RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4735:** Probability and Statistics 4

Mark Scheme for June 2011

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PO Box 5050
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1 (i)	$\sum_{x=0}^{n}\binom{n}{x}p^x q^{n-x}t^x$ $=\sum_{x=0}^{n}\binom{n}{x}(pt)^x q^{n-x}$	M1 A1 **2**	From $E(t^x)$ M1A0 $\sum$ without limits $G_X(t)$=q+pt M1 then argument A0
(ii)	$G_T(t) = (q+pt)^n(q+pt)^{2n}$ $= (q+pt)^{3n}$ So $T \sim B(3n, p)$	M1 A1 M1 A1 **4** **[6]**	Multiplying pgfs For B For parameters
2 (i)	H_0: $m_d = 0$, H_1: $m_d > 0$, (where d = high – low) D: -4 3 6 1 12 7 14 16 11 -9 10 Rank -3 2 4 1 9 5 10 11 8 -6 7 $P = 57$, $Q = 9$ $T = 9$ CV = 13 9 < CV so reject H_0 There is sufficient evidence at the 5% significance level to support the botanist's belief	B1 M1 A1 B1 B1 M1 A1 ft **7**	Or H_o:m_H=m_L , etc. Medians Ranking top down, -9,-10,8, ..M1A0 T=15 B0 [SR last 3 marks: z=-2;09 B1 <-1.96 etc M1A1] Or equivalent ft T
(ii)	The rank sum test is for independent samples, the H and L values are correlated	B1 **1** **[8]**	Accept data paired
3 (i)	$P(A\|B') = P(A\cap B')/P(B')$ => $P(A\cap B')$ = 1/8 AEF Use $P(A\cap B) = P(A) - P(A\cap B')$ To give $P(A\cap B)$ = 5/8 AEF	M1 A1 M1 A1 **4**	 May be implied Or equivalent
(ii)	$P(A\cap B\cap C) = {}^5/_8\times{}^1/_4 = {}^5/_{32}$ AEF	B1 √ **1**	Ft 5/8
(iii)	$P(B\cap C) = 3\lambda/4$ and $P(C\cap A) = 3\lambda/4$ Use formula for $P(A\cup B\cup C)$ And $P(A\cup B\cup C)=1$ Sub into formula for $P(A\cup B\cup C)$ and solve for λ giving $\lambda = 3/16$ AEF	M1 M1 B1 M1 A1 **5** **[10]**	For use of both conditional probs Allow one sign error
4 (i)	$M'(t) = 3({}^1/_4 + {}^3/_4 e^t)^2\times{}^3/_4\, e^t$ $E(X) = M'(0) = {}^9/_4$	M1 A1 A1 **3**	Allow one error
(ii)	mgf $({}^1/_{64} + {}^9/_{64}e^t +)$ ${}^{27}/_{64}e^{2t}$ $(+\ {}^{27}/_{64}e^{3t})$ $P(X = 2)$ = coefficient of $e^{2t} = {}^{27}/_{64}$	M1 A1 A1 **3**	Or PGF=$({}^1/_4 + {}^3/_4 z)^3$ expand find coefficient of z^2 27/64
(iii)	Sum of 3 obs of Y with mgf ¼ + ¾e^t has mgf of X y : 0 1 p: ¼ ¾ $Var(Y) = ¾ - (¾)^2 = {}^3/_{16}$	M1*dep A1 *M1A1 **4** **[10]**	 OR $B(1,{}^3/_4)$ Using $E(Y^2) - (E(Y))^2$ OR $1\times{}^3/_4\times{}^1/_4$ M0 if integration used

5(i)	Does not require a known probability distribution	B1 **1**	Or equivalent
(ii)	H_0: $m_A = m_B$, H_1: $m_A \neq m_B$	B1	Medians
	Ranks: A 1 2 3 5 6 10		
	B 4 7 8 9 11 12	M1	
	R_A = 27, 78 – 27 = 51, so W = 27	M1	
	OR: R_B = 51, 78 – 51 = 27	A1	
	5% CV = 26	B1	Use N(39,39) with cc B1
	27 > CV so do not reject H_0	M1	P(W≤27.5), Z=-1.84 or equivalent M1
	there is insufficient evidence at the 5% SL to indicate a difference in breaking strengths	A1 **7**	Not in CR etc A1
(iii)	B would have an extra rank 13	M1	
	W still 27 but CV now 27	B1	P(W≤27.5)=-2.07 M1A1
	H_0 is now rejected	A1 **3** **[11]**	In CR Ho rejected A1
6(i)	L=0, C = 1, choose 1C from 14 and 1 from 6 Others	M1	Or $^{14}/_{36}\times^{6}/_{35}\times 2$
	$14\times6\ /\ ^{36}C_2$ = 2/15 AG	A1	
	L = 1, C = 1, choose 1 from 16, 1 from 14	M1	Or $^{14}/_{36}\times^{16}/_{35}\times 2$
	$16\times14\ /\ ^{36}C_2$ = 16/45 AG	A1 **4**	
(ii)	Marginal C probs: 11/30 22/45 13/90	B1	
	E(C) = 22/45 + 26/90 = 35/45 = 7/9	M1	AEF
		A1 **3**	
(iii)	EITHER: 2×1/42 + 2/15 + 16/105	M1	Other: 0 1 2 M1
	OR: E(L) = 8/9, E(O) = 2 – 15/9	A1	p: $^{29}/_{42}$ $^{2}/_{7}$ $^{1}/_{42}$ A1
	= 1/3	A1 **3**	E(O)=$^{2}/_{7}$ + $^{2}/_{42}$=$^{1}/_{3}$ A1
(iv)	EITHER: Argument	B2	e.g The more Ls the fewer Cs
	OR: Use idea that for independence P($L \cap C$)= P(L)P(C)	M1A1	OR Use conditional probability
	Conclude that covariance is non-zero	B1 **3** **[13]**	OR Cov(L,C)=-136/405 M1A1 L,C not indep B1
7(i)	E(S) = ½ (E($\bar{U}_4$) + E($\bar{U}_6$))	M1	
	= ½ (μ +μ) = μ, so S is unbiased	A1	With conclusion
	Var(S) = ¼ (σ^2/4 + σ^2/6)	M1	
	= 5σ^2/48	A1 **4**	
(ii)	E(T) = (a +b)μ=μ , a+b=1	M1	
	Var(T) = $a^2\sigma^2/4 + b^2\sigma^2/6$	B1	
	Minimise $y = a^2/4 + b^2/6 = a^2/4 + (1-a)^2/6$	M1	
	EITHER by differentiation OR, completing square, OR from a sketch graph.	M1	
	Giving a = 2/5, b = 3/5	A1	
	Justify minimum value	B1	Allow from completion of square
	Variance = σ^2/10	A1 **7**	
(iii)	T is better since (both are unbiased and) Var(T)<Var(S)	B1 **1**	From calculated variances
(iv)	Sample mean of 10 observations (is also unbiased) with σ^2/10	M1	Or show that T = mean of 10 observations
	They have the same efficiency	A1 **2** **[14]**	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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