OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

OCR Report to Centres

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This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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EXAMINER'S REPORT

Overview - Pure Mathematics

In preparing mark schemes for these units, examiners have to take account of possible alternative approaches that candidates might adopt in answering questions. Alternative methods likely to occur with any frequency will have their own different scheme. Even then, examiners are occasionally delighted to come across an elegant solution to a problem that had not been anticipated. Sometimes different approaches, although not anticipated, deal as effectively with the question as the expected solution. An example of this occurred in unit 4724 where some candidates tackled Q1(i) by resolving the expression into partial fractions.

A vital technique in answering mathematics problems is the process of choosing an effective method in situations where different methods are possible. A candidate who is aware of the different approaches that can be used and is able to opt for the most effective in a particular case is showing the mathematical awareness that is welcome at this level.

The following gives one example from each unit at this session where different methods are possible:

4721	Q8(ii)	Different methods can be used in determining whether a stationary point is a maximum or minimum point.
4722	Q5(b)	Two different formulae can be used for the sum of an arithmetic progression, namely $S = \frac{1}{2}n[2a + (n-1)d]$ and $S = \frac{1}{2}n(a+l)$.
4723	Q2(i)	The question can be approached either by using logarithm properties immediately or by first simplifying $\frac{\mathrm{e}p^2}{q}$.
4724	Q9(i)	There are different procedures for finding the constants when the expression is resolved into partial fractions.
4725	Q2(ii)	This question can be answered by either finding the inverse of **AB** or by multiplying $\mathbf{B}^{-1}$ by $\mathbf{A}^{-1}$.
4726	Q8(i)	Some candidates attempted to use the given information to find the constants in the expression $y = \frac{ax^2 + bx + c}{dx + e}$ whereas others opted to use the information to write down $y = \frac{1}{2}x + 1 + \frac{k}{x+2}$ at the outset.
4727	Q1	Finding the vector perpendicular to two given vectors can be done by using the vector product, or by using the scalar product to set up two equations which require solution.

A candidate with good knowledge of alternative legitimate methods is less likely to make the mistake made by many candidates in 4723 Q4. Here the assumption that part (b) could be answered in a manner similar to the way used in part (a) was common and the integral $\frac{1}{3\mathrm{e}^x}(\mathrm{e}^x + 2)^3$ was noted on very many scripts.

4721 Core Mathematics 1

General Comments

This paper was completed to a generally very high standard. Many candidates secured over 60 of the 72 marks available. Almost all finished the whole paper and there were very few questions which were omitted by even a small number of candidates. There remains a significant minority of candidates who scored less than 10 marks. The answer book was appropriate and efficiently used with very few additional sheets needed, usually for restarting a long question.

A notable improvement is that candidates are getting better at spotting the connections between different parts of the questions and using their previous working to support the later parts of questions instead of unnecessarily adopting a whole new approach or repeating previous work. For example, most candidates realised their “completed square” in Q4(i) would provide them with the vertex in Q4(ii). Another improvement is that candidates are continuing to get better at choosing the appropriate method for solving quadratic equations. In some previous sessions, there has been an over-reliance on using the quadratic formula, or even completing the square. In this session, many candidates tried factorising first and only used other methods when appropriate. Where the formula was used, for example in Q7, it was usually correctly quoted and/or applied, although some candidates did make errors in recalling the formula.

Candidates showed good skills with basic differentiation and algebraic manipulation, although there remain concerns with arithmetical processing, particularly with negatives and fractions. Surds and indices presented problems for many candidates with a lot of marks lost in Q2(ii) and, to a lesser extent, in Q2(iii) and Q6.

There were some very good responses to the contextualised requests in Q9, although it was clear that some candidates found these extremely difficult. Some centres need to provide more opportunities for candidates to apply their techniques in different contexts and consider the appropriateness of their answers, for example by considering whether negative solutions are possible. On the whole, however, many candidates scored very highly on these questions.

Whereas the presentation of solutions is generally very good, there remain some problems with sketching graphs and describing transformations. Another area for development for many candidates would be the justification of their solutions. Unclear explanations cost some candidates a lot of marks in questions 8 and 10 in particular. Centres should help candidates to consider the validity of their assertions and how they justify these.

Comments on Individual Questions

1) Many candidates scored full marks on this opening question. Almost all were able to expand both sets of brackets correctly. There was, however, a significant number of errors at the simplification stage, especially where candidates had not bracketed their expansions and then did not subtract the negative terms correctly. Very few candidates had corrected this error at a later stage, suggesting a lack of checking.

2) (i) Almost all candidates secured the mark for this simple recall of fractional index notation; those who did not know this fact usually opted for –4 as the index.

(ii) Many candidates found this part difficult with only around a quarter scoring both marks. The mark scheme was generous in awarding a method mark for a clear correct use of either of the appropriate index rules. Even then, candidates’ work was often unclear and difficult to follow. A large number of candidates approached the question by trying to rationalise the denominator but this approach was seldom successful, with many going round in circles with complex expressions they were unable to simplify. Common incorrect answers with little or no working included $7^{-\frac{1}{7}}$ and $7^{-\frac{7}{2}}$.

(iii) This was generally more successful than part (ii). Most candidates rewrote $(49)^{10}$ as $(7^2)^{10}$, but a significant minority evaluated this as 7^{12} rather than 7^{20}. Most of those who did know how to deal with $(a^b)^c$ were then able to add the powers correctly to obtain the correct answer, but $7^4 \times 7^{20} = 7^{80}$ was a frequently seen error.

3) (i) Most candidates correctly rearranged the equation to make y the subject and then gave the correct gradient, usually in fraction form. Some candidates did erroneously just choose "3" from the original equation; other errors included inverting the gradient to $\frac{5}{3}$ and also giving $-\frac{3}{5}$ or $-\frac{5}{3}$. $\frac{3}{5}x$ was also seen as the answer but this was not accepted.

(ii) This proved quite challenging to a large number of candidates, with some being unable to start at all and others just quoting the mid-point formula. Most realised the need to set x and y to zero in order to find the coordinates of P and Q, but a large number incorrectly put x equal to zero to find P and made the corresponding error for Q. Even with the zero coordinates, the arithmetic of finding the mid-point where a fractional value was involved proved difficult to a significant number; half of $\frac{20}{3}$ was often calculated as $\frac{40}{3}$ or $\frac{40}{6}$. A small number of candidates subtracted rather than added the coordinates before their attempts at halving.

4) (i) As usual, the vast majority of candidates were able to spot the simple values of p and q in this familiar "completing the square" question, although some erroneously found q to be 10 or 20. About three-quarters of candidates went on to secure all four marks. Those who were not successful made the usual error of forgetting the factor of p when evaluating the constant.

(ii) The vast majority of candidates spotted the connection between the two parts of this question and most gave the vertex that corresponded with their expression in part (i), with only the occasional sign error. Some candidates restarted using differentiation and were often successful.

5) (i) Many candidates were familiar with the shape of this graph and over 60% secured both marks, which is high compared to some previous graph questions. Freehand axes sometimes made it difficult to judge candidates' interpretations of the behaviour of functions with increasing x; they should be advised to draw their axes with a ruler. Some candidates resorted to "plotting", which was rarely successful. Finite plots earned no marks; those with the correct shape established often earned one mark but errors such as the omission of (0, 0) lost the second mark.

(ii) The use of correct mathematical language to describe transformations continues to improve slowly. "Translation" is now seen much more often than the unacceptable alternatives such as "shift". The second mark was still often lost due to incorrect phrases such as "in/on/along the x-axis" rather than "parallel to the x-axis" or use of the correct vector. Translating to the left was a fairly common error, but only a very small number erroneously thought the translation was vertical.

(iii) Finding the equation of a graph after a stretch remains a challenging aspect of the specification with only around a third of candidates securing both marks. Another fifth of candidates correctly stretched in the x direction but used the wrong factor giving the incorrect answer $y = \sqrt{5x}$; this was awarded the method mark. The most common error was to stretch in the y direction instead of the required x direction.

6) This question proved very discriminating. Most candidates adopted the correct approach of differentiating first to find the gradient function but weaker candidates did not deal correctly with subtraction from a negative power. Substitution of $x = 2$ into $-12x^{-3}$ also proved challenging to some, but many obtained the correct gradient. Many then correctly found the gradient of the normal and used this in their equation, but a significant number found the equation of the tangent instead. Finding the value of y was arithmetically challenging for some, but many did so correctly and went on to secure full marks. A common arithmetical error was to simplify the gradient from $-\frac{12}{8}$ to $-\frac{4}{3}$. A small number of candidates did not give their answer in the correct form, either omitting the "= 0" from their equation or not using integers as specifically requested.

7) As usual, the vast majority of candidates was able to recognise a disguised quadratic, and many made the appropriate substitution $y = x^{\frac{1}{2}}$. Only a small number of candidates tried to "square the whole equation" by squaring or doubling each term, which of course gained no credit. Following the substitution, most candidates correctly completed the square or applied the quadratic formula to obtain $y = 3 \pm \sqrt{7}$,or at least $y = \frac{6 \pm \sqrt{28}}{2}$. A large number of candidates then stopped, either forgetting to reverse the substitution or not knowing how to proceed. Of those who did recognise the need to square, a significant minority did not expand correctly and, for example, expanded $(3 \pm \sqrt{7})^2$ as $9 + 7$. Candidates who went straight to the formula without stating the substitution and proceeded no further than $y = 3 \pm \sqrt{7}$ earned one mark. Centres need to remind candidates that, both to be sure of credit and to help their progress in their solution, they need to state clearly their substitution; this will also serve as a reminder to reverse the process at the end.

8) (i) Most candidates scored very highly on this familiar differentiation question involving positive integer powers of x. Almost all secured the first two marks for differentiating correctly and the vast majority put their derivate equal to zero, although this was not always as explicitly shown as it could have been. Errors came in after this, with large numbers putting $4x^3 = 32$ instead of -32 and many of those reaching $x^3 = -8$ concluding with either $x = 2$ or $x = \pm 2$. Substituting to find the y value was often marred by difficulties with dealing with the negative number arithmetic. The error was carried forward for a single incorrect value of x found but not if more than one value was found as the question clearly indicated there was only one stationary point. Another common error was to substitute back into $\frac{dy}{dx}$ instead of y and to get 0 again.

(ii) By far the most common approach was the expected one of finding the second derivate and substituting the value(s) of x found in (i). The method mark was awarded for any value(s) provided a conclusion had been stated, but the accuracy mark depended on the correct x value having been used. Centres should encourage candidates to be more thorough in their justification. Although statements like "$\frac{d^2y}{dx^2} = 48$ so minimum" were tolerated this series, many examiners felt that at least "> 0" or "positive" should be seen to truly deserve the method mark.

(iii) Again, most candidates realised the need to consider their answers to parts (i) and (ii) and correctly gave the answer $x > -2$ with no need for supporting working. Those who restarted using $\frac{dy}{dx} > 0$ were also usually successful. A follow-through mark was available to candidates with incorrect values of x provided their answer was consistent with their earlier work.

9) (i) Despite the possible added difficulty of the question being placed in context, many candidates made good progress with this question. Almost all found the correct expression for the area and the inequality was also usually given the correct way. Most candidates at least attempted to solve the resulting quadratic, many first dividing by four to make it easier to solve. For those who were successful in factorising, the most common approach was to choose the correct "between the roots" region to solve the inequality and obtain $-7 < x < 4$. Only a very small number then remembered the context of the question with x representing a length and restricted their answer to $0 < x < 4$ to earn the final accuracy mark, suggesting perhaps that candidates have had little experience of similar context-based material. Other errors here were solving the inequality as "$x < 4$ or $x < -7$" and then stating that "x can't be negative so $x < 4$", seeming unaware that this included negative values. Other candidates seemed to think x had to be an integer, both here and in part (ii).

(ii) The method for solving the linear inequalities arrived at in the later part of this question is well established and was very well done by the majority of candidates; the previously quite common error of only dealing with one "part" of the inequality was quite rare. This may be partly due to the fact that some candidates did not establish inequalities in the first place; many who did so did not arrive at the correct $20 < 10y + 6 < 54$. Finding the correct expression for the perimeter proved very problematic with common errors being: adding just the four sides shown; adding five of the sides; attempting to subtract the perimeters of the "big" and "small" rectangles, and finding the area of the shape instead. This suggests again a reliance on technique-driven learning and lack of practice of contextualised material.

10) (i) The majority of candidates were able to secure all three marks using the given equation to find the centre and diameter of the circle. There were occasional sign slips with the centre and a significant number only found the radius, thus losing a mark.

(ii) Despite the negative y value for the centre, most candidates were successful in finding the gradient of the radius and then correctly finding the equation of the required line; around 70% scored full marks and even those candidates with arithmetical errors were usually able to secure both method marks. As ever, the candidates who drew a sketch were particularly successful; indeed several candidates who had found the centre as (5, 2) in (i) were able to realise their error and go back and correct their work. Where errors occurred, it was usually due to inadvertently inverting the formula to find the gradient and so reaching

$\frac{1}{2}$ instead of 2.

(iii) Most candidates followed the instructions on the paper and found the length of their *CP*; a few tried to look at the square of the distances only which therefore received less credit. The last mark was not awarded so often as many candidates did not make a clear comparison; "$CP = \sqrt{20}$ so it's inside" was not accepted, partly because "$CP = \sqrt{20}$ so it's outside" was also regularly seen, so candidates were expected to justify their answer by direct comparison with the radius to gain the mark. A common error amongst those who did make their comparison clear was to use the diameter instead of the radius.

(iv) Most candidates realised the need to try and solve the equation of the circle and the line simultaneously and many were successful in establishing a quadratic equation. This was usually correct where candidates substituted directly, for those who chose to expand the circle equation first there were a number of algebraic errors mainly from the squaring of $2x$ term. Relatively few candidates secured the last two marks. Candidates were seldom clear in their reasoning, or gave spurious reasons. "It doesn't factorise so they don't meet" was a very common incorrect assertion; only a small number of candidates made the required reference to the discriminant and even then "–76 so no" or similar responses were often insufficiently clear. Whilst this is challenging, centres need to support candidates in explaining their reasoning with greater rigour. There were some elegant alternative solutions such as finding the shortest distance from the line to the centre and comparing with the radius.

4722 Core Mathematics 2

General Comments

Candidates seemed to find this paper accessible, and the majority were able to make an attempt at most, if not all, of the questions. Candidates seemed well prepared and were able to tackle the routine questions with relative ease, but some then struggled when asked to apply their knowledge to less routine questions. A number of candidates are still failing to show sufficient detail to make their intentions clear. Lack of brackets can result in candidates subsequently evaluating their expressions incorrectly, and examiners can only award method marks if it is clear that the correct method has been employed. This is especially true in questions where candidates are asked to show a given answer; full credit will only be given where each step has been shown explicitly. Additionally, on these types of questions, candidates are expected to do more than just verify the answer if they wish to gain full credit. Candidates should read the question carefully in order to ascertain the degree of accuracy required, and then give their final answer accordingly. There were a number of questions on this paper where exact answers were required, and this also required exact working throughout.

Candidates should be aware that if they make more than one attempt at a question, it is only the last complete solution that will be marked. Clearly it is in the candidate's best interests if they identify which their final solution is. This is particularly important if they have used extra sheets of paper or erroneously answered a question in the response box for another question. The final attempt should be clearly identified, and a line put through all other attempts. If a number of amendments have been made, candidates should consider rewriting their final solution on an additional piece of paper. This was particularly apparent on the graph sketching on Q6(ii) where it was sometimes unclear whether candidates had drawn the correct curve, and whether one or two trapezia were intended. Additionally, whilst a sketch graph is not expected to be drawn to scale, care should be taken to ensure that it clearly conveys the salient points which can be difficult to achieve without use of a ruler.

Comments on Individual Questions

1) (i) This proved to be a straightforward start to the paper and most candidates gained full marks. As always, the most successful candidates made effective use of brackets. The most common error was to omit to apply the power to $2x$ in its entirety. Candidates usually used the correct binomial coefficients, but this was not always shown explicitly which made it difficult to award credit.

(ii) This part of the question was not so well done. It was expected that candidates would change the signs on the relevant terms and then add the two expressions together. The most common error was to find the difference rather than the sum of the two expansions, though this could still gain one of the two marks. Some candidates attempted a full expansion of the second bracket rather than appreciating the link between the two expansions, and others changed the signs on all but the first term of their expansion.

2) (i) The vast majority of the candidates were able to successfully integrate the given function to obtain at least two of the algebraic terms, although the constant term sometimes disappeared. A few candidates lost a mark by leaving dx or the integral sign in their final answer.

(ii) Most candidates appreciated what was required and could make a reasonable attempt at finding the equation of the curve. There were some sign errors when attempting to evaluate *c*, with $-(3)^2$ becoming +9. Other candidates failed to write the equation as $y = ...$, or even failed to state a final equation at all, and thus lost the final mark. A few candidates attempted to use the equation of a straight line graph, and others attempted to use 3 and 11 as limits in a definite integration. Neither of these approaches gained any credit.

3) (i) Most candidates were clearly aware of the relationship between radians and degrees, but too many failed to give their answer as an exact value as requested in the question. Some candidates obtained an angle of 1.26 and then went on to write it as 1.26π, which suggested that they believed that π is the unit of measurement for angles in radians.

(ii) Candidates were expected to work exactly, and many could do so either using 0.4π in the formula for the area of a sector or by using an alternative method involving fractions of a circle. Most candidates could recall the correct formula for the area of a sector, though a few omitted the $\frac{1}{2}$, and others used the angle in degrees. There was also some confusion in rearranging the formula, with some errors resulting from using a calculator to divide by $15 \times \pi$, but omitting the necessary brackets. A common error was to use the area of the sector as 45 not 45π, again suggesting a lack of understanding of the notation for radian measure.

(iii) Many candidates were successful in this question, though some lost the final mark due to inaccuracies caused by working in decimals. Most candidates attempted to use the relevant formula for the area of a triangle; a few attempted to find the base and height but these were rarely successful. There was some confusion over which calculator mode to use and some candidates just found the area of the triangle and made no further progress.

4) This question was very well answered, with the majority of the candidates gaining full marks despite no hint being given in the question as to the method required. Candidates showed a clear appreciation of the need to use the appropriate identity, and the resulting quadratic was usually correctly simplified and solved. Some candidates lost the final mark through including an extra, incorrect solution such as 270° or through errors in finding the second angle from $\sin x = 0.75$. Whilst some candidates used an incorrect rearrangement of the required identity, and others used $\sin x = 1 - \cos x$, these errors were fewer than in previous sessions.

5) (a)(i) Most candidates could readily write down the two required values, though a few attempted to use it as a n^{th} term definition instead.

(a)(ii) Most candidates could give an acceptable mathematical description of the behaviour of the sequence, though some then spoilt their answer by adding an incorrect statement such as geometric.

(b) There were many good answers to this question, with candidates able to quote the correct formulae and then attempt to solve them. The more common approach was to state two equations in *a* and *d* and then solve them simultaneously, though some of the methods used were neither efficient nor easy to follow. The more successful, though less common, method was to start with $\frac{1}{2}n(a+l)$ as candidates then only had to find one variable at a time.

6) (i) This was generally done well, though some candidates were reluctant to work with surds and instead used a decimal approximation for $\sqrt{5}$. With the answer given, it was expected that the working would be convincing, including use of the 'big brackets', and in most cases it was. There were relatively few candidates who used incorrect x-coordinates or the wrong value for h.

(ii) Candidates were told that the approximation was an under-estimate, and were asked to justify this using a sketch graph. Examiners expected to see a correct sketch of the given function, along with two trapezia with their top vertices on the curve. Too many candidates simply quoted the text-book explanation for an under-estimate but made no attempt to relate it to this particular situation. It was common to see only one trapezium drawn or two trapezia of unequal widths or even two rectangles. Whilst the y-coordinates found in part (i) indicated an increasing curve, there were too many drawn with increasing gradients or non-zero intercepts. A number of candidates seemed to have the correct idea, but lacked the precision expected to be convincing. Candidates should also appreciate that a sketch graph does not preclude the use of a ruler.

(iii) The integration was generally done correctly, with very few errors in either the index or the coefficient. The use of limits was invariably correct, even if the preceding integration was not. The most common error was to give a non-exact final answer, despite the question specifically requesting an exact value.

7) (a)(i) Only the most able candidates gained any credit on this question. The majority of candidates simply relied on their calculators and thought that writing down the entire calculator display would give an exact answer. The most efficient method was setting up a right-angled triangle and using Pythagoras, but this was rarely seen. The more common approach was the use of trigonometric identities, but this tended to be the less successful method.

(a)(ii) This question was also very poorly done. A number of candidates did manage to find an exact value to get two of the three marks, but did not register the significance of the angle being obtuse. Only the very best scored full marks on this question. For those working in decimals, there was one mark available if they appreciated that the angle being obtuse would mean that cos β was negative and stated a value in the allowed range.

(b) Most candidates gained at least one mark for using the correct sine rule, and many then went on to obtain the expected surd value though a few struggled with the rearrangement. Any subsequent working was ignored, but a number of candidates did not appreciate that they had answered the question at this stage and continued to actually find the angle. Whilst the majority of candidates did give an exact answer as expected, there were a number who worked in decimals throughout, thus gaining only one mark.

8) (i) Most candidates were able to attempt f(2) and/or g(2), though a small minority used x = -2 instead. Quite a few candidates failed to explicitly equate their expression(s) to 0 until much further through the solution. Whilst the majority of candidates were able to solve the two simultaneous equations, it proved problematic for others. Assuming that a = -4, and using this to find b, was given partial credit. Some candidates assumed this from the start whereas others used it as a last resort when unable to solve the simultaneous equations in two unknowns. Some candidates still seem reluctant to use the factor theorem and attempt to use division or coefficient matching. With the unknowns as coefficients in the cubics, this was invariably unsuccessful, especially if still working in both a and b. Whilst it is to the benefit of the

candidate if they know several alternative methods, they must also be able to appreciate which will be the most efficient in a given situation.

(ii) Most candidates could make an attempt to factorise f(x). The most popular method was division, but the zero coefficient of x^2 caused problems for some. Coefficient matching and inspection tended to be more successful, if less common, and the more astute candidates simply spotted that f(1) = 0 and then used this. Having found the quadratic quotient correctly, some then failed to write f(x) in fully factorised form thus losing a mark. There was a variety of methods used to attempt to show that the two cubics shared a common factor, with division and the factor theorem being equally popular. Those using the latter method did not always show enough detail; simply stating g(-3) = 0 was not enough to be convincing. Even when verified correctly, (x + 3) was not always indicated to be a second common factor. There was also some confusion over roots and factors, with the two common factors being stated to be $x = 2$ and $x = -3$. A common oversight was for candidates to only attempt to factorise g(x), which gained no credit unless it was then used to find a common factor.

9) (a)(i) The vast majority of candidates were able to score full marks on this question, showing full detail of each step in their proof. Whilst some candidates were clearly trying to use GP formulae, the main reason for losing marks was not showing enough detail to be convincing on a 'show that' question.

(a)(ii) A pleasing number of elegant and fully correct solutions were seen, which used exact values throughout. A few candidates gained one mark for using the correct method, but working in decimals. However many candidates failed to gain any credit. The most common error was to write $\log_2(27x^3)$ as $3\log_2(27x)$, failing to notice that this contradicted the working in part (i). Others simply rewrote 6 as $\log_2 6$, sometimes as an attempt to inverse the $\log_2$ term on the other side of the equation and sometimes with the $\log_2$ still intact. A number of candidates seemed to think that $\log_2$ was a multiplier of $27x^3$ and just divided the 6 by it as an inverse operation. Some candidates did not consider the base of the logarithm carefully with e^6 and, less frequently, 10^6 being seen instead of 2^6.

(b)(i) Candidates seemed unfamiliar with the condition for convergence for a GP, or did not appreciate its relevance to this question, and fully correct solutions were in the minority. Some candidates managed to gain one mark for identifying that 2 was important. In some cases this came from $r < 1$, and in others from considering the denominator of the sum to infinity. The other end of the inequality was often omitted, or stated to be $0 < y$, based on not being able to take the logarithm of a non-positive number.

(b)(ii) Most candidates were able to gain a mark for correctly equating the sum to infinity to 3, but a number struggled to make further progress. Misconceptions with logarithm laws were apparent, especially a confusion over the subtraction / division law with the correct sum to infinity becoming $\log_2 27 - (1 - \log_2 y)$. Some candidates could attempt the correct solution method, but resorted to decimals, whereas others managed to produce elegant and concise solutions with exact working throughout.

4723 Core Mathematics 3

General Comments

The general performance of candidates on this unit was better than has been the case in some recent sessions. Partly this seemed to be due to the fact that more of the requests were accessible to candidates; partly too it seemed that candidates were better prepared and able to make significant progress with some of the non-routine questions that might have caused problems in the past. About 1.3% of candidates recorded full marks and a very pleasing 3.6% recorded at least 70 marks out of the total 72. Although a tiny handful of candidates did record a total of zero, there were not many candidates who seemed totally out of their depth; only 1% of candidates recorded a total of 10 or fewer.
Questions that posed most problems were Q4b, Q8(ii)(b) and Q9(iii).

Many candidates use graphical calculators. They are generally used very effectively in answering iteration questions, such as Q5(ii)(b) in this case. However, they seem not to be used so well where graphs are concerned. The graphs involved in Q5(i), and perhaps to a lesser extent in Q1, should be well known to candidates and they should not need a graphical calculator to see what they look like. It seemed though that many candidates did use their calculators for Q5(i); if there is not careful attention to the scales on the axes, the evidence of the calculator can be misleading with the result that the essential nature of the curve in question is missed.

Comments on Individual Questions

1) While some 86% of candidates earned at least three marks, full marks were recorded by only 50% of the candidates. These figures reflect the fact that the vast majority had no difficulty in determining the two critical values but that the process for dealing with the inequality was often superficial at best.

   Squaring both sides of the inequality or of the corresponding equation was slightly the more popular approach and, although there were some slips, most were able to reach the two values $\frac{4}{3}$ and 6. But many then seemed to have no strategy for dealing with the inequality. Sketches were sometimes drawn but often not used in any effective way. Some candidates appeared to assume automatically that the region between the two critical values must form the answer. Others followed $(3x-4)(x-6)>0$ by stating $x>\frac{4}{3}, x>6$. Meaningless conclusions such as $\frac{4}{3}>x>6$ did not receive any credit. There were also many candidates who concluded with $x=\frac{4}{3}, x=6$; whether they thought this represented the solution to the question or whether they had completely forgotten about the inequality was not clear.

2) (i) It was expected that many candidates would follow the advice given in the question and set out a clear solution along the lines of $\ln(\frac{ep^2}{q}) = \ln e + \ln p^2 - \ln q = \ln e + 2\ln p - \ln q = 1 + 2\times 280 - 300 = 261$.

   However such solutions were not so common. Candidates switched, apparently randomly, between logarithm properties and laws of indices, some managing to fill the available answer space with a number of unconnected statements, some correct, some not. Confusion was caused by the appearance of e in the expression; sometimes it was ignored and sometimes the juxtaposition of ln and e led to 'cancellation' and $\ln ep^2 = p^2$. The coincidental fact that $\frac{280^2}{300}$ is equal to 261 (correct to the nearest integer) led some to claim the answer from totally wrong working. The alternative approach of expressing $\frac{ep^2}{q}$ in the form e^n followed by the appropriate conclusion was credited provided full details were present.

(ii) Most candidates adopted an appropriate strategy of taking logarithms and rewriting $\ln 5^n$ as $n\ln 5$. There were sometimes errors on the right-hand side as 280×300 appeared instead of $280+300$. A few candidates tried logarithms to base 10 or to base 5 but success in these cases was limited. The final mark was not earned so often, apparently as the result of not reading the question carefully enough. The one integer 361 was required as the final answer but all too often solutions finished with $n>360$ or $n>360.4$ or $n>361$, none of which earned the final mark.

3) (i) This was answered well and candidates showed their knowledge of the various trigonometric ratios involved. With the answer given, some detail was expected and candidates who went straight from $\sec\theta\sin\theta$ to $\tan\theta$ on the left-hand side did not receive full credit. On this occasion, no justification for rejecting the value -6 was expected although many candidates did refer to the tangent of an acute angle needing to be positive.

(ii) Both requests were answered very well; indeed 62% of candidates recorded all seven marks for question 3. There were occasional errors in the identities used and a few arithmetic slips. Answers written down without evidence of 6 being substituted for $\tan\theta$ received no credit and nor did attempts which appeared to be based on the angle $80.5°$. A few candidates did not understand the two requests at all and tried to solve equations such as $\tan(\theta-45°)=6$.

4) (a) This was answered extremely well and 87% of candidates earned all four marks. The fact that the answer was given was a help to quite a few candidates. They had made a mistake in their indefinite integral, either with the power, often $\frac{3}{2}$ instead of $\frac{1}{2}$, or with the coefficient. In most cases they tracked back through the whole solution and made the necessary corrections.

(b) In complete contrast to part (a), this was not answered at all well. In fact only 28% of candidates recorded four marks and 62% recorded zero. Most candidates did not appreciate that the first step had to be the expansion of $(e^x+2)^2$ and there were many indefinite integrals involving $(e^x+2)^3$. Knowing what is a viable first step in any mathematical problem is a vital skill; on this occasion, many candidates did not choose appropriately. Those who did begin with the necessary expansion sometimes went wrong. Not all expanded to obtain three terms and $e^x\times e^x$ sometimes became e^{x^2}. Carelessness with brackets and signs also meant some errors occurring when the limits were applied.

5) (i) These two graphs should not have presented problems to candidates but, in fact, many attempts were not at all convincing. Most candidates presented an acceptable sketch of $y=14-x^2$ although not all were shown as also existing in the third and fourth quadrants. A few drew a straight line and there were cases of the intercepts on the x-axis being shown as (–7, 0) and (7, 0); this error with

the intercepts was not penalised and nor were sketches, otherwise correct, showing the logarithm graph crossing at (k, 0). There were many more difficulties with the sketch of $y = k\ln x$. Some were drawn in only the first quadrant; others passed through the origin. It was expected that the sketch would display the appropriate asymptotic behaviour with respect to the y-axis but many sketches showed the curve actually touching, or in a few cases crossing, the y-axis; in such instances, the mark was not earned. To earn the third mark, candidates had to have the two curves more or less correct in the first quadrant and to indicate in some way the single intersection of the two curves. Some candidates with two acceptable sketches did not earn this third mark because they did not refer in any way to the one intersection.

(ii)(a) This part presented problems to many candidates; 39% recorded zero for this part and many seemed to have no idea what to do. There were attempts to solve the equation by treating it as a quadratic. Others decided that $\sqrt{14}$ was a relevant value despite the fact that it is not an integer. There were many instances of a sign change being detected in the value of $14 - x^2$ as x increases from 3 to 4. Various rearrangements of the equation were used and some of the associated attempts were not very convincing. Many candidates did know what to do and produced the necessary evidence with ease although a few missed the final mark by not stating clearly as a conclusion what the two integers were.

(ii)(b) This part was answered well and 84% of candidates duly recorded all four marks. The individual iterate values were given and, in most cases, the appropriate conclusion, giving the value of α to exactly 2 decimal places, was made.

6) (i) The differentiation was carried out accurately in the majority of cases. There were some instances of differentiation leading to $\frac{3}{2}(3h^2+4)^{\frac{1}{2}}$ and others where –8 was retained or where –8h appeared. It was pleasing to note the ready application of the chain rule in so many cases although, strangely, the factor h sometimes disappeared when a correct unsimplified version of the derivative was simplified. Candidates with a correct derivative had no difficulty when substituting 0.6 and reached the value 12.17.

(ii) Many candidates showed a firm grasp of the idea of connected rates of change and were able to reach the correct answer without trouble; it was the case that 59% of candidates recorded all seven marks on question 6. A significant minority of candidates did not recognise what was required. Some tried to introduce a formula describing exponential decay and others merely evaluated $\frac{dV}{dh}$ for $h = 0.015$.

7) Examiners were delighted to see so many assured, accurate solutions to this question on functions. The relatively unstructured nature of the question had been expected to cause some difficulties but, in the event, this was not the case and 72% of candidates earned all seven marks. Solutions were not only accurate but set out clearly. Candidates had identified the three main steps – finding the value of a, finding an expression for g(x) and solving the equation – and proceeded through them systematically.

A few candidates were unsure what $g^{-1}(x)$ meant, believing it indicated either a derivative or a reciprocal. There was some carelessness, even, occasionally, in attempts at solving the equation $12 - a = 8$. There were some instances where the functions were composed the wrong way round. The only other problem occurred near the end of the solution when the equation $(2x+5)^3+4=68$ appeared. The majority adopted the sensible approach of dealing with $(2x+5)^3=64$ by going straight to $2x+5=4$ but there were some candidates who expanded $(2x+5)^3$ and were then faced with an awkward cubic equation; success then was not so common.

8) (i) This familiar request was answered well with 84% of candidates recording all three marks. The only error to occur with any frequency was a value of $36.9°$ for α; values of 25 and 7 for *R* were also noted occasionally.

(ii)(a) The fact that solutions were requested for values other than the usual $0°$ to $360°$ together with the fact that $5\sin(\theta+53.1°)$ was equal to a negative value meant that this equation presented some difficulties for many candidates. Nevertheless, 40% of candidates did manage to find the two values required. Many more were able to find one value, $-64.7°$, but not the other. The problem arose because candidates did not list enough possibilities for $\theta+53.1°$, perhaps excluding the relevant value, $191.5°$, because it was beyond $180°$. There seemed to be some suspicion of negative signs and the answer $41.6°$ sometimes appeared followed by a second value, $138.4°$, which happened to be correct but obtained through faulty working.

(ii)(b) This was a challenging request and many candidates made no progress beyond an initial step of $-37 \le 5k\sin(\theta+53.1°)+c \le 43$. The key to progress was a recognition that the value 43 corresponded to the greatest value, 1, for $\sin(\theta+53.1°)$ and that the value –37 corresponded to –1. Solution of the two simultaneous equations $-5k+c=-37$ and $5k+c=43$ completed the solution. Some elegant alternative approaches were seen, usually based on relevant curve transformations. Partial credit was available to candidates identifying the greatest and least values of 1 and –1 but persisting with inequalities.

9) (i) Though a majority of candidates realised the need to use the product rule, details were often wrong with the derivative of $\ln(2y)$ being the main problem. With the outcome given in the question, there were many sensible attempts at correction but also some more imaginative devices to reach $\ln(2y)$. Some candidates neglected the term $-y$ altogether or only brought it into the picture at a late stage when its presence was required to reach the conclusion. Attempts that rewrote $\ln(2y)$ as $\ln 2+\ln y$ were seen as were attempts that rewrote $y\ln(2y)-y$ as $y[\ln(2y)-1]$; such adjustments usually led to success. A concise, accurate response such as 'Using the product rule, $\frac{d}{dy}(y\ln 2y-y)=1\times\ln 2y+y\times\frac{2}{2y}-1=\ln 2y+1-1=\ln 2y$' would have earned the three marks immediately but such confident and clear answers were seldom noted.

(ii) Many candidates made at least some progress with this part. The first step of expressing x or x^2 in terms of y was attempted, usually successfully, by most who also stated the correct integral $\int \pi\ln(2y)\,dy$. There was then a pleasing recognition by many that the result of part (i) was relevant and they were able to integrate correctly. Some candidates ignored part (i) and attempted the integration by other means; apart from a few cases where integration by parts was used, such attempts did not succeed. There were also difficulties with the lower limit of the integration where 0 often appeared rather than $\frac{1}{2}$. A suitably simplified version of the answer was expected but there were errors involving signs and difficulty for some in dealing with $\frac{1}{2}e^4\ln(e^4)$.

A significant number of candidates proceeded as if rotation was about the *x*-axis and, inevitably, they could not earn any marks.

(iii) This part was a challenge to many who could not see what was required; indeed, there were sometimes comments about infinite cylinders. For those trying to find the volume of a cylinder, there was some confusion between radius and height so that answers involving e^8 appeared. However, it should be noted that some 11% of candidates answered all three parts accurately and duly recorded eleven marks.

4724 Core Mathematics 4

General Comments

Although there were a few non-familiar questions in the paper, there were plenty of requests where standard techniques were being tested. There are many excellent mathematicians whose presentation of the work is crystal clear and who give every indication that the paper is an enjoyable challenge. Unfortunately, as mentioned in previous reports, there were some candidates who struggled with the examination many of whom could benefit by giving the questions more thought, writing clearly to avoid miscopying their work and taking more care over signs.

Comments on Individual Questions

1) (i) Most were able to factorise the denominator but a significant number detected no difference between the $1-x$ in the numerator and the $x-1$ in the denominator, cancelling so as to give a final answer of $\frac{1}{x-2}$. One group of candidates, unexpectedly, decided to work with partial fractions, but the idea worked and $\left(\frac{0}{x-1}\right)+\frac{-1}{x-2}$ was soon produced. A further group (whose reasoning was obscure) inverted the fraction and divided x^2-3x+2 by $-x+1$, obtaining $-x+2$ with a subsequent final answer of $\frac{1}{-x+2}$; they were fortunate that the division process had zero remainder.

(ii) Partial fractions were, again, evident in many solutions – but the standard method worked well, provided care was taken with the algebra. Common denominators $(x-1)(x-3)(x-4)$ or $(x-1)(x-3)^2(x-4)$ were used, the former resulting in a numerator of $3x-9$ and the latter $3x^2-18x+27$; provided the denominators were not expanded, subsequent cancellation produced the answer.

2) The majority realised that the integral needed to be shown as $\int 1.\ln(x+2)\,\mathrm{d}x$ and moved rapidly to $x\ln(x+2)-\int\frac{x}{x+2}\,\mathrm{d}x$. Relatively few coped with the next stage. Changing the numerator of the integral into $x+2-2$ or using the simple substitution $u=x+2$ would both work; changing $\frac{x}{x+2}$ into $\frac{x}{x}+\frac{x}{2}$ was the most popular method which did not work.

3) (i) The first five marks were awarded for expansions of either $(1+4x)^{-\frac{1}{2}}$ or $(1+4x)^{\frac{1}{2}}$. In general, these were found satisfactorily; rarely was an unsimplified term wrong (except for 3 appearing instead of 3!) but there was much incorrect simplification. Although the use of $(1+4x)^{\frac{1}{2}}$ could have been used to produce the final answer, this was rarely (if ever) seen; however, most simple errors from the expansion of $(1+4x)^{-\frac{1}{2}}$ were followed through in the final answer. A not insignificant number of candidates wrote $(1+x)^2$ for $1+x^2$, either by misreading or miscopying.

(ii) Centres prepared candidates well for the sorts of expansion needed in part (i) but should also emphasise the restrictions required for the expansions to be valid.

4) In general, it was only the final request in the question which caused the problem. Most indicated, or implied, that the original equation could be written in the form $\int e^{2y}\,dy = -\int \tan x\,dx$ and performed the integration satisfactorily. Although examiners may not always insist on '+c' being added to all integrations, they obviously require it when boundary conditions are given.
The manipulation of the equation into the required format was not well done, although many indicated a partial understanding of the principles involved; probably many found the existence of the term involving ln within a natural logarithm confusing.

5) (i) Most candidates were successful in this part, though a few favoured the sine instead of the cosine in the scalar product equation.

(ii) This was not a standard problem but it was designed to test candidates' geometrical understanding of the meaning of $|\mathbf{a}-\mathbf{b}|$. Very few coped; most just resorted to $|\mathbf{a}-\mathbf{b}| = |\mathbf{a}| - |\mathbf{b}|$. A respectable minority indicated that the length of AB was required, and then used a simple application of the cosine rule to determine it. A very few worked with $(\mathbf{a}-\mathbf{b}).(\mathbf{a}-\mathbf{b})$ and generally were successful although the difference between **ab** and **a.b** was not always clear.

6) This was quite a successful question with most candidates coping with individual aspects. They were expected to show a relationship (in any suitable form) between du and dx and then transform the given integral into $\int f(u)\,du$.

They then had to manipulate a function of the type $\frac{au+b}{u}$ and integrate it – and finally use appropriate limits to confirm the given value. Irrespective of the capabilities of candidates, the required answer always appeared on the last line – but, needless to say, solutions were carefully scrutinised and any errors or carelessness duly noted.

7) This question differentiated well between candidates and the mark scheme was designed to cope with minor discrepancies, provided the basic understanding of the method required was evident. The most common start was to square – and even that produced errors involving $-\sin^2 3x$, $\sin^2 6x$, $\sin^2 9x$ and the omission of the term $-2\sin 3x$.
Most realised that the integral of $\sin^2 3x$ involved changing it into f(cos 6x), or $\sin^2 6x$ into f(cos 12x) or $\sin^2 9x$ into f(cos 18x) and many of these attempts were successful. The integrations of the cos 6x, cos 12x or cos 18x terms were investigated, as was the integral of the $-2\sin 3x$ term (if present). Finally, the substitution of the limits was checked.

Integration using parts was frequently successful but any integration to $\frac{(1-\sin 3x)^3}{3}$ or similar was instantly disregarded.

8) (a) This was very well done with only the occasional lack of understanding. Most candidates, however, did leave it to the end of the work to substitute the values of x and y, whereas it would have been a slightly simpler exercise had these values been substituted immediately the differentiation had been effected.

(b)(i) The crux of this problem was the manner in which the phrase 'tangent is parallel to the y-axis' was translated. Some latitude in expression was allowed here (for example $\frac{dy}{dx} = \infty$) providing the intention was clear. This did not have to be stated explicitly; if $\frac{dy}{dx} = \frac{3t^2 + 1}{4t}$ was given followed by $t = 0$, this was accepted.

(b)(ii) Most indicated that $\frac{dy}{dt} = \frac{dx}{dt}$ and successfully solved the subsequent quadratic equation. Some candidates, having obtained $t = \frac{1}{3}, 1$, gave the final answer as $\frac{1}{3} < t < 1$; this extra step was ignored. Surprisingly, there were many candidates unable to differentiate accurately $2t^2 - 1$ and/or $t^3 + t$. There were fewer candidates, however, on this occasion who developed their own questions and automatically worked out the cartesian version of the curve.

9 (i) In general, candidates understood this aspect of the syllabus and were very successful. It is appreciated that tutors/teachers have particular principles in respect of what they consider to be satisfactory methods in solving examination questions, but it should be said that the majority of errors arose when candidates used the comparison of coefficients method for the derivation of these partial fractions. As the equations involved all of A, B and C, any error in determining one of them would have repercussions in finding the others.

(ii) The integration of two of the fractions involved natural logarithms and candidates were generally successful. The integration of the third fraction, $\frac{C}{(x-2)^2}$, proved an obstacle; the index -3 often appeared, and it was C rather than $-C$ that completed it.

10 (i) The reasoning behind the solutions was rather flimsy and carelessly expressed but, in the majority of cases, it was felt that candidates understood the principles.

(ii) It was pleasing to see how well many candidates were able to give a correct (though not necessarily the simplest) vector equation for the line through O and A.

(iii) This part could be approached in a few different ways and many candidates gave a good indication of their ideas. There was, however, a substantial number of candidates who made no attempt at this part.

4725 Further Pure Mathematics 1

General Comments

The majority of the candidates found this paper accessible. Most were able to demonstrate sound knowledge of a good range of syllabus topics, with most candidates producing completely correct solutions to several questions. Completely correct solutions to all questions were seen and there was no evidence of candidates being under time pressure. Candidates now seem used to the printed answer booklets and few wrote their answers in the wrong place. Most candidates had sufficient space with fewer candidates requiring additional sheets. There were more errors in basic algebraic techniques seen than in previous years, with few appearing to check their working. This often meant that quite a few marks were lost, even though the method required was known.

Comments on Individual Questions

1) (i) Most candidates answered this part correctly, with only minor arithmetic slips seen.

(ii) The majority of candidates multiplied by the conjugate of the denominator, but a significant minority evaluated (5 + 4i)(5 – 4i) to get 25 + 4.

2) (i) Most candidates answered this part correctly, with only minor arithmetic slips seen.

(ii) Candidates who used $\mathbf{B}^{-1}\mathbf{A}^{-1} = (\mathbf{AB})^{-1}$, generally answered this part correctly. Those who found both inverse matrices often omitted one or both determinants, while a minority thought that $\mathbf{B}^{-1}\mathbf{A}^{-1} = (\mathbf{BA})^{-1}$.

3) Correct solutions using one of the many methods available were seen, the most popular being the sum and product of the roots, or the expansion of two linear brackets. Arithmetic errors or sign slips could have been picked up if candidates had solved their equation as a check.

4) Most candidates made reasonable progress with this question. The standard results were well known, with the most common error being giving the last term as 2, rather than $2n$. These candidates should have realised that something had probably gone wrong, as there is then no common factor. Those who saw the factor of ½ n often left the last term as 2, rather than correctly getting 4. Very few candidates checked their answer was correct, for example, when $n = 1$.

5) This question proved challenging for a good number of candidates. Most established the result when $n = 1$, but then added $k + 1$, the sum to k terms or the sum to $k + 1$ terms to the sum to k terms. Those who added correctly the $(k + 1)$th term often then omitted enough working to justify the derivation of the sum to $k + 1$ terms. Centres should remind candidates that a clear statement of the induction principle is required; it is not sufficient to just say "True by Induction" or something similar.

6) (i) A correct quadratic equation was found by most candidates, although some omitted " = 0". There were many errors in basic algebra seen, for example $(u + 1)^2 = u^2 + 1$ and $5(u + 1)^2 = 5u^2 + 10u + 1$.

(ii) Many candidates saw that the value required is the product of the roots of the quadratic equation in part (i), while others rearranged to obtain an expression in $\alpha + \beta$ and $\alpha\beta$. A significant minority then used the values from their quadratic equation from part (i), rather than the values from the original equation.

7) (i) Most candidates realised that C_1 was a circle, but the centre was often in the wrong quadrant. Many were not careful enough to demonstrate that the circle touches the x-axis, crosses the y-axis twice and does not pass through the origin. That C_2 was a horizontal line through $y = 4$ was often not seen, the most common errors being a line through $y = 8$, a half line starting at the y-axis, or even another circle.

(ii) Those candidates who had a reasonable sketch were able to find the points of intersection, but some gave the answers as coordinates (-1, 4) and (7, 4) or (-1, 4i) and (7, 4i) and not as complex numbers as requested.

(iii) Most realised that the region required was inside the circle, but fewer that it was above C_2.

8) (i) The given result was established by a large majority of candidates.

(ii) Most used the method of differences correctly though some did not show sufficient terms to see which terms cancelled. A reasonable number of candidates who had a correct expression for the required sum, then tried to simplify and made an error. This was not penalised in this part, but meant that the solution in part (iii) went astray.

(iii) The majority of candidates were able to find an equation for N from their answer to part (ii).

9) (i) While the misspelling of “shear” as “sheer” or “shere” was not penalised, “skew” is not an acceptable synonym. The best way of indicating the direction of a shear is to give the image of one point or state which axis is invariant.

(ii) **Z** = **YX** was seen more frequently than **Z** = **XY**, the incorrect order of transformations. Most were then able to attempt to find **Y,** with only minor slips seen.

(iii) The description of the transformation was generally well done, although some candidates gave a pair of transformations for **Y**.

10) (i) Most candidates could demonstrate a correct method for finding det **D,** with algebraic slips, for example $a(a^2 - 1) = a^3 - a^2$, causing some loss of marks in this part. However this type of error can lead to a significant loss of marks in part (ii).

(ii)(a) In all parts candidates were expected to firstly evaluate their determinant correctly. When a correct non-zero value for the determinant was found the unique solution can be stated.

(ii)(b) When a zero value for the determinant is found, non-uniqueness can be stated, and then an attempt to solve the equations must be seen. It is helpful to the examiners if some indication of the method of solution is given and that when a pair of inconsistent equations is found these are clearly indicated.

(ii)(c) When the solution leads to a repeated pair of equations, these should be clearly indicated or a clearly equivalent statement should be given, so that consistency has been established.

4726 Further Pure Mathematics 2

General Comments

Most candidates were able to access all questions. Although the last part of the last question was answered by the smallest number of candidates, the same was not true for the first part. This indicated more the difficulty of the question rather than candidates running out of time.

In the last report we commented on the need for full working to be shown to achieve a given answer in a "show that..." question. This remains an issue and comments are offered in the body of the report.

There are occasions when candidates are not able to complete their answer to a part question within the space allotted. In such situations, candidates are required to complete their working on separate additional sheets. The use of additional multi-page booklets should be avoided and candidates should avoid using space that is available but allocated to answers for another part question. It is also helpful if the candidate writes in the answer space that an extra sheet has been used.

Comments on Individual Questions

1) Bearing in mind the space allocated to this question and the marks available, it is expected that candidates will have a feel about where to start. To start with the definition for $\cosh x$ and use the formula for $\cosh 2x$ before inverting is rather more than required. It was sufficient to say $\operatorname{sech} 2x = \dfrac{1}{\cosh 2x} = \dfrac{2}{e^{2x}+e^{-2x}}$. A number of candidates failed to manipulate the algebra correctly or to make the substitution correctly to give the correct integral. Of those that did get this far, a few failed to convert their answer back into an expression for *x*.

2) (i) Most candidates knew how to find the equations of the tangents.

   (ii) In this part, however, there were many who did not understand that differentiation was required to find a maximum. Many appealed to symmetry (which was not present). As a result, nearly 50% earned no marks for this part.

   (iii) There was some very poor algebra seen here. Most gained the two marks for the substitution of *x* and *r* but were unable to eliminate fully to give a required result. Two marks was a common outcome.

3) (i) There were two problems here. There is no identity for $\tanh(x+y)$ in the formula book so this is not an appropriate place to start the question; nor is "Osborn's Rule" by which candidates could claim an adaptation of $\tan(x+y)$. Secondly, taking $\tanh 2x = \dfrac{\sinh 2x}{\cosh 2x} = \dfrac{2\sinh x\cosh x}{\cosh^2 x+\sinh^2 x}$ and then writing down the answer as stated is a situation where not enough working is being seen. It was necessary to explain that the numerator and denominator had to be divided by $\cosh^2 x$ to obtain full marks.

   (ii) Using the results of part (i), a cubic in $\tanh x$ was usually obtained. However, the solution of that equation was not always found. For some, the process of finding the three roots and proceeding to the required answer seemed straightforward; others reverted to re-expressing the cubic in terms of exponentials, creating for themselves a huge task. Just a few got to a quadratic in e^{2x} and hence to the answer but most got completely lost in the algebra involved.

4) (i) This part was done well with 90% of candidates achieving full marks. Some of the

diagrams left a little to be desired but, providing the staircase diagram was clearly seen, these were accepted.

(ii) The sketches for this part were not so good and many did not fulfil the conditions for the marks. For full marks, it was necessary to demonstrate divergence so it was necessary to identify, at the very least, the starting value. Arrows on the lines were accepted as were the points on the *x*-axis marked by the values or $x_1, x_2,$ etc.

(iii) Many completed the iterations from part (i) to produce the root. Others made an error in the differentiation of the function, or made an error in the use of their calculator, but still produced the root. In neither of these cases were marks awarded. This part was for a demonstration of an understanding of the Newton-Raphson method. This required the formula to be stated and the correct iterations shown to a point where two consecutive iterations agreed to 4 decimal places.

5) (i) The formula book gives the derived function of $\sinh^{-1} x$ so it was surprising how many candidates wrote down the wrong function. Perhaps even more surprising was the fact that some candidates turned $\sinh^{-1} x$ into its logarithm equivalent (from the formula book?). This rarely achieved the correct result as the algebra required was usually overwhelming. The major error was the failure to use the chain rule when differentiating the second term.

(ii) A majority of candidates assumed symmetry correctly though, once again, the sketches left much to be desired and it was rarely evident that the *y*-axis was an asymptote. A mark was often lost by not reading the question properly. The demand was for the range of values for which $x \neq 0$ so this included the part in the first quadrant as well.

6) (i) It was evident that some candidates worked backwards to correct their mistakes; this was accepted of course providing they backtracked far enough. Often there were signs incorrect and *n* missing though they reappeared later.

(ii) A few candidates ignored the formula of part (i) and evaluated I_5 from scratch. Others used the formula in part (i) to find I_1. This gave the right value but was not accepted as that formula was only valid for $n > 1$.

7) (i) The four values here were usually given correctly, though the most popular error was to give them the wrong way round, i.e. $a = 1$ and $c = 2$, etc.

(ii) The majority of candidates did not understand the connection between f(*n*) and the series given in part (i); many more candidates got no marks than those who got full marks.

(iii) Most candidates were able to write down the first two terms of $\ln(1+x)$ but completely misunderstood what *x* could or could not be. Consequently, those who wrote down $f(n+1)-f(n)$ correctly and then expanded $\ln(1+n)$ could not be awarded any marks. Most of those candidates also found difficulty in expanding $\ln n$ and so wrote it as $\ln(1-(1-n))$. The requirement of the question was that $\ln n - \ln(n+1)$ should be written as

$-\ln\left(\frac{n+1}{n}\right) = -\ln\left(1+\frac{1}{n}\right) = \left(\frac{1}{n}-\frac{1}{2n^2}\right)$; those that did so usually obtained the correct result.

However, another common error was to write $\ln n - \ln(n+1)$ as

$\ln\left(\frac{n}{n+1}\right) = \ln\left(\frac{n+1-1}{n+1}\right) = \ln\left(1-\frac{1}{n+1}\right) = -\left(\frac{1}{n+1}+\frac{1}{2(n+1)^2}\right)$. This produced a valid algebraic approximation but not the one given. It would not be possible to obtain the desired result because this way there is no *n* in the denominator. In spite of this, 4 marks out of 5 were given for a correct fraction.

8) (i) Candidates who started with the information about the asymptotes and wrote $\frac{1}{2}x+1+\frac{A}{x+2}$ usually found that A = 8 and combined the two terms to give the correct answer. However, a majority of candidates started with the information about q(x) and p(x) and wrote $\frac{Ax^2+Bx+C}{Dx+E}$. This resulted in an enormous amount of algebra, including a long division and most foundered, sometimes after pages of work.

(ii) A small number of candidates differentiated the equation of C_1 to give values of x at turning points from which the correct range of values for y was obtained. The majority, however, rewrote the equation as a quadratic in x and used the condition for real roots. Candidates using an incorrect C_1 were able to obtain most of the marks in this part.

(iii) A few misread the question and wrote $y=\left(\frac{\mathrm{p}(x)}{\mathrm{q}(x)}\right)^2$ giving a lot of algebra which was fruitless. Candidates using an incorrect C_1 were also able to obtain most of the marks in this part.

4727 Further Pure Mathematics 3

General Comments

Overall this paper was found to be of a similar standard to those of recent years. Most candidates were able to attempt all questions, and time did not appear to be a major issue. Differential equations and the easier questions on vectors and group work were topics on which most candidates scored well. The topics that candidates were least secure in were Argand diagrams and the properties of sub-groups. The very best scripts used clear, precise mathematical language in solutions and proof, often showing a good understanding of necessary and sufficient conditions. Candidates should be aware that, when the examiner asks for a proof or demonstration, it is important to give the final answer in the form requested. A significant proportion of candidates were poorly prepared for this paper; these candidates tended to show a complete lack of knowledge of whole topics. There was again evidence that weaker candidates were insecure in certain Core Mathematics topics.

Comments on Individual Questions

1) The best answers to this problem were precise and used the vector product of the direction of l_1 and the normal to p. Other candidates used scalar products to find two simultaneous equations, but this method did lead to more mistakes and some had no technique for dealing with three unknowns. Candidates were penalised if, for their final answer, they wrote $l_2 = \cdots$, rather than $\mathbf{r} = \cdots$.

2) This question produced a good spread of marks.

   (i) Part (i) was generally well answered. Most could find the argument of z^4 and knew that they needed to divide it by 4. Many then went on to find all four angles correctly, but some either neglected to give their answer in the requested form or miscalculated the modulus. A modulus given as $\sqrt[4]{4}$ was quite commonly seen, but at this level candidates were expected to use the standard simplified form of $\sqrt{2}$.

   (ii) In this part, the quality of sketches was generally quite poor. Few showed the roots to be positioned at the ends of perpendicular diameters of a circle. Many failed to show the moduli and arguments of z and z^4 in correct proportions and many were unaware that an Argand diagram requires equal scales on horizontal and vertical axes.

3) The questions on differential equations were generally well answered. The vast majority of candidates were able to correctly find the integrating factor and most then produced a right-hand side requiring integration by parts. Those who did get this far quite often then made sign or calculation errors. Sometimes candidates appeared to be differentiating a product rather than integrating by parts. The final answer was only accepted in standard notation and not when the solution contained a fraction within a fraction.

4) (i)(ii) This question was very well answered with virtually all candidates correctly answering these two parts.

   (iii) In part (iii), most identified H as being the isomorphic group, though not always with substantiating correspondences. Some gave only one of the isomorphisms. Those who tried both were insufficiently precise about the pairings of c, d with r, r^3. Without precision here, the isomorphisms are not properly specified. A common solution, which did not fully address the question asked, was to simply show that the orders of elements corresponded: partial credit was given for this.

5) This question differentiated well between candidates of differing abilities. The strongest candidates wrote a clear concise 5 or 6 line proof in part (i) and demonstrated their breadth of mathematical ability with part (ii) which required little more than general algebraic confidence in applying Core Mathematics 2 techniques.

(i) In this part, most candidates were able to make some progress. About two-thirds used exponential expressions for both sin θ and cos θ; the rest merely used the term for sin θ, having given the original expression as powers of this. A very common error was to omit i from the denominator of the sin θ term. Even when this was included, sign errors crept in upon taking powers. Most gained marks for the binomial expansions and for grouping terms. Several candidates converted back to multiple angles too soon and then, unsuccessfully, tried to use compound angles to simplify the resulting expression.

(ii) Here there was a significant proportion of candidates who failed to use part (i), or who did not apply the standard procedure for dealing with an equation where $f(x)g(x) = 0$. Whilst many, correctly, established sin $\theta = 0$, the solution to this was often only given as $\theta = 0$. Similarly, for cos $\theta = 0$, the solution was often given as $n\theta/2$ (which is incorrect for even values of n). Solving $\sin^3\theta - \sin^5\theta = 0$ was a valid, alternative approach sometimes seen.

6) (i) This part generally proved a very successful question for candidates, with the vast majority being able to correctly solve the auxiliary equation and find the correct particular integral. A small number produced an incorrect auxiliary equation of the form $m^2 + 4 = 0$ or, having stated the correct equation, gave Ae^{-4x} or $(Ax + B)e^{-4x}$ as the complementary function. Almost all got the correct particular integral, though a few, mistakenly, tried axe^{2x}. Candidates usually knew how to combine particular integral and complementary function to produce their general solution.

(ii) This also proved a good source of marks for most candidates who had correctly solved part (i). However, a number took insufficient care whilst solving the simple equation $-4B + 2 = 6$ with a significant number stating the incorrect result that $B = 1$.

7) Many candidates demonstrated a poor understanding of vectors believing that **u**, **v** and **w** should be interpreted as u**i**, v**j** and w**k**, or even that **u**, **v** and **w** were identical to **i**, **j** and **k**. There were also many instances of poor vector notation, such as writing M in place of $\overrightarrow{OM}$, and these appeared to greatly contribute to candidates' misconceptions and errors.

(i) Few candidates gave well-presented and fully coherent arguments in order to show the given result, often gaining their marks from an argument that lacked clarity. The best candidates, again, wrote clear mathematical language that communicated their steps precisely.

(ii) Some candidates could not find their way into this question, while others tended to either use the equation of the line UM as a starting point, or to compare vectors $\overrightarrow{UM}$ and $\overrightarrow{UG}$. Those who gained all 5 marks on this part usually scored the final two marks by repeating the full working two more times, and it was rare to see an elegant use of the symmetry of the situation. Some candidates found the point of intersection of lines *VN* and *WP*, showing that it was point *G*. However, many lost marks by relying upon their knowledge of medians and the centroid of a triangle rather than deducing such by vector means.

(iii) Many of the candidates who used notation well were able to score full marks on this part or at least gain part marks for one of the two terms given correctly.

(iv) This final part was often tackled better than the preceding ones with candidates gaining 2 or 3 of the marks available. The errors often occurred when trying to evaluate the vector product where sign errors in one of the elements were regularly observed.

8) (i) Whilst the very strongest candidates regularly produced concise, thorough proofs, many candidates were unaware of the requirements for a subgroup. Some wasted time dealing with associativity, which is not a requirement (since it follows from *R* being a sub-set of a group), although they were not penalised for so doing. In dealing with closure, a common error was to consider only $R(\theta)R(\theta)$. Inverse, identity and closure were often addressed, but frequently without the candidate demonstrating that their matrix was a member of *R*. For instance, the identity I was not always linked with $\theta = 0$, nor the inverse $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ with $-\theta$.

(ii) Most candidates realised that the group had something to do with $2\pi \div 6$, but many believed that they needed reflections and rotations. Some tried to use θ as $\pi/6$ or $2\pi/3$ and some got no further than listing the six angles of rotation.

Overview - Mechanics

A high proportion of the scripts in each unit contained work of a commendable standard, by candidates who had evidently applied themselves diligently to their course and who had been well taught.

It remains the case that many scripts contain work which is disorganised, and therefore difficult to comprehend. This is understandable when a candidate is unsure of how to address a particular question. However, when an entire paper consists of an assemblage of numbers with no clear origin (possibly leading to a correct answer) it is apparent that some important mathematical lessons about clarity, logic and communication have not been learnt.
Without evidence that correct methods and formulae are being used with appropriate values, examiners cannot award marks. The usefulness of a clear annotated diagram in alerting the examiner to the candidate’s thinking cannot be over emphasised.. Its creation also helps ensure the candidate reads the question accurately, and gleans from the question paper all the relevant details given.

4728 Mechanics 1

General Comments

Many good scripts were submitted, with candidates well prepared across the full range of topics within the syllabus. One particularly pleasing aspect of the work was the ability of candidates to devise a successful strategy for their solution.

Questions which had several sections were at times wrongly answered because some candidates assumed an answer to an earlier part would be relevant in a later one. A second common source of error was the failure to work to sufficient accuracy to achieve answers correct to 3 significant figures. Sign errors were also common, often arising from confusion relating to direction when employing suvat formulae.

Comments on Individual Questions

1) (i) Very few wrong uses of Pythagoras Theorem were seen. A large majority of candidates also calculated the angle correctly. The small minority of wrong values arose from selection of an inappropriate trigonometric ratio, rather than finding the wrong angle.

(ii) The common incorrect value for E was 15, an answer from part(i). The most common wrong value for the angle arose from adding 90° to the angle found in (i). Candidates were given full credit for either adding their angle in (i) to 180° or for subtracting it from 180°. A few candidates did not understand that “state” implies a brief response, without significant calculations.

2) (i) Nearly all candidates found both values correctly. Marks were lost either through finding only one quantity, or using an incorrect sign with the value of g.

(ii) A majority of solutions were obtained through use of ascent and descent times, and usually gained full marks, even though the exact answer was not given. An accumulation of small errors through premature approximation led to the loss of the final mark for a small minority of candidates. A small number of solutions used the calculation of a distance associated with a time of 0.4s, and incorporated it with the distance found in part (i) at 0.5 s. At best this gained 1 mark for the height above the ground of the particle at 0.9s.

Relatively few (quicker) solutions used the height calculated at 0.9 s, and the greatest height (H) found from 02 = 72 -2gH.

3) (i) Many correct solutions were seen, some using very precise mathematics. Confusion about how best to manage the sign associated with deceleration was the origin of almost all solutions which did not gain full marks.

(ii) Many different ways to find A’s distance were seen. Solutions relying on the equivalence of area and distance being more successful than ones relying on constant acceleration formulae. A minority of scripts showed a direct calculation of the area between the graph lines for the motion of A and B. A very common error was to ignore the 1 m difference in distances at the instant the baton was exchanged, and some who did include it used the measurement incorrectly to give an answer of 10.6 m.

4 (i) The most common source of error was to place the block on a slope inclined at 30° to the horizontal. By using plausible quantities associated with this erroneous configuration, candidates could obtain the given value of μ. However, a majority of solutions used the correct configuration and gained full marks.

(ii) The increase in the value of the normal component of reaction when the tractive force was removed was often overlooked. Confusion was also apparent when finding the speed when 14 N force was removed: should it be u or v, was a positive or negative?

5 (i) Often this was calculated correctly, though the value 2.94 (arising from a sign error with 0.4 x 2.45) was common.

(ii) Virtually all candidates appreciated which quantities had to be calculated for the solution. Rounding the intermediate value of Q's mass to 0.67 kg unfortunately gave the wrong answer.

(iii) The usual errors in this type of question were less common this session. Candidates correctly found the distances P moved with its two different accelerations. In very many scripts the initial height of P above the surface was omitted.

6 (i) This was usually answered correctly, though some equated the component of tension and the mass.

(ii) Though many correct values were calculated, H = 4.11 (from 6.4sin40) was a very common error. It was unclear whether this was a consequence of (i) being based on 6.4cos40, or a simple mis-understanding of the relevance of the tension in the horizontal part of the string.

(iii) This was evidently the hardest item on the paper, but about half of the candidates answered it correctly. The most common error was the inclusion of H in the calculations of friction and the normal component of the reaction. The printed answer led to much confusion as ad hoc improvisation were made.

(iv) The printed answer in (iii) was intended to help candidates answer this item, however it was often omitted by those who could not resolve (iii). Usually a comparison was made of the maximum possible frictional force with relevant component of the weight of P. The inequality made some candidates deduce P would move. The word friction was used interchangeably for the value of μR and the force needed to hold P at rest.

7 (i) Almost all candidates answered this correctly.

(ii)(a) Though most values were correct, 3.37 (from 3.1 + 3 x 0.32) and 2.83 (from 3.1 - 3 x 0.32) were fairly common errors.

(ii)(b) Very few solutions were seen involving constant acceleration formula, and the most common reason for a wrong final value was having the wrong value of V0.

(ii)(c) Correct answers were common, candidates almost always rightly interpreting "immediately before" as t = 0.3 though sometimes t = 0.2, 0.29 etc. were used. An error in the value of V0 did not lead to a loss of marks here.

(iii) The majority of candidates gave a correct solution, though sign errors for the "after" momentum, or 0.5 x 3.1 for the "before" momentum were quite common.

4729 Mechanics 2

General Comments

Examiners commented on how pleasing it was to see that the majority of candidates were well prepared for the rigours of this module. Only a minority found some of the questions inaccessible.
As usual, those candidates who drew relevant force diagrams and informed examiners of what they were doing performed well. It should be noted, that in questions where the answer is given, candidates should show enough detail to demonstrate to examiners that they have achieved the required result.

Comments on Individual Questions

1) (i) This was answered extremely well with nearly all candidates correctly calculating the speed after impact with the wall as 1.2. The majority of candidates appreciated that to calculate the impulse the difference in momenta needed to be considered but a significant number did not appreciate the change of direction due to the impact.

(ii) This part was nearly always correctly given as 5.82(4) with only a few candidates either calculating $\frac{1}{2}(0.8)(4-1.2)^2$ rather than the correct $\frac{1}{2}(0.8)(4^2-1.2^2)$.

2) (i) Another well answered question with the majority of candidates scoring full marks. Nearly all calculated the driving force correctly by considering the ratio of the power and velocity and many went on to apply Newton's second law correctly to arrive at the answer of 0.125 – it was noted that a small minority either did not include the constant resistance force in their $F = ma$ calculation or added the resistance.

(ii) Most candidates appreciated that the acceleration was now zero and that the weight needed to be resolved parallel to the plane and the majority did this correctly. A significant number of candidates omitted g from their weight component.

3) (i) A very common wrong answer seen was 392N. This error arose from candidates assuming the force exerted on the beam was vertical, even though the question clearly stated the "force exerted on the beam at *Q* by the rod is in the direction *PQ*". Another common error was the failure to deal with trigonometry of the problem correctly and therefore having the angle between the rod and the horizontal as 30 (rather than between the rod and the vertical).

(ii) This part proved difficult for a significant number of candidates. Some who found part (i) tricky made no attempt at this part. Candidates could either resolve vertically and horizontally or take moments to find the components of the required force. Combining these components was usually well done, however some candidates did lose the final mark by not making it clear (either in words or via a diagram) the direction in which the force exerted on the beam at *A* acted; examiners wanted to see an indication that it was below the horizontal (or the downward vertical) - *to the horizontal* was not sufficient for this mark.

4) (i) This part was usually well answered by the majority of candidates. A common error seen was for a minority to use the 14.4 ms^{-1} vertically and as well as horizontally.

(ii) This part was answered well by those who approached the request by attempting to find two equations, by considering horizontal and vertical motion, connecting the initial velocity and the time for the whole flight. It was very common to see 0.2 rather than -0.2 used. A less successful approach was for those who considered the motion in two parts – the first part up to the maximum height and then from the maximum height to the point of impact with the wall.

5) (i) This part was either answered very well with the strongest candidates easily scoring full marks or close to full marks, while at the other end of the spectrum a number of candidates scored no marks at all. It needs to be stressed that a clear diagram with all relevant forces in place would assist candidates set up the relevant equations of motion. The most common mistakes were incorrectly calculating an angle or assuming that the angle was one of 15, 30, 45 or 60. It was surprising to see a significant number of candidates who wrongly assumed that the tensions in the strings were equal and only considering horizontal motion.

(ii) This part proved difficult for a significant number of candidates. The request relied upon the fact that the least possible speed of *P* is when the tension in the lower string is zero. For those who did appreciate this, it was all too common to see a final answer of angular speed when the request was for linear speed. Some candidates did not appreciate that the situation as described was different from part (i) and used the tensions found in (i) to, unsurprisingly, find the speed to be 4.8 $m s^{-1}$.

6) (i) This part was answered extremely well by nearly all candidates and it turned out to be a good source of marks for the weakest candidates. However some candidates did lose marks by not showing sufficient working in solving the momentum and restitution equations simultaneously.

(ii) This, in contrast to part (i), was answered poorly. While many scored the first two marks for two relevant equations involving v_B and v_C, a number did not appreciate that $v_B \geq 0.2$ for there to be no further collision between *A* and *B;* many simply stated that the velocity of *B* only had to be positive for no such collisions to occur (or that somehow the speed of *C* was relevant at this point). While many tackled the whole problem without inequalities this did cause to be a problem at the end when many guessed incorrectly that $e \geq {}^{2}/_{3}$. A significant number of candidates who correctly arrived at $0.44 - 0.36e \geq 0.2$ (or equivalent) could not solve this inequality correctly.

7) (i) Answered well by the majority of candidates. The most common error was in the inability to deal with the relevant position of the centre of mass of the triangle.

(ii) This part was well done. It was pleasing to see that candidates persevered even though they may not have been successful in part (i).

(iii) This question proved to be the most difficult on the paper. The position of the centre of mass from *AB* (or *CD*) was required to answer the request. However it was common to see candidates assuming this to be 2.5 cm.

4730 Mechanics 3

General Comments

As usual the candidates for this unit showed a wide spread of ability. There were a number of well prepared and very able candidates who scored full marks, or almost full marks. The majority of candidates found this a challenging paper that allowed them to show what they could achieve.

Many candidates presented their work in a neat and orderly fashion. However, a considerable number of scripts were written untidily, and it was sometimes very hard to follow the thread of the mathematical argument being presented on these scripts.

As is usual on this paper there were a number of questions that could be solved by a variety of methods. In particular, candidates showed a wide variety of approaches on questions 2 and 5, with many carrying out complex calculations successfully. It was a pleasure to mark these when the work was presented clearly and neatly.

In some questions significant number of candidates might have done better if they had drawn clear diagrams for themselves, with forces or velocities, etc, marked on carefully. This would have helped clarify thoughts, and avoid errors with signs.

Comments on Individual Questions

1) (i) Almost all candidates found this an easy starter to the paper for 2 marks.

(ii) A small number of candidates found the normal reaction at to be 22.5 N rather than 40 N, but were still able to gain marks for the vertical component of the force. A considerable number of candidates gave the components of the force exerted on *BC* at *B*, instead of on *AB*, and just a few gave the components of the force exerted on *AB* at *A*.

(iii) Candidates who had made mistakes earlier benefitted from some follow through marks here. Many candidates worked out the horizontal distance of the line of action of the centre of mass from *A* rather than the distance from *A* to the centre of mass of *AB*, and some who tried to calculate the correct distance from this got into difficulties with trigonometry.

2) (i) Very few candidates gave the complete answer, that the impulse is perpendicular to the plane because the plane is smooth.

(ii) Almost all candidates correctly worked out that the mass has a speed of 7 m s^{-1} when it first hits the plane. A variety of methods were then used to find u and I; with many candidates being totally successful whether they used a triangle approach, or started by considering the conservation of velocity parallel to the plane, or simultaneous equations for perpendicular components of I. A few candidates got into difficulties by omitting mass from some terms of momentum equations.

3) (i) This question was done correctly by many candidates. Almost all realised that they had to use $v\,dv/dx$ for acceleration, and only a few failed to get the ‘200’ in the integral. A larger number slipped up by forgetting the constant of integration. The final demand in this question was to show that v^2 must be less than 3920, which was usually done by pointing out that $e^{-0.005x}$ is always greater

than 0. Many candidates instead pointed out that this expression tends to 0, and so v^2 tends to 3920, which is not quite the same thing.

(ii) This was a simple matter for candidates who had the correct expression for v^2 in part (i). Marks were not awarded if candidates had an answer to part (i) that simplified this demand.

4) (i) Candidates generally did this piece of bookwork extremely well, with most scoring all 6 marks. Unfortunately, some candidates omitted finding the expression for *R* in terms θ, and so were unable to gain the marks for this, even though many of those did equivalent work in arriving at their answer to part (ii).

(ii) Most candidates gained full marks for this part; those who had not arrived at the correct expression of *R* in part (i) were generally able to gain 3 of the 4 marks on follow through, providing they had a similar expression.

5) (i) The majority of candidates realised that the maximum speed of *P* would be at the equilibrium position, and first found that this was for an extension of the string of 0.375 m, or for a total length of string of 1.155 m. They then used an energy equation to find the maximum speed of *P*, with only a few making mistakes with signs. A considerable number of other candidates took the extension of the string as *x* (or sometimes the total length of the extended string as *x*) and worked out expressions for KE, PE and EE in terms of *x* to gain an equation for v^2. They then either completed the square to identify the maximum value of v^2, and hence *v*, or else used calculus. A few candidates took more complicated approaches, like finding the KE at the point when the string was fully extended, or at the equilibrium position, and based their solution on changes of KE, PE and EE from that position, often successfully.

(ii) The most common approach here was to work out the EE gained and the PE lost between the particle being at *O* and being at its lowest point, with candidates taking as their unknown either the extension of the string from its natural length, or the extension from the equilibrium position, or the total length of the extended string. Only a very few of these omitted to add on the original length of the string, or string plus extension to equilibrium position, when it was appropriate. Some of those who had found an equation for v^2 in terms of *x* in part (i) used this to very quickly and easily find this answer.

6) (i) This question was generally done well, though some candidates found the component of A's speed parallel to the line of centres, and others took the 7.56 J as a gain in KE.

(ii) While most arrived at 1.2 m s^{-1}, not all could give a convincing reason as to why this velocity component was to the left, and a few thought it was to the right.

(iii) Some follow through was allowed here for candidates with a wrong value or sign for the velocity component of A parallel to the line of centres; some candidates with the right answer in part (ii) made a mistake with the signs here. Some also omitted the mass from one or more of the terms.

(iv) Those who had got parts (ii) and (iii) correct generally succeeded with this part too.

7) (i) Most candidates successfully showed that the total elastic potential energy at each position was 24.6 J, though a few made errors in the extensions, the calculations, the formula (forgetting the '2' in the denominator), and a small

number subtracted to get 24 J. Most candidates then failed to give a convincing argument as to why neither string becomes slack in the subsequent motion, with most just pointing out that neither string is slack when $AP = 2.1$ m or when $AP = 2.9$ m. Acceptable explanations almost always referred to the speed or KE of the particle.

(ii) Some candidates did not get the extensions of the two strings quite right, but most had an attempt at applying Newton's second law, with not all the signs being right. A small number did not use the correct formula for tension.

(iii) Very few gave anything other than SHM here, but the question did ask for a reason, and just quoting the result found in part (ii) was not considered sufficient. Most of those with SHM equations in part (ii) successfully found T, but many did not realise that the question asks for a half of the period

(iv) Most gained M1 for successfully using the correct formula, though some wrongly used $x = a\sin wt$. A second M1 was given to those who realised the distance travelled was more than 4 times 0.4 m (the amplitude) but less than 5 times. Only a few candidates gained the correct answer, since most added their value for x to 4 x 0.4 m.

(v) Many of those who had succeeded with part (iv) were also successful with part (v), either using $v = -a\omega\sin\omega t$ or $v^2 = \omega^2(a^2 - x^2)$.

4731 Mechanics 4

General Comments

The candidates' work on this paper was generally of a very high standard. Most candidates demonstrated a sound understanding of the mechanical and mathematical principles involved, and presented their solutions clearly and concisely. They were particularly competent at applying calculus to find centres of mass and moments of inertia, and using energy to investigate stability of equilibrium. Topics which were found more challenging included the course for closest approach, small oscillations and, for a rotating body, applying the work-energy principle and finding the force acting at the axis.

Comments on Individual Questions

1) In part (i) most candidates found the moments of inertia and applied the conservation of angular momentum correctly. There was some confusion about using the side length or the semi-side length in the standard formulae, and some candidates used kinetic energy instead of angular momentum.
In part (ii) the constant angular deceleration problem was solved correctly by almost every candidate. Many candidates used a two stage strategy, finding the deceleration first, instead of applying the appropriate equation to find the time in a single step.

2) The method for finding the centre of mass of a solid of revolution was very well understood, and most candidates carried it out correctly. Some candidates used the formulae for finding the centre of mass of a lamina.

3) In part (i) almost all candidates found the course for interception correctly.
For the course of closest approach in part (ii), a common error was to place the right-angle in the wrong vertex of the velocity triangle. Most candidates found the minimum value of V correctly in part (iii).

4) About three-quarters of the candidates found the moment of inertia of the lamina correctly.

5) Parts (i) and (ii) were answered well; a fairly common error was to use the weight in the equations instead of the moment of the weight.
In part (iii), candidates needed to consider the work done against the couple and the changes in potential and kinetic energy. About a half obtained the correct answer; there were many sign errors in the work-energy equation, and some omitted the work done against the couple.
Another fairly common error was to assume that the maximum angular speed occurs when the rod is vertical instead of in the position given by part (ii).
Most candidates obtained a correct equation in part (iv) and then applied the sign-change method successfully.

6) Establishing the position and stability of equilibrium in part (i) was very well done.
In part (ii) the method of differentiating the energy equation to obtain the given equation of motion was quite well understood, and about half the candidates earned full marks for this part.
Many had an incorrect expression for the kinetic energy, and some tried to answer the question without considering kinetic energy at all.

7) Parts (i) and (ii) were answered very well.
In part (iii) about half the candidates correctly found both components of the force acting at the axis. There were often sign errors in the equations of motion, and many used a radius of a instead of $\frac{1}{2}a$ when calculating the accelerations.
Most candidates understood how to use their expressions from part (iii).to answer part (iv).

Overview - Statistics

Much good numerical work was seen on all the Statistics units, although questions that require verbal answers continue to reveal misunderstandings. There seems to be much "teaching to the test", but examination questions are designed to reward candidates who can understand the ideas. Some candidates often answer the previous year's question (which they have no doubt recently practiced) rather than the one in front of them.

A significant number of candidates wrote their answers in the wrong spaces in the Answer Book. Candidates are reminded not to *rub out* any part of their diagrams. Crossing-out should be used, and NOT erasing.

Conclusions to hypothesis tests. Most Centres have taken note of the need to give conclusions in a form that is not too assertive. (Thus the flat statement "the mean pH of the soil is not 6.1" loses a mark.) However, if a null hypothesis is not rejected, it is incorrect to say that "there is evidence that the null hypothesis is true". (A correct statement would be "There is insufficient evidence that the mean pH of the soil is not 6.1".)

4732 Probability and Statistics 1

General Comments

Candidates generally found this paper a little more difficult than has usually been the case. However, some showed a good understanding of a high proportion of the mathematics involved. Questions which proved difficult included 6, 7(ii)(b), 8(ii)(b) and 9(ii)(b). Even question 1 seemed to cause candidates more problems than usual. Only question 9(ii)(b) used any significant pure mathematics, but few candidates were able to reach the point at which the relevant formula came into play. There were several questions that required an interpretation to be given in words, and these were often answered poorly.

A significant number of candidates lost marks by premature rounding (eg in question 1) or by giving their answer to fewer than three significant figures without having previously given a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or the intermediate answer correct to several more significant figures.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 1, 5(ii) and 8(i) (for binomial tables). In question 1(i) a few candidates quoted their own (usually incorrect) formulae for r, rather than using one from MF1. Some thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Others used the less convenient versions, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}}$ and $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1. A few used these formula correctly but clearly spent far more time than was necessary wading through a great deal of tedious arithmetic. However, most of those who used these formula completely misunderstood them, interpreting them as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{\sqrt{\{(\Sigma x-\bar{x})^2\}\{(\Sigma y-\bar{y})^2\}}}$ and $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$.

A few candidates used the convenient versions for calculating r but then, when calculating b, ignored their previous values for Sxy and Sxx and started again with the less convenient version, usually making errors. In question 5(ii), Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes miscopied as $\frac{6\times\Sigma d^2}{4(4^2-1)}$ or $\frac{1-6\times\Sigma d^2}{4(4^2-1)}$. Additionally, a few candidates found $(\Sigma d)^2$, or found Σd^2 correctly, but squared this value before substituting.

In question 8(i), some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Responses to this question also gave evidence that many students (understandably!) prefer to use the binomial formula rather than the tables. However, most of these candidates made mistakes and the very few who launched out on the full, correct method became lost in a deluge of numbers. Centres should be aware that questions are sometimes asked in which the use of the formula is laborious, whereas the use of the tables is simple.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) (i) This question was answered incorrectly by more candidates than is usual. Some miscalculated, eg, Σxy. Errors such as those mentioned above were common.

(ii) Some candidates chose the wrong totals from part (i) to substitute into the formula for b. Many made a sign error while calculating a. Some failed to give the regression equation (as required by the question), going straight from their values of a and b to calculating the estimate. Many substituted 280 000 instead of 280. Others failed to multiply their estimated value of y by 1000.

(iii) Many candidates did not appreciate the point here, giving answers such as "More tourists would mean more sales", "A lower value of r means less money spent by tourists.", "Since –0.8 is not perfect negative correlation, this statement is untrue.", "r is close to –1 so sales would have increased." or "r = –0.8 just means that the points are close to a straight line."

2) A large number of candidates appeared to have little familiarity with the method of coding. Some candidates assumed that the mean was 1.5. Others gave answers such as 1.5 + 1.4 = 2.9 or $\frac{2.9}{50}$. Some candidates found $\frac{1.4}{50}$ = 0.028 but failed to add 1.5. A few found the variance correctly but added 1.5 either before or after finding the square root.

A large number of candidates missed the point and attempted to find Σx and Σx^2. These candidates usually succeeded in finding the mean from $\frac{76.4}{50}$ but for the standard deviation, most made errors such as $\Sigma x^2 = 0.05 + 50\times1.5^2$.

3) (i) Surprisingly few candidates used the "best" method for the lower quartile (ie the median of all the values below the median, leading to the answer 23). The other acceptable method was to find the $\frac{n+1}{4}$th value (leading to the answer 22.5, although some who used this method gave the incorrect answer of 22), but many candidates used an incorrect method, finding the $\frac{n}{2}$ th value (also leading to the incorrect answer 22).

(ii) Some candidates gave the correct answers of 0 and 0, although many gave 30 and 30. A few gave answers such as 31 and 32.

(iii) Many candidates used one of the two acceptable methods (see (i) above) giving answers of 38, 39 or 40, 40.75. Many others used the incorrect method, using $\frac{3}{4}n$ instead of $\frac{3}{4}(n + 1)$, giving answers of 38, 38.5, 39. But many candidates (presumably attempting $\frac{3}{4}(n + 1)$) gave answers such as 42, 42.5.

Some of those who used a correct method were confused as to whether the possible values for the upper quartile were 8 and 9 or 38 and 39. A few gave a range rather than individual values.

(iv) Many correct answers were seen. Some common incorrect answers referred to "spread" or "skew" or "range" or "maximum value" or ease of reading values.

Some candidates referred to frequencies or the mode but without stating that they were referring to frequencies of <u>classes</u> or to the modal <u>class</u>.

(v) As for part (iv), many correct answers were seen. But again, some common incorrect answers referred to "spread" or "skew" or "range" or "maximum value" or ease of reading values. A few stated that the mean could be found.

4) (i) Almost all candidates answered this question correctly.

(ii) Most candidates also answered this question correctly. Occasionally $\frac{5}{6}\times\frac{4}{5}$ was seen. Some candidates correctly found P(RB) and P(BR) but multiplied these two.

(iii) Most candidates did not understand the significance of "Given that . . . " and found P(RBR or RRR), obtaining an answer of $\frac{2}{3}$. A few used the formula $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$, usually correctly. This formula, although acceptable, is not required for this module. In fact this method is longer than necessary in this case.

5) (i)(a) The answers to this part and the next can be written down without calculation.

Some ignored the instruction "Write down . . . " and used the formula for r_s. But almost all candidates answered this question correctly, most without working.

(i)(b) Again some candidates used the formula for r_s. But almost all candidates answered this question correctly, most without working. However, a few gave the answer 0.5, some without working and some with incorrect working.

(ii) Most candidates used the formula and obtained the correct answer. Some made errors such as those mentioned above.

(iii) Some candidates ignored the instruction to use "everyday language" and commented only on "correlation". Most candidates were able to give a reasonable interpretation for $r = 1$. Some just stated that "They agreed" or "They strongly agreed" which were insufficient. For $r = -1$, many used the word "opposite" or "reversed" thus scoring the mark. The third case, with $r = 0$, was found more difficult. Some gave inadequate answers such as "They disagreed", "They had different views", "They had not much in common", "Half way between (a) and (b)" and "completely different orders".

6) Most candidates found P(score = 1) = 0.18, but many then gave muddled methods. Some found the expected loss on scores of 5 and 6 (£106) but subtracted this from £300 instead of from the expected gain on 1, 2, 3 and 4 (£216). Candidates who explained their method clearly were less likely to give a muddled method such as this. Many candidates showed that they could work out E(X) for a discrete probability distribution, but did not know how to apply this in a given context. A disappointing number found E(X) for the scores on the die (3.3) instead of for the gain or loss.

7) (i)(a) Some candidates either just arranged 5 letters (5!) or chose 5 out of 7 without arranging them (${}^7C_5$).

(i)(b) A good number of candidates answered this part correctly, some by the elegant method of $\frac{2}{7}\times$ (i)(a). Many candidates gave methods involving choosing either 7 or 5 letters, rather than 6, which is the correct approach. Other incorrect methods included $^6C_4\times2$, $6!\times2$, $^5C_3\times2$, $^5P_3\times2$, $4!\times2$, $4!+1$.

(ii)(a) This part was answered well although a few candidates found $^7P_5$ instead of $^7C_5$.

(ii)(b) This part was found difficult by most candidates. Some correctly found $^5C_3$ but divided by an incorrect denominator such as $^7C_5$. Others correctly found $\frac{5}{7}\times\frac{4}{6}$ but went no further. A few used the formula $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$ (as in question 4(iii)) either with combinations or fractions. These candidates generally succeeded, although this is a more complicated method than is necessary here. Some incorrect methods seen were $\frac{1}{7}\times\frac{1}{6}$, $\frac{1}{7}\times\frac{1}{5}$ and $\frac{6}{7}-\frac{1}{7}$. A few candidates found a binomial probability.

8) (i) Many candidates answered this part correctly. Some common incorrect answers using the tables were these: 1 – 0.2855 (ie $P(X > 16)$ and 0.2855, (ie $P(X \le 16)$. A few attempted the long method using the formula but generally made arithmetical errors or omitted a term. However, candidates who used the formula often just found $P(X = 16)$ or $1 - P(X = 15)$.

(ii)(a) Many candidates answered this part correctly. Surprisingly, many candidates made elementary errors in finding the binomial probability such as omitting power(s) or the coefficient. 0.3×0.7 was not uncommon.

(ii)(b) This part was found difficult by many. Many candidates used only 4, 4, 2 or 4, 3, 3 but not both. Some correctly found, eg, $(P(X = 4))^2 \times P(X = 2)$ but omitted to multiply by 3, or multiplied by 3!. A few found, eg, $2\times P(X = 4) + P(X = 2)$.

Many missed the fact that only three values of Y were required or that the maximum value of Y is 4, so combinations such as 2, 2, 2, 2, 2 and 5, 3, 2 were seen. Some candidates misread the question and used B(30, 0.6) from part (i), which led them into reams of turgid calculations, most of which yielded zero to 5 or 6 decimal places.

9) In this question some candidates used decimal powers based on times, such as $0.9^{00.05}$ or $0.1^{599.95}$ (from 0600 – 0005). Others used numbers of minutes such as 55 or 29 as powers. A few candidates used the binomial distribution with $n = 24$, for example.

(i)(a) Most candidates stated or implied “geometric distribution” Some candidates found $0.9^4\times0.1$. Other found $0.1^5 \times 0.9$.

(i)(b) Some common errors were $1 - 0.9^6$ or $1 - 0.9^4$. Candidates who used the “long” method often included a bogus extra term such as $0.9^{-1}\times0.1$ (ie supposed $P(X = 0)$) or $0.9^5 \times 0.1$.

(ii)(a) Many candidates found 0.05 and $0.95^2\times0.05$ correctly but failed to add them, presumably misunderstanding “the probability that . . . either 0030 or 0130” to mean two separate probabilities. Others multiplied instead of adding these two terms.

(ii)(b) Many candidates substituted figures into the formula for the sum of a GP, but without writing down any series to be summed. Others had $r = 0.05$ or 0.95.

4733 Probability and Statistics 2

General Comments

There were many very good candidates for this paper, but also a somewhat larger number than usual who could not answer routine questions well or were under-prepared. Centres are advised to act on advice given in this report and the reports from previous series.

There has been an increase in the number of candidates using calculators that work out, for instance, Poisson probabilities exactly for any parameter. Candidates who get the right answer with no working will generally be given full credit but those who get a wrong answer from a calculator without showing any working, even if it may have come from a standard mistake or is nearly right, are likely to lose several marks. Full working is expected, and also correct notation (so that calculator syntax does not get method marks).

Major conceptual misunderstandings of the meaning of probability density functions, and of the Central Limit Theorem, continue to be widespread.

Several candidates introduced continuity corrections when they were not needed. Continuity corrections should be used only when the original distribution is discrete.

Comments on Individual Questions

1) Many candidates were familiar with the requirements of the specification; "the use of random numbers" is required, so pulling numbers (or even actual CDs!) out of a hat is not a method that gains credit. "Select CDs using random numbers" is acceptable; "select numbers randomly" is not. And the members of the population to be sampled should first be numbered sequentially (not randomly).

2) (i) Questions involving finding the parameters of a normal distribution are generally standard, and many candidates were well prepared for this. Some nevertheless failed to find a z-value. There were a few problems with signs, and rather more with square roots; it was necessary to use a variance involving 40 to score more than 1 mark on this question. Some made algebraic mistakes, but many right answers were seen.

2) (ii) The Central Limit Theorem remains poorly understood. Some think that it gives information about the variance, which it does not. The correct answers were that it is needed because you do not know the distribution of V, and that its use is justified because the sample is large.

3) This is an entirely standard question on testing the parameter of a binomial distribution, and as usual it was poorly done. Some candidates attempted to use a normal approximation, which is not valid. Others attempted to find $\mathrm{P}(\le 7)$ or $\mathrm{P}(= 7)$, either of which lost all the remaining marks. Those who attempted to find the critical region often got it wrong, saying for example that $\mathrm{P}(\le 7) > 0.95$ so that the critical region is ≥ 7.

4) (i) Many correctly said that crystals have to occur independently of one another. The marking of this type of question requires candidates to do more than quote learnt phrases ("they must occur randomly, singly, independently and at constant average rate" scores 0); the context must always be mentioned (here "crystals"), and this is a context in which "singly" is meaningless. As emphasised in previous reports, "singly" is at best confusing and at worst wrong; it should not be mentioned.

4) (ii) Almost always right.

4) (iii) Most used Po(2.368) but some approximated to Po(2.4) so that they could use tables. In this question they were expected to use the formula. Those who used a calculator without showing any working were taking a high risk.
A common error was failure to subtract from 1.

4) (iv) This was generally well done, with a perhaps higher proportion of correct continuity corrections than usual.

5 This is a multi-step problem involving the calculation of unbiased estimates before carrying out a hypothesis test for the mean of a normal distribution. Many candidates made the usual mistakes: not using the factor $n/(n–1)$ when estimating the variance, omitting the factor of $\sqrt{80}$, or not stating the hypotheses correctly. In particular, too many candidates made the serious error of treating the population mean as 6.2 instead of 6.1. Those who wrote, for example, $H_0 : \mu = 6.2$ or N(6.2, …) were likely to lose a lot of marks.
As usual, the final conclusion needs to be contextualised ("the mean pH of the soil is 6.1") and not to be too assertive (the following wording is recommended: "there is insufficient evidence that … is not 6.1"). It is wrong to say "there is significant evidence that the mean pH of the soil is 6.1"

6) (i) Many did this well, though some failed to state both necessary conditions (if inequalities are used, they must be $np > 5$ and $nq > 5$, and not npq). Apart from the usual errors with the continuity correction and the square root of npq, several candidates mistakenly divided the variance by an extra 32.

6) (ii) Most knew what to do, though a few ignored the instruction in the question ("use a suitable approximation") and found the probability from an exact binomial distribution, losing most of the marks. A surprisingly common error was 1% = 0.1, which meant that a Poisson approximation was not valid.

7) (i) This was poorly answered. Many drew both sides of a parabola (many candidates ignore the range in which the PDF is zero, which remains baffling). Some failed to draw the parabola through the origin.
The interpretation was again very poorly answered, although this type of question has been asked very often. Many candidates have little idea of what a PDF represents; they do not realise that x and X are essentially the same and seem to think X is an "event" which "occurs", or not, depending on the value of x. Answers should focus on the word "value", for instance "the value of X is more likely to be close to a than close to 0".

7) (ii)(a) Many fully correct answers were seen. Those who did not succeed generally failed to use the condition that the total area was 1, and so did not get the second simultaneous equation connecting k and a. Most who got both equations could solve them correctly.

7) (ii)(b) Most knew what to do here; only a few forgot to subtract 4.5^2.

8) (i) Those who remembered to use a factor of $\sqrt{18}$ in the standard deviation often got the right answer, although many failed to give a region as their answer. The choice of symbol is important here; the correct answer is $\overline{X} > 33.1$, and not $C >$ 33.1 or even $X_C > 33.1$ (because C stands for the critical value and not the test statistic).

8) (ii) Almost everyone correctly identified a Type I error.

8) (iii) As this is a 5% significance test, the appropriate distribution is B(20, 0.05). Far too many used B(20, 0.2), presumably because there were 4 rejections out of 20, but this is completely wrong.
The interpretation caused much confusion. As the probability (0.0159) is low, the assumption that $\mu = 30$ is very implausible. However, many said "I think $\mu = 30$ as the chance of its being rejected is so low". Those who thought in terms of a hypothesis test generally thought correctly.

8) (iv) Pleasingly many correct answers were seen. In finding the probability of a Type II error it is always necessary to use the critical region. Many failed to give their final answer to 4 significant figures as requested; it is necessary here because of the possible issue of ill-matching.

4734 Probability and Statistics 3

General Comments

Many candidates scored full, or almost full marks. There were very few weak candidates. Fewer candidates than previously lost marks for making their conclusions to signifience tests over-assertive. The statement of hypotheses has also improved, with far fewer candidates not stating them in terms of the population parameters.
Question 1(ii) was the hardest question. Over half the candidates scored no marks. Question 5(iii) was also found to be difficult. Both involved the normal distribution.

Comments on Individual Questions

1) (i) Most answered this question correctly.

(ii) A common wrong answer was: $npq = 50$, so mean = $np = 50/0.86 = 58.1$. Many others used a variance of 50/100. Several candidates scored the first method mark, but were unable to solve their equation.

2) Some candidates treated this as a 2-sample *t*-test, for which two-thirds of the marks were made available. However a paired *t*-test was expected, the word 'pair(s)' appearing twice. Many lost their only mark by not using the word 'population' in their assumption.

3) Over half the candidates scored full marks, but many pooled samples when they should not have done. This led to a test statistic of 1.95998 and a critical value of 1.96. Thus different conclusions were obtained depending on the number of significant figures used in the test statistic. Some pointed out that because the values were so close, a larger sample should be taken. Two-thirds of the marks were available to these candidates, and most obtained them.

4) (i) Most scored full marks, but some used a *z*-value instead of *t*.

(ii) Over half the candidates scored full marks. Almost all made some progress.

5) (i) Most scored full marks.

(ii) Almost all stated 'normal' but some did not give the parameters. Most gave adequate justifications.

(iii) As usual, modulus caused difficulties. Some ignored modulus and found $P(Y - X) \geq 3$. The best candidates used a continuity correction and were successful. Those who did not were allowed three-quarters of the marks.

6) (i) Almost all the candidates answered this question correctly.

(ii) The main error was to omit '+F(1)' when finding $F(t)$ for $1 < t \leq 4$. Some did not state 0 for $t < 0$ and/or 1 for $t > 4$.

(iii) Some did not score the first two marks for the justification of the use of $F(y^2)$, but most did. They have learned the relevant procedure and can distinguish between Y and y. Those who started $G(Y) = P(Y \leq y)$ were almost always successful.

7) (i) Almost all the candidates scored most, or all, of the marks.

(ii) Many could not construct the correct contingency table. They were allowed limited credit. Those who did usually scored high marks.

(iii) Most correctly stated *A*, usually with an acceptable reason.

4735 Probability and Statistics 4

General Comments

There were 47 candidates, similar to recent years. There were many excellent scripts, but no-one obtained full marks.
The question on estimators (Q7) was answered better than in previous years.

Comments on Individual Questions

1) (i) Candidates using pgfs were always successful. Those answering in words often omitted either 'independent' or 'same probability'.

(ii) Many scored only one mark, for ${}^{15}C_5 \times p^5 \times q^{10}$.

2) (i) Most knew what to do, and the integration by parts was usually well done.

(ii) Just under half the candidates scored this mark, usually saying that the denominator became $(2 - -t)^2$,or that the power of e became $-x(2 + t)$.

(iii) Most knew that multiplication of the mgfs was needed, but addition, subtraction and division were all seen.

3) (i) Three out of seven was the lowest mark in this part. Most scored six, losing the mark for the necessary assumption.

(ii) Just under a quarter of candidates scored this mark.

(iii) Most candidates answered this part correctly, often saying that the t-test was more powerful.

4) (i) Two-thirds of candidates scored full marks on this part.

(ii) Almost all the candidates answered this part correctly.

(iii) Almost all candidates scored at least two out of three. Those who scored two did not simplify their answer to $1/p$.

(iv) Most answered this part correctly.

(v) Almost all knew that the coefficient of t^6 was required. Most were able to find it.

5) (i) Three-quarters of candidates answered this part correctly.

(ii) Eight out of nine was the mode, candidates losing the mark for the assumption.
Some did not calculate $m(m + n + 1) - R_m$. Most obtained the correct CV.

6) (i) Few used the easy method, ie $P(S = 0) = 0.08/0.4 = 0.2$, hence $a = 0.2 \times 0.6 = 0.12$ and similarly for b. Most others set up two equations and solved them correctly. A few did not consider independence, so only three marks, for finding b, were available to them.

(ii) Almost all the candidates answered this part correctly.

(iii) A few failed to subtract the square of the mean from $E(T^2)$ and $E(S^2)$, and some used covariance. Those who found Cov = 0 went on to score full marks.
Most candidates scored full marks, the two main methods being roughly equal.

7) (i) Many candidates seemed unaware that $f(x) \geq 0$.

(ii)-(v) All these parts were almost always answered correctly.

8) (i) Just over half the candidates scored full marks. Of those who did not, most knew one of the necessary conditions for $P(A \cap B)$, but not both..

(ii) Two-thirds of the candidates answered this part correctly.

4736 Decision Mathematics 1

General Comments

Some candidates did not use time wisely, for example copying out the simplex tableau again before carrying out the iterations in question 4 or copying out the table of shortest distances, sometimes more than once, in question 5. There were many instances of students losing marks for simple arithmetic errors.

Candidates need to make sure that they have used an appropriate form for their answers and should be aware that working done on the question paper is not available to the examiners for marking. The answer booklet had appropriate space for each answer and most candidates used the space well. A few candidates used additional sheets, but when they did so they usually indicated this in the appropriate space in their answer booklet.

The presentation of some answers was poor, and in particular the handwriting of some candidates was very difficult to read. Candidates need to read questions carefully and make sure they attempt written answers concisely whilst still including the essential information.

Comments on Individual Questions

1) (i) Generally answered well, although some candidates had arcs missing or drew a straight line through *D* when joining *C* to *E* while others failed to include an arc *CE*.

(ii) Nearly all the candidates could use Dijkstra's algorithm correctly. A few recorded extra temporary labels. Most candidates gained marks on this question with many fully correct responses. Quite a few candidates obtained the right final answers to part (ii), despite earlier mistakes with the graph. Some candidates omitted to state the shortest route.

(iii) Answered well, although many candidates failed to convert 1200 seconds into minutes. A minority of candidates ignored the fact that the algorithm had quadratic order, despite it being clearly stated in the question.

2) (i) The vast majority of the candidates were able to draw appropriate graphs that satisfied the given requirements.

(ii) Most candidates were able to represent the information graphically but several could not see how to use their graph to construct the sessions. Many candidates mentioned that the graph was semi-Eulerian, which it was, but this did not answer the question. Some candidates just repeated the information given in the question and several gave no response. This was a good practical example of using graphs, and sorting the classes into two sessions was not difficult if the graph was used. In part (ii)(c), few candidates gave a full explanation, most recognised that the classes clashed in the current allocation, but for a full explanation they also needed to explain why there would always be a clash. Candidates tended to focus on whether the vertices were odd or even, or whether the graph was semi-Eulerian or not, rather than the practical issue that M1, D1 and S2 formed a cycle of three classes each of which was joined to the other two and so any allocation into two sessions would have at least two of these clashing.

3) (i) Many candidates were able to calculate the equations of the boundary lines, but often they made errors with the inequalities. Weaker candidates were not able to find the equations of the boundary lines.

(ii) Candidates who successfully found the correct boundaries on (i) often gained full marks on this question, but some candidates only managed to identify (3,1), or read off the coordinates from the graph in decimal form.

(iii) The candidates who persevered with this question were usually able to calculate the value of *P* for their vertices, and several of them achieved the optimum of 12 at (4/3, 8/3), to within a suitable tolerance.

(iv) Several candidates chose (3, 1) because it was the only integer-valued vertex of the feasible region. Other candidates realised that they needed to find an integer-valued point within the feasible region and tried out points that were 'near' the optimal vertex. Very few recognised that there were only four integer-valued feasible solutions, although there were some interesting proposals as to why (2, 2) would be the optimal integer-valued point, suggesting some engagement with the ideas.

4) (i) The first part of the question was answered quite well by good candidates, but some candidates failed to change the ≤ signs to = and many candidates failed to state that s, t, u ≤ 0.

(ii) Generally done very well, although some had the wrong signs on the objective row. Pleasingly most noticed that there was no coefficient of y in the second constraint.

(iii) The explanations were not always concise enough, candidates often stated that the *z* column was chosen because it had the 'most negative' value in the objective row and did recognise that the *z* column must be chosen because it had the only negative value. Some candidates did not refer to the objective row when explaining why the *z* column had to be chosen, and some interchanged rows and columns. Most candidates calculated the ratios of RHS ÷ entry in *z* column although some did not then say that the pivot corresponds to the row with the smallest ratio.

(iv) The basic processes required for this question seems to be understood by most candidates. This part was answered well by candidates who had a correct initial tableau, although numerical errors were evident, as well as the loss of the correct structure in some cases. A valid simplex tableau needed four basis columns and four non-basis columns, non-negative values on the RHS and the value of *P* should increase, or at worst stay constant, from one iteration to the next. Most candidates showed their calculations for the rows although sometimes the pivot row operations were missed out and sometimes the operations were difficult to read.

(v) Even where the first iteration was error-free there were often arithmetic errors or loss of structure in the second iteration and it was not clear that some of the candidates knew to interpret the tableau when they had gone wrong or needed to interpret their results. Some candidates were unable to correctly state the values of *P*, *x*, *y* and *z* from their final tableau, and some gave *x*, *y* and *z* but omitted *P*.

(vi) The main problem seemed to be in giving correct explanations. Although they could read off the values of *s*, *t* and *u*, very few candidates were able to explain what they meant in terms of the original constraints. Some candidates often failed to read off their values for s, t and u correctly. Some had them all as 0, and some had negative values.

5) (i) Candidates needed to select an appropriate method, which should have been the route inspection algorithm, and then apply that method showing all their working. Some candidates used nearest neighbour or tried to construct a minimum spanning tree. Of those who attempted route inspection, most were able to identify the odd nodes and put them together as three pairings, but arithmetic errors meant that often at least one of the totals was wrong. Candidates did not seem to have appreciated that the table gave them the shortest distances between pairs of nodes. A number of candidates claimed that 1200 + 220 = 1440.

(ii) Not answered particularly well, but good candidates often gained full marks.

(iii) Generally answered well. Some candidates dropped the final mark as they stated that the shortest route was 'less than' this distance, when all we know at the moment is that it is 'less than or equal to' this distance. Some candidates did not use the nearest neighbour method correctly and either slipped up near the start or failed to close the route by returning to *M*. Quite a few candidates had a correct route but used incorrect weights when adding up the total. *WP* was often recorded as 110, from the table, rather than 170, from the network. The 110 route involves travelling via *S*. This issue does not arise when we use a network of shortest distances based on a complete graph.

(iv) Generally answered well. There were two possible trees and either was accepted, but not a mixture of the two. Several candidates added up the weight of the tree incorrectly.

(v) Most candidates who attempted this part realised that they needed to add the two shortest arcs joined to *W* to give a lower bound. Many added 30+170 to their tree, although the question had said to use the table of shortest distances, albeit that some of these are indirect distances.

6) (i) Several candidates made errors in their calculations but were still able to achieve reasonable marks on this part for passing through the algorithm with one cycle round the loop. Some candidates went beyond the stopping point by misreading the box 'Is *W*-*X* between -0.05 and 0.05?' Efficient use of a calculator should have enabled candidates to achieve sufficiently accurate values to gain the marks.

(ii) Those candidates who attempted this question usually answered it quite well. There were some errors in the initial calculations, usually on the value of *Y*. Having achieved $Z = 0$, some candidates claimed that the algorithm had stalled instead of using the other part of the loop to take them back to the position they were in at the start of part (i).

(iii) Most candidates who progressed to this part were able to gain marks for their working. Some candidates achieved full marks on this part and several of those who attempted it achieved all but the final accuracy mark.

(iv) Those candidates who attempted this part often gave a response that was awarded a mark. Some candidates seemed concerned about negative numbers and thought that the algorithm would not work for negative inputs. This was clearly not true since the candidates already knew that $X = -0.2$ achieved convergence in one pass.

4737 Decision Mathematics 2

General Comments

Most candidates were able to complete the paper in the time allowed. Candidates need to read the questions carefully as sometimes marks were lost through answering a different question to the one that had been asked.

Comments on Individual Questions

1) (i) Almost all candidates were able to draw the bipartite graph. Some added working to their graph in constructing their answers to later parts, but this was ignored here.

(ii) Most candidates were able to write down the alternating path 5-*D*-4-*E*. Some candidates gave a longer alternating path, contrary to what had been asked, or went on to find the complete matching in this part. Candidates who drew a diagram for the incomplete matching needed to also write it down, as asked in the question.

(iii) Some candidates appeared to have started again in this part, but most were able to augment the incomplete solution to obtain the complete matching.

2) (i) Almost all candidates knew how to modify the table to a form that was appropriate for the Hungarian algorithm, although confusion between 'subtract from 10' and 'subtract 10' or 'subtract by 10' caused some candidates to drop this mark even though they did the correct thing in part (ii). Changing the sign of every entry meant that the table had negative entries, which is not appropriate for the Hungarian algorithm. Subtracting each entry from the maximum in its row does achieve the equivalent of modifying and then reducing rows, but this is a two-stage modification.

(ii) Because this was a 'show that', candidates did need to give some indication of how the tables had been formed.

(iii) Most candidates could carry out the augmentation, some insisted on doing all their working on one table and crossing out entries, which sometimes made it difficult to know if their answers were correct or not. Several candidates then continued with the 'usual' request by finding a matching and writing down its total weight, others identified Hilary as being the most likely to succeed (which was true, but not what the question had asked). A tiny number of candidates thought that Ieuan would be the least likely because he was last, or because he 'may not get a turn', however the question had said that each cadet will have one attempt.

3) Several very good answers. The setting up of the table was usually done competently, although some candidates insisted on using letters instead of stage and state variables, and some candidates labelled the actions by just using increasing order, rather than having the action label correspond to the state label of the vertex for the next stage (the stage that the arc had come from, when working backwards). Some candidates appeared to have answered the question on the diagram and then attempted, with varying degrees of success, to transfer their answer to the table. Only a few candidates worked forwards instead of backwards, although some candidates ended up with a mixture of forwards and

backwards methods that included stage 4.Candidates whose tables had the right structure, even if it was mislabelled, were usually able to carry out the dynamic programming well.

Most candidates chose to work with journey times and find the minimum journey time, a few tried to maximise or carry out a maximin or a minimax. Some candidates worked with arrival times and found the latest (maximum) arrival time for each vertex. Either approach was acceptable. The latest take-off time should have been 2am and the places passed through should have been *A* at 7am, *E* at 10am and *H* at 1pm.

4) (i) Generally done well. Some candidates did not show their working or they found the row min and column max values but did not locate the maximum of the row minima and the minimum of the column maxima, and some confused the play-safe strategies with the play-safe values.

(ii) Usually done well. A few candidates deleted the wrong row or column and one or two made arithmetic slips.

(iii) The majority of candidates achieved the correct values, some just added the scores instead of averaging them.

(iv) Many good answers, although some candidates confused the rows and columns and some tried to set the expected values equal to zero rather than identifying where the lower boundary achieved its maximum value – the value that can be expected no matter what strategy the other team use.

(v) A few candidates interchanged parts (iv) and (v) and some repeated their answer from part (iv), although using *q* rather than *p*. Some equated the expected value to the optimum value that was already known from part (iv), and this was usually successful.

5) (i) Usually done well, apart from candidates who only answered one of the two requests.

(ii) Many correct answers. Some candidates failed to deal with the lower capacities for the arc *FI* that flowed across the cut from sink to source.

(iii) Usually done well, by referring to the maximum that can flow into *JG* from arc *KJ*. Tracing the flow of 3 backwards meant that the flow in *HK* had to be 0 and the flow in *IL* had to be 3.

(iv) Some rather lengthy and confused explanations, often attempting to argue from the maximum for *EH*. The most efficient answers referred to the flows in arcs *IF* and *IL* to deduce that the flow out of *I* must be 5. Considering the flows leaving *H* meant that the flow in *EH* had to be 7, and a similar argument at *F* meant that the flow in *FC*, and hence in *CB*, had to be 3.

(v) Using the flows found in parts (iii) and (iv), together with the restriction on arc *ED* meant that there was only one feasible flow of 10 litres per second. Most candidates were able to find this flow. Some candidates tried to show the flow using excess capacities and potential backflows, this was rarely successful. A few went straight to the maximum flow of 11.

(vi) The labelling procedure was not well done by many candidates. In this particular case candidates had to take account of the minimum arc capacities and this confused most of them.

(vii) Despite having problems with the labelling procedure, most candidates were able to find the flow augmenting path and the maximum flow. The intention was that candidates could then refer to the cut from part (ii) to show that the flow was maximal, some found a new cut of 11, such as the cut *X* = {*B*, *C, E*, *F*}, *Y* = {*A*, *D*, *G*, *H*, *I*, *J*, *K*, *L*}. Inevitably, some candidates tried to find the value of their cut using the flow diagram, so that anything they chose would give 11.

6) (i) The network was usually drawn correctly, apart from missing arrows and the use of extra, unnecessary, dummies.

(ii) Candidates could usually carry out the forward and backward passes on this fairly simple network. Some candidates were unsure how to deal with the dummy activity, particularly on the backward pass, where the dummy came into play. The early event time at the end of the dummy was 8 (being the larger of 5+3 and 8+0) and the late event time at the start of the dummy was also 8 (being the smaller of 12-3 and 8-0). Some candidates completed the passes but did not write down the minimum completion time (12 hours) and the critical activities. A number of candidates thought that activity *G* was also critical, the labels at the vertices at each end of *G* had equal values, but the difference exceeded the activity duration.

(iii) Several candidates realised that starting activity *D* at time 5 required Sally and Tariq sharing *A* and *B* and Tariq doing *C* on his own. Some candidates completely ignored the issue of who could do which activity and some thought that, because the early start time of *D* was 5, there was no problem. Some candidates gave descriptions that involved all the activities, sometimes going as far as giving the answer to part (vi) in answering this part.

(iv) Many candidates seemed to have convinced themselves by writing out lists or durations without actually explaining why Sally would be busy for 18 hours. A few candidates misunderstood what happened with shared activities and tried to manipulate the figures to get 18. Tariq was busy on his activities for 15 hours.

(v) Many candidates assumed that this was something to do with the time needed for the critical activities. Apart from activities *A* and *B*, all of Sally's activities depended on the completion of *A*, *B* and *C*. This meant that Sally would have to wait for Tariq to complete activity *C* so she could not finish until 20 hours at the earliest.

(vi) There were many very good answers, with working set out neatly. Some candidates missed out on one of the precedences or gave solutions that took too long. Some candidates repeated the solution from part (iv), despite having already said that this took Sally 20 hours. Other candidates gave confused descriptions or drew diagrams without saying which activities were being done by which person, ignored the restrictions on who could do which activity or split activities so that they were partially completed and then finished off later.

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