

GCE Examinations  
Advanced / Advanced Subsidiary

## Core Mathematics C2

Paper J

Time: 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

### INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**



*Written by Shaun Armstrong*

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1. A geometric progression has first term 75 and second term  $-15$ .
- (i) Find the common ratio. [2]
- (ii) Find the sum to infinity. [2]

2. Find the area of the finite region enclosed by the curve  $y = 5x - x^2$  and the  $x$ -axis. [6]

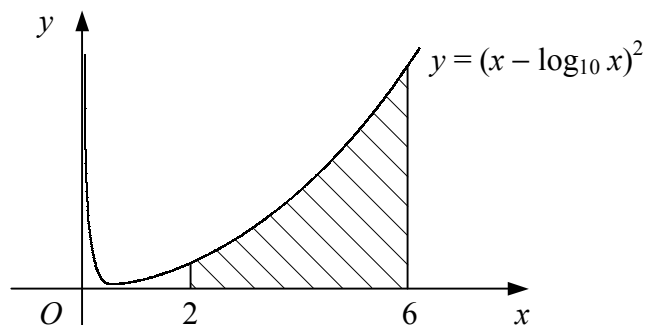
3. During one day, a biological culture is allowed to grow under controlled conditions. At 8 a.m. the culture is estimated to contain 20 000 bacteria. A model of the growth of the culture assumes that  $t$  hours after 8 a.m., the number of bacteria present,  $N$ , is given by

$$N = 20\,000 \times (1.06)^t.$$

Using this model,

- (i) find the number of bacteria present at 11 a.m., [2]
- (ii) find, to the nearest minute, the time when the initial number of bacteria will have doubled. [4]

4.



The diagram shows the curve with equation  $y = (x - \log_{10} x)^2$ ,  $x > 0$ .

- (i) Copy and complete the table below for points on the curve, giving the  $y$  values to 2 decimal places.

$x$	2	3	4	5	6
$y$	2.89	6.36			

[2]

The shaded region is bounded by the curve, the  $x$ -axis and the lines  $x = 2$  and  $x = 6$ .

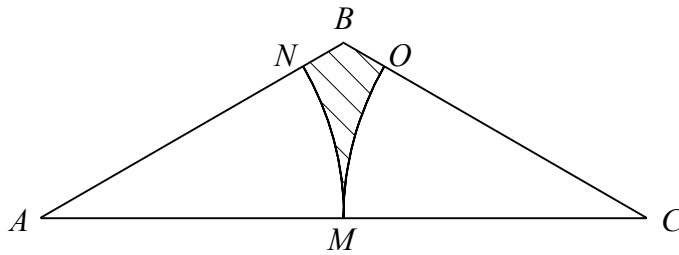
- (ii) Use the trapezium rule with all the values in your table to estimate the area of the shaded region. [3]
- (iii) State, with a reason, whether your answer to part (b) is an under-estimate or an over-estimate of the true area. [2]

5. (i) Given that  $\sin \theta = 2 - \sqrt{2}$ , find the value of  $\cos^2 \theta$  in the form  $a + b\sqrt{2}$  where  $a$  and  $b$  are integers. [3]

- (ii) Find, in terms of  $\pi$ , all values of  $x$  in the interval  $0 \leq x < \pi$  for which

$$\cos 3x = \frac{\sqrt{3}}{2}. \quad [5]$$

6.



The diagram shows triangle  $ABC$  in which  $AC = 8$  cm and  $\angle BAC = \angle BCA = 30^\circ$ .

- (i) Find the area of triangle  $ABC$  in the form  $k\sqrt{3}$ . [4]

The point  $M$  is the mid-point of  $AC$  and the points  $N$  and  $O$  lie on  $AB$  and  $BC$  such that  $MN$  and  $MO$  are arcs of circles with centres  $A$  and  $C$  respectively.

- (ii) Show that the area of the shaded region  $BNMO$  is  $\frac{8}{3}(2\sqrt{3} - \pi)$  cm<sup>2</sup>. [4]

7. (i) Expand  $(2 + x)^4$  in ascending powers of  $x$ , simplifying each coefficient. [4]

- (ii) Find the integers  $A$ ,  $B$  and  $C$  such that

$$(2 + x)^4 + (2 - x)^4 \equiv A + Bx^2 + Cx^4. \quad [2]$$

- (iii) Find the real values of  $x$  for which

$$(2 + x)^4 + (2 - x)^4 = 136. \quad [3]$$

**Turn over**

8. (i) The gradient of a curve is given by

$$\frac{dy}{dx} = 3 - \frac{2}{x^2}, \quad x \neq 0.$$

Find an equation for the curve given that it passes through the point (2, 6). [6]

- (ii) Show that

$$\int_2^3 \left( 6\sqrt{x} - \frac{4}{\sqrt{x}} \right) dx = k\sqrt{3},$$

where  $k$  is an integer to be found. [6]

9. The polynomial  $f(x)$  is given by

$$f(x) = x^3 + kx^2 - 7x - 15,$$

where  $k$  is a constant.

When  $f(x)$  is divided by  $(x + 1)$  the remainder is  $r$ .

When  $f(x)$  is divided by  $(x - 3)$  the remainder is  $3r$ .

- (i) Find the value of  $k$ . [5]
- (ii) Find the value of  $r$ . [1]
- (iii) Show that  $(x - 5)$  is a factor of  $f(x)$ . [2]
- (iv) Show that there is only one real solution to the equation  $f(x) = 0$ . [4]