OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4725/01

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

THURSDAY 14 MAY 2015: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4725/01
List of Formulae (MF1)
Insert for Question 6(i)

OTHER MATERIALS REQUIRED:
Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KN) 5313492959 109335/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 The complex number $x + \mathrm{i}y$ is denoted by z. Express $3zz^* - |z|^2$ in terms of x and y. [3]

2 Find $\sum_{r=1}^{n} \left(3r^2 - 5\right)$, expressing your answer in a fully factorised form. [4]

3 The matrix A is given by $\mathrm{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$, where a is a constant.

(i) Find A^{-1}. [2]

The matrix B is given by $\mathrm{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$.

(ii) Given that PA = B, find the matrix P. [3]

4 Prove by induction that, for $n \geqslant 1$,
$\sum_{r=1}^{n} r\left(3r + 1\right) = n\left(n + 1\right)^2$. [5]

5 The loci C_1 and C_2 are given by $|z + 2| = 2$ and $\arg\left(z + 2\right) = \frac{5}{6}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [4]

(ii) Find the complex number represented by the intersection of C_1 and C_2. [2]

(iii) Indicate, by shading, the region of the Argand diagram for which

$|z + 2| \leqslant 2$ and $\frac{5}{6}\pi \leqslant \arg\left(z + 2\right) \leqslant \pi$. [2]

6 The matrix M is given by $\mathbf{M} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$.

(i) The diagram in the Printed Answer Book and the Insert shows the unit square $OABC$. The image of the unit square under the transformation represented by M is $OA'B'C'$. Draw and label $OA'B'C'$, indicating clearly the coordinates of A', B' and C'. [3]

(ii) The transformation represented by M is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a possible pair of transformations P and Q and state the matrices that represent them. [4]

7 (i) Use an algebraic method to find the square roots of the complex number $5 + 12\mathrm{i}$. You must show sufficient working to justify your answers. [5]

(ii) Hence solve the quadratic equation $x^2 - 4x - 1 - 12\mathrm{i} = 0$. [5]

8 (i) Show that $\frac{3}{r-1} - \frac{2}{r} - \frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$. [2]

(ii) Hence find an expression, in terms of n, for $\sum_{r=2}^{n} \frac{4r+2}{r(r^2-1)}$. [6]

(iii) Hence find the value of $\sum_{r=4}^{\infty} \frac{4r+2}{r(r^2-1)}$. [2]

9 The matrix D is given by $\mathrm{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

(i) Find the values of a for which D is singular. [6]

(ii) Three simultaneous equations are shown below.

$$x + 3y + 4z = 3$$
$$2x + ay + 3z = 2$$
$$y + az = 0$$

For each of the following values of a, determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 1$ [4]

10 The cubic equation $x^3 + 4x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u. [3]

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$. [7]

END OF QUESTION PAPER

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