GCE

Mathematics

Advanced GCE

Unit **4732:** Probability and Statistics 1

Mark Scheme for January 2011

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1i	38 61	B1 B1 2	Reversed: B1B0	
ii	Paper 2 Higher median or curve is to right	B1 B1dep 2	Indep of reason or similar Higher average or mean or midpoint Paper 2: half ≤ 61, cf paper 1: half ≤ 38 Paper 1: more students scored lower marks (or lower than eg 40)	Ans "Paper 1", ignore reason: B0B0 unless reversed in (i) More scored higher mks Highest & lowest mks are higher For each cf, the corresponding mark is higher in p2. None get 0-10 Some get 100 Eg 25 scored > 69 in p1, cf 65 scored > 69 in p2 NOT Marks are higher NOT marks seem higher NOT everyone gets higher mks NOT Curve steeper Ignore irrelevant or incorrect SC: If reversed in (i): (ii) p1 because median higher B1B1ft

iii	55, 25
	73, 46
	Paper 1 IQR = 30
	Paper 2 IQR = 27
	
	Suggestion correct or p2 less varied
		M1
		
		A1
		A1
		
		B1f
		indep
		
		4
			M1 one pair of quartiles
			
			p2 more consistent or less spread out
			Allow "p2 has smaller range (or smaller variance)" if IQRs found
			
			"It" is less varied: assume p2: B1
				Allow 55±1, 25±1 Not necessarily subtracted
				     73±1, 46±1
				     30±1
				     27±1
				
				p1 more varied or more spread out or less consistent
				Little difference or similarly varied
				
				NOT p2 IQR smaller than p1 unless also says less varied oe
				
				If quartiles found but not IQRs: max M1A0A0B1
				If no quartiles calculated can still score B1
				
				Steeper curve alone M0A0A0B0
				
				If IQRs wrong, with p1 < p2, ft "suggestion wrong": B1f
				
				Ignore irrelevant or incorrect

iv	$37 (\pm 3)$	B2 2	B1 for $163 (\pm 3)$	Not necessarily integer. B1 for 78-80 mks for min grade A on p2 SC: ans 105 – 110: B1 (from p1 10 mks hier instead of lower)
v	37.5 28.2	B1 B1 2	cao or sd the same	NOT eg 37.51 Ignore all working
Total		**12**		
2				SC:Consistent use of incorrect $(1 – 0.2)$ score M-marks only SC:Consistent 0.8 insted of 0.2, no A-marks: max M0M2M2M2 "Consistent" means in every part attempted
2i	$0.8^2 \times 0.2$ $= \frac{16}{125}$ or 0.128	M1 A1 2		
ii	$0.8^2 \times 0.2 + 0.8^3 \times 0.2 + 0.8^4 \times 0.2$ $= \frac{976}{3125}$ or 0.312 (3 sfs)	M2 A1 3	1 term omitted or wrong or extra: M1	Using $P(X \le 5)$ & $P(X \le 2)$; three methods: $1 – 0.8^5 – (1 – 0.8^2)$ or $0.672 – 0.36$: M2 Allow M1 for $1 – 0.8^5 – (1 – 0.8^3)$ or $0.672 – 0.488$ or $1 – 0.8^4 – (1 – 0.8^2)$ or $0.5904 – 0.36$ $0.8^2 – 0.8^5$: M2 Allow M1 for $0.8^3 – 0.8^5$ or $0.8^2 – 0.8^4$ $0.2 + 0.8 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2 + 0.8^4 \times 0.2 – (0.2 + 0.8 \times 0.2)$: M2 One term omitted or wrong or extra: M1 But NB If include $0.8^{-1} \times 0.2$ in both $P(X \le 5)$ & $P(X \le 2)$, get correct ans but M1M0A0 M0 for eg $1 – 0.8^5 – 0.8^2$ or $0.672 – 0.64$
iii	0.8^4 $= \frac{256}{625}$ or 0.4096 or 0.410 (3 sfs)	M2 A1 3	$1 - (0.2 + 0.8 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2)$ 1 term omitted or wrong or extra: M1 $1 – 0.8^4$ or 0.590 M1 or 0.8^3 or 0.512 or 0.8^5 or 0.328: M1 Allow 0.41	$1 – (0.2 + 0.8 \times 0.2 + 0.8^2 \times 0.2 + 0.8^3 \times 0.2)$ M2 0.2×0.8^4 M0 $1 – 0.8^n (n \ne 4)$ M0

iv	$0.2 \times 0.8 \times 0.2$ $\times 2$ $= 0.064$ or $^8/_{125}$	M1 M1 A1 3	or $0.2 \times 0.8^0 \times 0.8 \times 0.2$ or $0.2 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.2$	or 0.032 NOT $n \times 0.2^2 \times 0.8$ except $n = 2$ Fully correct method except allow M0M1 for $(0.2 + 0.8 \times 0.2) \times 2$, must see method Attempt 0,3 and/or 3,0, as well as 2,1and/or 1,2; max M1M0A0 Careful: $0.2 \times 0.8 \times 0.2 + 0.2 \times 0.8^{-1} \times 0.128 = 0.064$ M1M0A0 Careful: $0.8 \times 0.8 \times 0.2 \div 2 = 0.064$: (ie P($X = 3$) ÷ 2) M0M0A0
Total		**11**		
3i	$\dfrac{7351.12 - \frac{86.6 \times 943.8}{12}}{\sqrt{(658.76 - \frac{86.6^2}{12})(83663 - \frac{943.8^2}{12})}}$ or $\dfrac{540.03}{\sqrt{33.80 \times 9433}}$ $= 0.9564\ldots$ or 0.956 or 0.96	M1 M1 A1 3	 Must see at least 2 sfs	1st M1 for correct subst in any correct S formula 2nd M1 for all correct subst'n in any correct r formula 0.96 or correct better, no working: M1M1A1 eg $0.958 \rightarrow 0.96$ with correct working M1M1A0 without working: M0M0A0

ii	Strong (or high or good or close etc) relationship (or corr'n or link) between amount spent on advert & profit	B1 1	Allow Almost complete relationship
			      or Very positive corr'n
			      or Very reliable relationship
			      or Near perfect relationship
			 between spend on advert & profit
			
			 oe, in context
				Must state or imply "strong" or "good" or equiv & in context but NOT Strong *agreement* between etc
				
				 NOT High spend on ads produces high profits
				 NOT The more spent on adverts, the higher the profit
				 NOT Positive corr'n between spend on ads & profits
				 NOT There is a relationship between spend on ads & profit
				 NOT There is a great relationship between etc
				 NOT ans involving "proportion(al)"
				
				 Ignore irrelevant or incorrect
				 If incorrect r (< 0.9) in (i), no ft for ans "weak rel'nship" here; but correct ans here scores B1 even if inconsistent with their r

iii				Allow without context
	Relationship may not continue	B1	Can't extrapolate	Examples:
				Can't predict future; Things can change
			Any indication that pattern may not continue	May be recession ahead; Economic situation may change
				Cost of advertising may increase
				If spend too much on ads, profit may be reduced as a result
			Must state or imply referring to future	Advertising may not be as successful in the future
				Item may go out of fashion
				NOT Spending on adverts may not bring high profits
				NOT Spending more on adverts may not bring higher profits
				(Since these just restate the question)
				NOT More money spent on ads will not affect profit
	Corr'n not imply causation	B1 2	Increase in profit may not be due to increase in spend on advertising.	Both variables may be affected by a third
				Other factors may affect profits
				Advertising not the sole factor affecting profits
			Variables may be increasing separately	
				Two different categories of reason needed, as given above.
				Two reasons which both fall under the same category: only B1
				NOT Because corr'n not equal to 1
iv	$b = \dfrac{7351.12 - \frac{86.6 \times 943.8}{12}}{658.76 - \frac{86.6^2}{12}}$	M1	or $\dfrac{S_{xy}}{Sxx}$	ft values of S_{xy} & S_{xx} if clearly shown in (i)
	$= 15.9788$ or 16.0	A1		
	$y - \frac{943.8}{12} = \text{"}16.0\text{"}(x - \frac{86.6}{12})$	M1	or $a = \frac{943.8}{12} - \text{"}16.0\text{"} \times \frac{86.6}{12}$	
	$y = 16x - 37$ or better	A1 4	$(y = 15.9788x - 36.664)$	Coeffs not nec'y rounded, but would round to 16 & 37
				These marks can be earned in (v) if not contradicted in (iv)
				If x on y line found: M-marks only $(x = 2.71 + 0.0572y)$
v	"16" × 7.4 – "37"	M1		"16" × 7400 – "37": M0A0
	81400 to 81750		81.4 thousand to 81.7 thousand: M1A1	
		A1f 2	but 81.4 to 81.7 alone: M1A0	ft their (iv)
Total		**12**		

4i	0.4×0.7	M1	or 0.6 + prod of 2 probs	1– prod of 2 P's or 0.4×0.3
	$0.6 + 0.4 \times 0.7$	M1	Condone $0.6 \times 0.7 + 0.6 \times 0.3 + 0.4 \times 0.7$ or $0.6 \times 0.6 + 0.6 \times 0.4 + 0.4 \times 0.7$	$1 - 0.4 \times 0.3$
	$= 0.88$	A1 3		
ii	$p + (1 - p) \times p = 0.51$ or $2p - p^2 = 0.51$	M1	or $p^2 + p \times (1 - p) + (1 - p) \times p$	Condone $p + p \times 1 - p$ M1, but $p + qp = 0.51$ M0
	$p^2 - 2p + 0.51 = 0$	A1	Correct QE = 0 Condone omission of "= 0"	or $(1 - p)^2 = 0.49$ M1A1
	$(p - 0.3)(p - 1.7) = 0$ or $p = \frac{2 \pm \sqrt{4 - 4 \times 0.51}}{2}$ oe	M1	Correct method for their 3-term QE	$1 - p = \pm 0.7$ M1 must have $\pm$
	$p = 0.3$	A1 4	Not $p = 0.3$ or 1.7	Correct ans from correct but reduced wking or T & I or verification or no wking: 4 mks
				Ans $p = 0.3$ or 1.7 from correct but reduced wking or T & I or no wking: M1M1M1A0
				Ans $p = 0.3$ following correct wking except other solution incorrect: BOD 4 mks
				(eg $p = \frac{2 \pm \sqrt{4 - 4 \times 0.51}}{2}$ so $p = 0.3$ or -1.3 so $p = 0.3$: 4 mks))
				$p = 0.3$ from wrong wking but correct verification: BOD 4 mks
				$p = 0.3$ from wrong wking alone: M0A0M0A0
Total		**7**		

5			Consistent use of $\frac{1}{3}$ or MR of 30% (eg 0.2): ("Consistent" as in Qu 2) (i) B1B0B1B1 (iia) B0 (iib) $0.7901 - 0.4609$ or $^5C_2(\frac{2}{3})^3(\frac{1}{3})^2$ M1; $= 0.329$ (3 sf) A1 (iii) $p =$ "0.3292" M1; $^7C_3(1 - \text{"0.3292"})^4(\text{"0.3292"})^3$ M1; $= 0.253$ (3 sf) A1 ie max 8/10	
5i	Binomial or B (5, 0.3) Prob of gift same for all pkts Whether pkt contains gift is indep of other pkts	B1 B1 B1 B1 4	 Prob of gift is constant or fixed or consistent or same oe Obtaining a gift is indep Each time receive a gift is indep Context needed for 3rd & 4th B-mks	Allow mis-spellings but NOT "Biometric" Condone B~(5, 0.3) or B(0.3, 5): B1B1 but B($X = 0.3$, $n = 5$): B1B0 NOT: prob of success const; NOT prob stays same each go One box doesn't affect another. Pkts indep. Gifts indep She buys packets separately Prob of a gift is indep Prob of gift indep of one another & const: B1B1 NOT: Each week is indep NOT: Number of gifts received is indep NOT: Events indep If Geo(0.3) stated, can score max B0B0B1B1 If Geo(5, 0.3) stated, can score max B0B1B1B1
iia	0.8369	B1 1	or 0.837	
b	$0.8369 - 0.5282$ or $^5C_2(0.7)^3(0.3)^2$ $= 0.3087$ or 0.309 (3 sf)	M1 A1 2		
iii	$p =$ "0.3087" $^7C_3(1 - \text{"0.3087"})^4(\text{"0.3087"})^3$ $= 0.235$ (3 sf)	M1 M1 A1 3	(iib) used in a calc'n eg "0.3087" × 3	or B(7, "0.3087") stated or $1 - $ "0.3087" used instead of "0.3087" $n = 35$ or 15: max M1M0A0
Total		**10**		

6i	$7! \div 3!$ $\qquad\qquad$ $7! \div 2!$	M1	But NOT $^7P_4$ or $7!/(7-4)!$ if seen	$\frac{7!}{3!+2!}$: M1M0
	$\div 2!$ $\qquad\qquad$ $\div 3!$	M1dep		$\frac{7!}{3! \times n!}$ any n: M1M0
	$= 420$	A1 $\quad$ 3		
iia	$^5C_3$ or $^{10}C_4$ seen	M1	or 10 or 210	$\frac{^5C_3 \times ^{10}C_4}{anything}$ $\quad$ M1M1A0
	$^5C_3 \times ^{10}C_4$	M1		
	$= 2100$	A1 $\quad$ 3		$^5P_3 \times ^{10}P_4$ or 60×5040 or 302400: SC B1
b	$^4C_2 \times ^9C_4$ or $^4C_3 \times ^9C_3$ $\qquad$ or 756 or 336	M1	$\frac{3}{5}$ or $\frac{4}{10}$ oe	Not from incorrect wking
	$^4C_2 \times ^9C_4 + ^4C_3 \times ^9C_3$ $\qquad$ or 1092	M1	$\frac{3}{5} \times (1 - \frac{4}{10})$ or $(1 - \frac{3}{5}) \times \frac{4}{10}$	SC $\frac{1}{5} \times \frac{9}{10}$ or $\frac{4}{5} \times \frac{1}{10}$ $\quad$ M1
	$\div 2100$ or $\div$ (iia) dep $\geq$ one M1 scored	M1dep	$\frac{3}{5} \times (1 - \frac{4}{10}) + (1 - \frac{3}{5}) \times \frac{4}{10}$	$\frac{1}{5} \times \frac{9}{10} + \frac{4}{5} \times \frac{1}{10}$ $\quad$ M1
	$= \frac{13}{25}$ or 0.52	A1 $\quad$ 4	$= \frac{13}{25}$	$(= \frac{13}{50}$ $\quad$ A0)
			$\frac{3}{5}$ or $\frac{4}{10}$ oe $\qquad$ M1	Not from incorrect wking
			$\frac{3}{5} + \frac{4}{10} - \frac{3}{5} \times \frac{4}{10}$ $\qquad$ M1	ie P(WA or GA or both) Must be correct figures
	"2100" $-$ ($^4C_3 \times ^9C_4$ or $^4C_2 \times ^9C_3$) or "2100" $-$ (504 or 504) $\quad$ M1		$\frac{3}{5} + \frac{4}{10} - \frac{3}{5} \times \frac{4}{10} - \frac{3}{5} \times \frac{4}{10}$ $\qquad$ M1	ie P(WA or GA but not both) Must be correct figures
	"2100" $-$ ($^4C_3 \times ^9C_4 + ^4C_2 \times ^9C_3$) $\quad$ M1		$= \frac{13}{25}$ $\qquad$ A1	
	$\div$ "2100" or (iia) dep $\geq$ M1 $\qquad$ M1			SC : $^4P_2 \times ^9P_4 + ^4P_3 \times ^9P_3$: M1
				$\div$ (iia) $\qquad$ M1dep
				Careful: 336 or 756 can be obtained by incorrect methods.
Total		**10**		

7i	$(0 \times a) + 2 \times (1 - a)$	M1		or $2(1 - a)$	Condone $2 \times 1 - a$ NB $2 \times (1 - a) \div 2$: M0A0
	$= 2 - 2a$ or $2(1 - a)$ oe	A1	2	Not ISW	Eg $E(X) = 2 - 2a$; $2 - 2a = 1$; $a = 0.5$: M1A0
ii	$(0 \times a) + 2^2 \times (1 - a)$	M1		or $4 - 4a$ oe	Condone $2^2 \times 1 - a$
	$- \text{``}(2 - 2a)\text{''}^2$	M1		$- (\text{i})^2$ dep contains a; ISW; Indep mk	$4 - 4a - 4 \pm 8a \pm 4a^2$ or $4 - 4a - 4 \pm 4a^2$ or equiv M1M1A0
					$4 - 4a - 2(1 - a)^2$ M1M1A0
	$= 4 - 4a - 4 + 8a - 4a^2$			or $4(1 - a) - 4(1 - a)^2$	
	$= 4a - 4a^2$	A1	3	$4(1 - a)(1 - (1 - a))$	Must see this line, correctly obtained
	$(= 4a(1 - a))$ **AG**				
					Careful:
					$4 - 4a - (2 - 2a)^2 = 4 - 4a -(4 - 4a^2) = - 4a + 4a^2 = 4a(1 - a)$
	<table><tr><td>$-2 + 2a$</td><td>$2a$</td></tr><tr><td>a</td><td>$1 - a$</td></tr></table> M1			Correct table oe	M1M1A0 only
	$\text{Var}(X) = a(-2+2a)^2 + 4a^2(1 - a)$ M1				
	$4a^3 - 8a^2 +4a + 4a^2 - 4a^3$				
	$4a - 4a^2$ A1				
Total		**5**			
8i	EDCBA	B1	1	A 5	NOT just 5, 4, 3, 2, 1
				B 4	
				C 3	
				D 2	
				E 1	
iia	$1 - \dfrac{6\Sigma d^2}{5(5^2 - 1)} = 0.9$	M1			$1 - \dfrac{6 \times 2}{5(5^2 - 1)}$
	$1 - \dfrac{6 \times \Sigma d^2}{5 \times 24} = 0.9$ or $0.1 = \dfrac{6 \times \Sigma d^2}{5 \times 24}$	A1	2	One correct step or better & nothing incorrect for A1	$= 1 - \dfrac{6 \times 2}{5 \times 24}$ or $1 - \dfrac{12}{5 \times (5^2 - 1)}$ One correct step or better & nothing incorrect for A1
	$(\Sigma d^2 = 2$ **AG**)				$(= 0.9$ **AG**$)$
b	d^2: 0, 0, 0, 1, 1 any order	M1		or d: 0, 0, 0, 1, -1 any order	May not be seen
	BACDE or similar	A1	2	Any two adjacent dogs interchanged	
					If **clearly** comparing second race with third; DECBA or similar:
					B1, but must be clear
Total		**5**			

Total 72 marks