8    Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

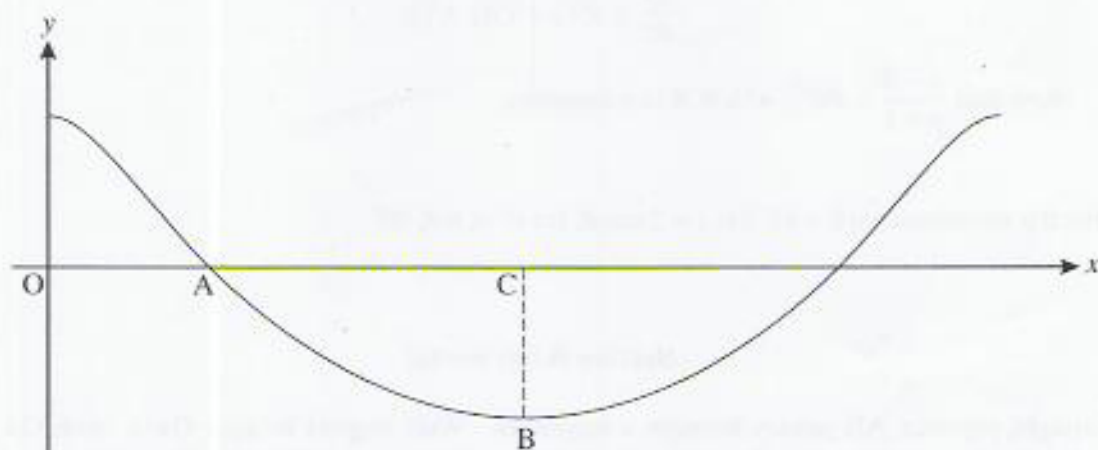This is shown in Fig. 8. B is a minimum point, and BC is vertical.



**Fig. 8**

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$.                    [5]

(ii) Find $\dfrac{dy}{dx}$ in terms of $\theta$. Find the gradient of the track at A.                    [4]

(iii) Show that, when the gradient of the track is 1, $\theta$ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$                    [2]

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$.                    [7]