OCR
Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find the equation of the tangent to the curve $y = \dfrac{5x+4}{3x-8}$ at the point $(2, -7)$. **[5]**

2 It is given that θ is the acute angle such that $\cot\theta = 4$. Without using a calculator, find the exact value of

(i) $\tan(\theta + 45°)$, **[3]**

(ii) $\operatorname{cosec}\theta$. **[2]**

3 The volume, V cubic metres, of water in a reservoir is given by

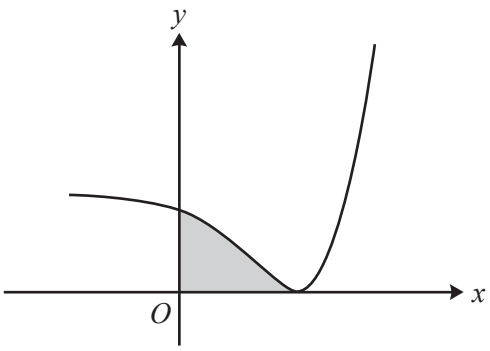
$$V = 3(2 + \sqrt{h})^6 - 192,$$

where h metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. **[5]**

4 It is given that $|x + 3a| = 5a$, where a is a positive constant. Find, in terms of a, the possible values of

$$|x + 7a| - |x - 7a|. \quad \textbf{[6]}$$

5

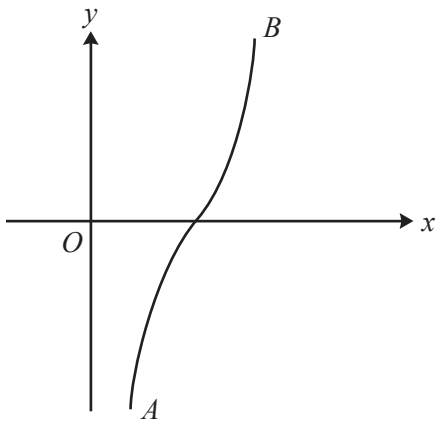


The diagram shows the curve $y = e^{3x} - 6e^{2x} + 32$.

(i) Find the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0. **[4]**

(ii) Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes. **[4]**

6



The diagram shows the curve $y = 8\sin^{-1}\left(x - \frac{3}{2}\right)$. The end-points A and B of the curve have coordinates $(a, -4\pi)$ and $(b, 4\pi)$ respectively.

(i) State the values of a and b. **[2]**

(ii) It is required to find the root of the equation $8\sin^{-1}\left(x - \frac{3}{2}\right) = x$.

(a) Show by calculation that the root lies between 1.7 and 1.8. **[3]**

(b) In order to find the root, the iterative formula

$$x_{n+1} = p + \sin(qx_n),$$

with a suitable starting value, is to be used. Determine the values of the constants p and q and hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. **[5]**

7 **(i)** Find the exact value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$. **[4]**

(ii) Use Simpson's rule with two strips to show that an approximate value of $\int_1^9 (7x+1)^{\frac{1}{3}} \mathrm{d}x$ can be expressed in the form $m + n\sqrt[3]{36}$, where the values of the constants m and n are to be stated. **[3]**

(iii) Use the results from parts **(i)** and **(ii)** to find an approximate value of $\sqrt[3]{36}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. **[2]**

Question 8 begins on page 4.

8 The functions f and g are defined as follows:

$$\mathrm{f}(x) = 2 + \ln(x+3) \text{ for } x \geqslant 0,$$

$$\mathrm{g}(x) = ax^2 \text{ for all real values of } x, \text{ where } a \text{ is a positive constant.}$$

(i) Given that $\mathrm{gf}(\mathrm{e}^4 - 3) = 9$, find the value of a. **[3]**

(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(iii) Given that $\mathrm{ff}(\mathrm{e}^N - 3) = \ln(53\mathrm{e}^2)$, find the value of N. **[5]**

9 It is given that $\mathrm{f}(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

(i) Show that $\mathrm{f}(\theta) = \cos\theta$. Hence show that

$$\mathrm{f}(4\theta) + 4\mathrm{f}(2\theta) \equiv 8\cos^4\theta - 3.$$ **[6]**

(ii) Hence

(a) determine the greatest and least values of $\dfrac{1}{\mathrm{f}(4\theta) + 4\mathrm{f}(2\theta) + 7}$ as θ varies, **[3]**

(b) solve the equation

$$\sin(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$. **[4]**

END OF QUESTION PAPER