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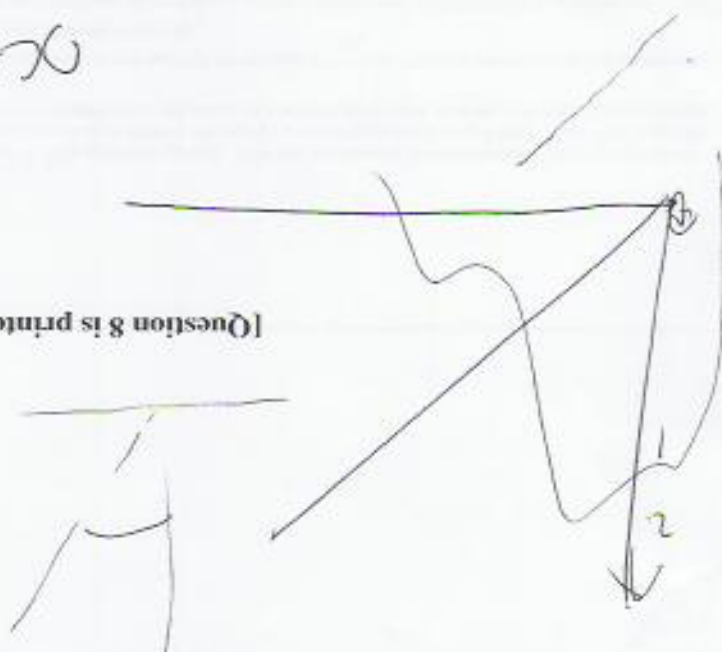
A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates $(-200, 100, 0)$ and B has coordinates $(100, 200, 100)$, where units are metres.

- (i) Verify that $\vec{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. [3]

- (ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. [6]

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

- (iii) Find the coordinates of the point where the pipeline meets the layer of rock. [4]
 (iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. [5]



[Question 8 is printed overleaf.]

$$x+1 = 2x^2 - 3x + 1$$

$$1 - 2 - 2x + 2x^2$$

$$(-2x)(1-x)$$

from $x=0$

$$1 - 2x$$

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- (i) Express $\frac{3}{(y-2)(y+1)}$ in partial fractions. [3]

- (iii) Hence, given that x and y satisfy the differential equation

$$\frac{dy}{dx} = x^2(y-2)(y+1),$$

show that $\frac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. [5]

- 6 Solve the equation $\tan(\theta + 45^\circ) = 1 - 2 \tan \theta$, for $0^\circ \leq \theta \leq 90^\circ$. [7]

Section B (36 marks)