OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4723/01

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

FRIDAY 12 JUNE 2015: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CW/CGW) 6020418119 110927

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Find the equation of the tangent to the curve $y = \dfrac{5x + 4}{3x - 8}$ at the point $(2, -7)$. [5]

2 It is given that θ is the acute angle such that $\cot\theta = 4$. Without using a calculator, find the exact value of

(i) $\tan(\theta + 45°)$, [3]

(ii) $\operatorname{cosec}\theta$. [2]

3 The volume, V cubic metres, of water in a reservoir is given by

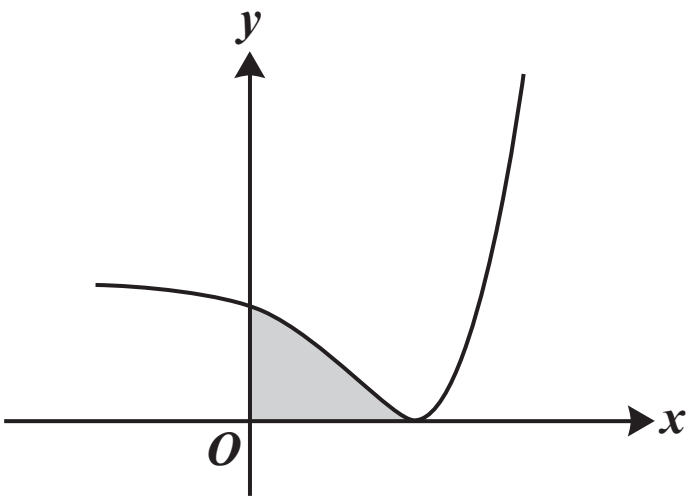
$$V = 3(2 + \sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. [5]

4 It is given that $|x + 3a| = 5a$, where a is a positive constant. Find, in terms of a, the possible values of

$|x + 7a| - |x - 7a|$. [6]

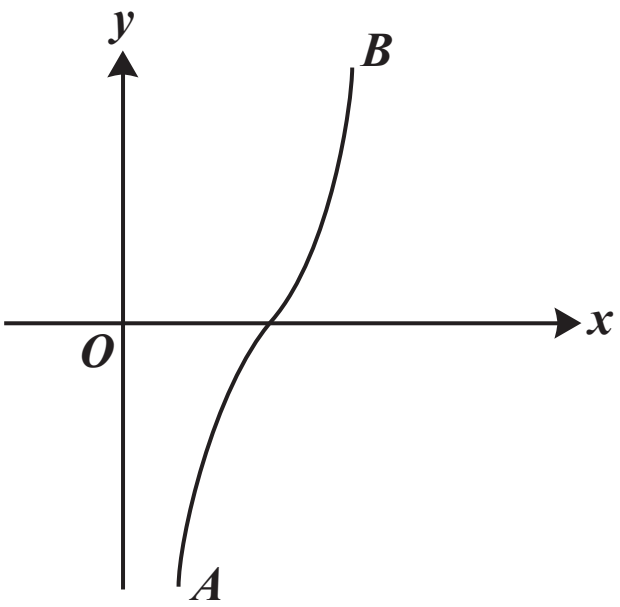
5 The diagram below shows the curve $y = e^{3x} - 6e^{2x} + 32$.



(i) Find the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0. [4]

(ii) Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes. [4]

6 The diagram below shows the curve $y = 8\sin^{-1}\left(x - \frac{3}{2}\right)$. The end-points A and B of the curve have coordinates $(a, -4\pi)$ and $(b, 4\pi)$ respectively.



(i) State the values of a and b. [2]

(ii) It is required to find the root of the equation $8\sin^{-1}\left(x - \frac{3}{2}\right) = x$.

(a) Show by calculation that the root lies between 1.7 and 1.8. [3]

(b) In order to find the root, the iterative formula

$$x_{n+1} = p + \sin(qx_n),$$

with a suitable starting value, is to be used. Determine the values of the constants p and q and hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [5]

7 (i) Find the exact value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$. [4]

(ii) Use Simpson's rule with two strips to show that an approximate value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$ can be expressed in the form $m + n\sqrt[3]{36}$, where the values of the constants m and n are to be stated. [3]

(iii) Use the results from parts (i) and (ii) to find an approximate value of $\sqrt[3]{36}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. [2]

8 The functions f and g are defined as follows:

$\mathrm{f}(x) = 2 + \ln(x+3)$ for $x \geqslant 0$,

$\mathrm{g}(x) = ax^2$ for all real values of x, where a is a positive constant.

(i) Given that $\mathrm{gf}(\mathrm{e}^4 - 3) = 9$, find the value of a. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) Given that $\mathrm{ff}(\mathrm{e}^N - 3) = \ln(53\mathrm{e}^2)$, find the value of N. [5]

9 It is given that $f(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

(i) Show that $f(\theta) = \cos\theta$. Hence show that

$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3$. [6]

(ii) Hence

(a) determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies, [3]

(b) solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$. [4]
