

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS  
A2 GCE  
4726/01  
MATHEMATICS  
Further Pure Mathematics 2  
QUESTION PAPER  
MONDAY 22 JUNE 2015: Morning  
DURATION: 1 hour 30 minutes  
plus your additional time allowance  
MODIFIED ENLARGED 24pt**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4726/01**

**Printed Answer Booklet for Questions 6(ii) and 6(iii)**

**List of Formulae (MF1)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

**Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or Printed Answer Booklet for Questions 6(ii) and 6(iii) and the paper provided by the centre. Please write clearly and in capital letters.**

**IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.**

**Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Answer ALL the questions.**

**Read each question carefully. Make sure you know what you have to do before starting your answer.**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

## **INFORMATION FOR CANDIDATES**

**The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**

**YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**

**The total number of marks for this paper is 72.**

**Any blank pages are indicated.**

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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- 1 By first expressing  $\tanh y$  in terms of exponentials, prove that  $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ . [3]
- 2 It is given that  $f(x) = \ln(1 + \sin x)$ . Using standard series, find the Maclaurin series for  $f(x)$  up to and including the term in  $x^3$ . [4]
- 3 By first completing the square, find the exact value of  $\int_{\frac{1}{2}}^1 \frac{1}{\sqrt{2x-x^2}} dx$ . [5]
- 4 It is given that  $I_n = \int_0^1 x^n e^{-x} dx$  for  $n \geq 0$ .
- (i) Show that  $I_n = nI_{n-1} + k$  for  $n \geq 1$ , where  $k$  is a constant to be determined. [3]
- (ii) Find the exact value of  $I_3$ . [3]
- (iii) Find the exact value of  $990I_8 - I_{11}$ . [3]

**5 It is given that  $y = \sin^{-1} 2x$ .**

**(i) Using the derivative of  $\sin^{-1} x$  given in the List of Formulae (MF1), find  $\frac{dy}{dx}$ . [1]**

**(ii) Show that  $(1 - 4x^2) \frac{d^2 y}{dx^2} = 4x \frac{dy}{dx}$ . [3]**

**(iii) Hence show that  $(1 - 4x^2) \frac{d^3 y}{dx^3} - 12x \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} = 0$ . [2]**

**(iv) Using your results from parts (i), (ii) and (iii), find the Maclaurin series for  $\sin^{-1} 2x$  up to and including the term in  $x^3$ . [3]**

**6 It is given that the equation  $3x^3 + 5x^2 - x - 1 = 0$  has three roots, one of which is positive.**

**(i) Show that the Newton-Raphson iterative formula for finding this root can be written**

$$x_{n+1} = \frac{6x_n^3 + 5x_n^2 + 1}{9x_n^2 + 10x_n - 1}. \quad [3]$$

**(ii) A sequence of iterates  $x_1, x_2, x_3, \dots$  which will find the positive root is such that the magnitude of the error in  $x_2$  is greater than the magnitude of the error in  $x_1$ . On the graph given in the Printed Answer Book or Answer Booklet provided, mark a possible position for  $x_1$ . [1]**

**(iii) Apply the iterative formula in part (i) when the initial value is  $x_1 = -1$ . Describe the behaviour of the iterative sequence, illustrating your answer on the graph given in the Printed Answer Book or Answer Booklet provided. [2]**

**(iv) A sequence of approximations to the positive root is given by  $x_1, x_2, x_3, \dots$ . Successive differences**

**$x_r - x_{r-1} = d_r$ , where  $r \geq 2$ , are such that  $d_r \approx k(d_{r-1})^2$  where  $k$  is a constant.**

**Show that  $d_4 \approx \frac{d_3^3}{d_2^2}$  and demonstrate this numerically when  $x_1 = 1$ . [4]**

**(v) Find the value of the positive root correct to 5 decimal places. [2]**

**7 It is given that  $f(x) = \frac{x^2 - 25}{(x - 1)(x + 2)}$ .**

- (i) Express  $f(x)$  in partial fractions. [4]**
- (ii) Write down the equations of the asymptotes of the curve  $y = f(x)$ . [2]**
- (iii) Find the value of  $x$  where the graph of  $y = f(x)$  cuts the horizontal asymptote. [2]**
- (iv) Sketch the graph of  $y^2 = f(x)$ . [2]**

**8 It is given that  $f(x) = 2 \sinh x + 3 \cosh x$ .**

- (i) Show that the curve  $y = f(x)$  has a stationary point at  $x = -\frac{1}{2} \ln 5$  and find the value of  $y$  at this point. [4]**
- (ii) Solve the equation  $f(x) = 5$ , giving your answers exactly. [5]**

**9 The equation of a curve in polar coordinates is  $r = 2 \sin 3\theta$  for  $0 \leq \theta \leq \frac{1}{3}\pi$ .**

- (i) Sketch the curve. [2]**
- (ii) Find the area of the region enclosed by this curve. [4]**
- (iii) By expressing  $\sin 3\theta$  in terms of  $\sin \theta$ , show that a cartesian equation for the curve is**

$$(x^2 + y^2)^2 = 6x^2y - 2y^3. \quad [5]$$

**END OF QUESTION PAPER**

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