GCE

Mathematics

Advanced GCE **4726**

Further Pure Mathematics 2

Mark Scheme for June 2010

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

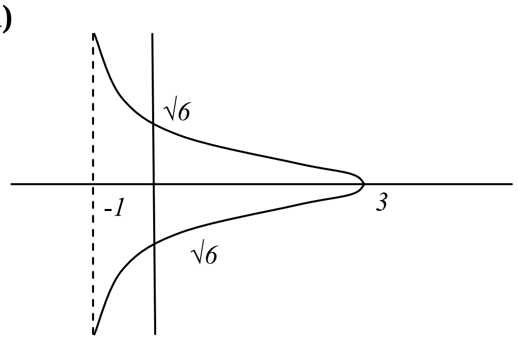
Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1	Derive/quote $g'(x) = p/(1+x^2)$	B1	
	Attempt $f'(x)$ as $a/(1+bx^2)$	M1	Allow any a, b=2 or 4
	Use $x = ½$ to set up a solvable equation in p, leading to at least one solution	M1	
	Get $p = {}^5/_4$ only	A1	AEEF
2	Reasonable attempt at $e^{2x}\,(1+2x+2x^2)$	M1	3 terms of the form $1+2x+ax^2$, $a \neq 0$
	Multiply out their expressions to get all terms up to x^2	M1	(3 terms) x (minimum of 2 terms)
	Get $1+3x+4x^2$	A1	cao
	Use binomial, equate coefficients to get 2 solvable equations in a and n	M1	Reasonable attempt at binomial, each term involving a and n (an=3, $a^2n(n-1)/2=4$)
	Reasonable attempt to eliminate a or n	M1	
	Get n=9, a=⅓ cwo	A1	cao
			SC Reasonable $f'(x)$ and $f''(x)$ using product rule (2 terms) M1
			Use their expressions to find $f'(0)$ and $f''(0)$ M1
			Get $1+3x+4x^2$ cao A1
3	Quote/derive correct $dx = 2dt/(1+t^2)$	B1	
	Replace all x (not $dx=dt$)	M1	From their expressions
	Get $2/(t-1)^2$ or equivalent	A1	
	Reasonable attempt to integrate their expression	M1	
	Use correct limits in their correct integral	A1√	Must involve √3
	Clearly tidy to √3+1 from cwo	A1	A.G.
4 (i)	Get a = -2	B1	May be quoted (from correct working)
	Get b = 6	B1	May be quoted (from correct working)
	Get c = 1	B1	May be quoted (from correct working)

(ii)



B1 Correct shape in $-1<x\leq 3$ only (allow just top or bottom half)

B1 90^0 (at x=3) (must cross x-axis i.e. symmetry)

B1 Asymptote at x= -1 only (allow −1 seen)

B1√ Correct crossing points; $\pm\sqrt{(b/c)}$ from their b,c

5 (i) Reasonable attempt at parts — M1 Leading to second integral
Get $e^x(1-2x)^n - \int e^x.n(1-2x)^{n-1}.-2\,dx$ — A1 Or $(1-2x)^{n+1}/(-2(n+1))e^x - \int (1-2x)^{n+1}/(-2(n+1))e^x dx$

Evidence of limits used in integrated part — M1 Should show ±1
Tidy to A.G. — A1 Allow $I_{n+1} = 2(n+1)I_n - 1$

(ii) Show any one of $I_3=6I_2-1$, $I_2=4I_1-1$, $I_1=2I_0-1$ — B1 May be implied
Get $I_0(=e^{½}-1)$ or $I_1(=2e^{½}-3)$ — B1
Substitute their values back for their I_3 — M1 Not involving n
Get $48e^{½} - 79$ — A1

6 (i) Reasonable attempt to differentiate sinh $y = x$ to get dy/dx in terms of y — M1 Allow ±cosh$y$ dy/dx = 1
Replace sinh y to A.G. — A1 Clearly use $\cosh^2 - \sinh^2 = 1$
SC Attempt to diff. $y = \ln(x+\sqrt{(x^2+1)})$
using chain rule M1
Clearly tidy to A.G. A1

(ii) Reasonable attempt at chain rule — M1 To give a product
Get $dy/dx = a \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$ — A1
Reasonable attempt at product/quotient — M1 Must involve sinh and cosh
Get d^2y/dx^2 correctly in some form — A1√ From $dy/dx = k \sinh(a\sinh^{-1}x)/\sqrt{(x^2+1)}$
Substitute in and clearly get A.G. — A1
SC Write $\sqrt{(x^2+1)}dy/dx = k \sinh(a\sinh^{-1}x)$
or similar
Derive the A.G.

7 (i) Get 5.242, 5.239, 5.237 — B1√ Any 3(minimum) correct from previous value
Get 5.24 — B1 Allow one B1 for 5.24 seen if 2 d.p.used

(ii) Show reasonable staircase for any region — B1 Drawn curve to line
Describe any one of the three cases — B1
Describe all three cases — B1

(iii) Reasonable attempt to use log/expo. rules — M1 Allow derivation either way
Clearly get A.G. — A1
Attempt f′(x) and use at least once in correct N-R formula — M1
Get answers that lead to 1.31 — A1 Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.

(iv) Show f′(ln36) = 0 — B1
Explain why N-R would not work — B1 Tangent parallel to Ox would not meet Ox again or divide by 0 gives an error

8 (i) Use correct definition of $\cosh x$	B1	
Attempt to cube their definition involving e^x and e^{-x} (or e^{2x} and e^x)	M1	Must be 4 terms
Put their 4 terms into LHS and attempt to simplify	M1	
Clearly get A.G.	A1	
		SC Allow one B1 for correct derivation from $\cosh 3x = \cosh(2x+x)$
(ii) Rewrite as $k\cosh 3x = 13$	M1	
Use ln equivalent on $13/k$	M1	Allow $\pm$ln or $\ln(13/k \pm \sqrt{(13/k)^2 - 1})$ for their k or attempt to set up and solve quadratic via exponentials
Get $x = (\pm)\ \frac{1}{3}\ln 5$	A1	
Replace in $\cosh x$ for u	M1	
Use $e^{a\ln b} = b^a$ at least once	M1	
Get $\frac{1}{2}(5^{\frac{1}{3}} + 5^{-\frac{1}{3}})$	A1	
9 (i) Attempt integral as $k(2x+1)^{1.5}$	M1	
Get 9	A1	cao
Attempt subtraction of areas	M1	Their answer – triangle
Get 3	A1√	Their answer − 6 (>0)
(ii) Use $r^2=x^2+y^2$ and $x=r\cos\theta$, $y=r\sin\theta$	B1	
Eliminate x and y to produce quadratic equation (=0) in r (or $\cos\theta$)	M1	
Solve their quadratic to get r in terms of θ (or vice versa)	A1√	
Clearly get A.G.	A1	$r>0$ may be assumed
Clearly show θ_1(at B)$=\tan^{-1}\frac{3}{4}$ and θ_2 (at A) $=\pi$	B1	
		SC Eliminate y to get r in terms of x only M1
		Get $r = x + 1$ A1
		SC Start with $r=1/(1-\cos\theta)$ and derive cartesian
(iii) Use area $= \frac{1}{2}\int r^2\, d\theta$ with correct r	B1	cwo; ignore limits
Rewrite as $k\operatorname{cosec}^4(\frac{1}{2}\theta)$	M1	Not just quoted
Equate to their part **(i)** and tidy	M1	To get $\int$ = some constant
Get 24	A1	A.G.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553