $3 \cos x + \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$
 $R \cos \alpha = 3, R \sin \alpha = 1$ M1
 $\therefore R = \sqrt{3^2 + 1^2} = \sqrt{10}$ A1
 $\tan \alpha = \frac{1}{3}, \alpha = 18.4$ (3sf) A1
 $\therefore 3 \cos x + \sin x = \sqrt{10} \cos(x - 18.4)^\circ$
- (ii) $6 \cos^2 x + 2 \sin x \cos x = 0$ M1
 $2 \cos x(3 \cos x + \sin x) = 0$ M1
 $\cos x = 0$ or $3 \cos x + \sin x = \sqrt{10} \cos(x - 18.4) = 0$ A1
 $x = 90, 270$ or $x - 18.4 = 90, 270$
 $x = 90, 108.4$ (1dp), $270, 288.4$ (1dp) A2 (8)

7. (i) $= \int_1^4 (x + \frac{2}{x}) dx = [\frac{1}{2}x^2 + 2 \ln|x|]_1^4$ M1 A1
 $= (8 + 2 \ln 4) - (\frac{1}{2} + 0) = 7\frac{1}{2} + 2 \ln 4$ M1 A1
- (ii) $= \pi \int_1^4 (x + \frac{2}{x})^2 dx = \pi \int_1^4 (x^2 + 4 + 4x^{-2}) dx$ M1
 $= \pi [\frac{1}{3}x^3 + 4x - 4x^{-1}]_1^4$ M1 A1
 $= \pi[(\frac{64}{3} + 16 - 1) - (\frac{1}{3} + 4 - 4)] = 36\pi$ M1 A1 (9)

8. (i) $P = 30 + 50e^{0.002 \times 30} = 83.1$ M1
 \therefore population = 83 100 (3sf) A1
- (ii) $30 + 50e^{0.002t} > 84, e^{0.002t} > \frac{54}{50}$ M1
 $t > \frac{1}{0.002} \ln \frac{54}{50}, t > 38.5 \therefore 2018$ M1 A1
- (iii) $30 + 50e^{0.002t} = 26 + 50e^{0.003t}, 50e^{0.003t} - 50e^{0.002t} = 4$ M1
 $e^{0.003t} - e^{0.002t} = 0.08, e^{0.002t}(e^{0.001t} - 1) = 0.08$ M1
 $e^{0.001t} - 1 = 0.08e^{-0.002t}$
 $0.001t = \ln(1 + 0.08e^{-0.002t})$
 $t = 1000 \ln(1 + 0.08e^{-0.002t})$ A1
- (iv) $t_1 = 69.887, t_2 = 67.251, t_3 = 67.595$ M1 A1
 $\therefore 2047$ A1 (11)

9. (a) (i)  (ii) 

- (b) $y = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2} \therefore p = \frac{1}{2}$ M1 A1
 $x = 0 \Rightarrow y = 1 \therefore q = 1$ B1
- (c) $y = \frac{2x-1}{x-1}, y(x-1) = 2x-1, x(y-2) = y-1$ M1
 $x = \frac{y-1}{y-2}$
 $\therefore f^{-1}(x) = \frac{x-1}{x-2}$ M1 A1 (11)

Total (72)

Performance Record – C3 Paper G

| Question no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|--------------|----------------|-----------|-----------------|--------------|-----------------|--------------|-------------|--|-----------|-------|
| Topic(s) | Simpson's rule | functions | differentiation | trigonometry | differentiation | trigonometry | integration | exponentials and logarithms, numerical methods | functions | |
| Marks | 5 | 6 | 6 | 8 | 8 | 8 | 9 | 11 | 11 | 72 |
| Student | | | | | | | | | | |
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