RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4723**

Core Mathematics 3

Mark Scheme for June 2010

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1	**(i)**	Attempt use of product rule	M1	producing … + … form
		Obtain $3x^2e^{2x} + 2x^3e^{2x}$	A1 **2**	or equiv
	(ii)	Attempt use of chain rule to produce $\dfrac{kx}{3+2x^2}$ form	M1	any constant k
		Obtain $\dfrac{4x}{3+2x^2}$	A1 **2**	
	(iii)	Attempt use of quotient rule	M1	or equiv; condone u/v confusions
		Obtain $\dfrac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$	A1 **2**	or (unsimplified) equiv

[If $...+c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of $...+c$.]

$\boxed{6}$

2	**(i)**	Obtain one of $\pm\ln(\pm x \pm 4)$	M1	
		Obtain correct equation $y = -\ln(x-4)$	A1 **2**	or equiv; condone use of modulus signs instead of brackets
	(ii)	State, in any order, S, S and T	M1	or equiv such as S^2, T or 2S, T
		State T, then S, then S	A1 **2**	or equiv (note that S, S, T^9 and S, T^3, S are alternative correct answers)

$\boxed{4}$

3	**(i)**	Use $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	
		Attempt to express equation in terms of $\sin\theta$	M1	using $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or equiv
		Obtain or clearly imply $6\sin^2\theta - 11\sin\theta - 10 = 0$	A1 **3**	or $-6\sin^2\theta + 11\sin\theta + 10 = 0$
	(ii)	Attempt solution to obtain at least one value of $\sin\theta$	M1	should be $s = -\frac{2}{3}, \frac{5}{2}$
		Obtain –41.8	A1	allow –42 or greater accuracy
		Obtain –138	A1 **3**	or greater accuracy; and no others between –180 and 180
		[Answer(s) only: award 0 out of 3.]		

$\boxed{6}$

4	**(i)**	Either: Integrate to obtain $k\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $k\ln 3 = \ln 81$ and hence $k = 4$	A1	**3**	AG; accurate work required
		Or 1: (where solution involves no use of a logarithm property)			
		Integrate to obtain $k\ln x$	B1		
		Obtain correct explicit expression for k and conclude $k = 4$ with no error seen	B2	**3**	AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$
		Or 2: (where solution involves verification of result by initial substitution of 4 for k)			
		Integrate to obtain $4\ln x$	B1		
		Use at least one relevant logarithm property	M1		
		Obtain $\ln 81$ legitimately with no error seen	A1	**3**	AG; accurate work required
	(ii)	State volume involves $\int \pi(\frac{4}{x})^2 \, dx$	B1		possibly implied
		Obtain integral of form $k_1 x^{-1}$	M1		any constant k_1 including π or not
		Use correct process for finding volume produced from S	M1		$\int (k_2 2^2 - k_3 y^2)\, dx$, including π or not with correct limits indicated; or equiv
		Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$	A1	**4**	or exact equiv
			$\boxed{7}$		

5	**(i)**	Attempt process for finding both critical values	M1		squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
		Obtain -4	A1		
		Obtain $\frac{2}{3}$	A1		
		Attempt process for solving inequality	M1		table, sketch, …; needs two critical values; implied by plausible answer
		Obtain $-4 \le x \le \frac{2}{3}$	A1	**5**	with $\le$ and not $<$
	(ii)	Use correct process to find value of $\lvert x+2 \rvert$ using any value	M1		… whether part of answer to **(i)** or not
		Obtain $2\frac{2}{3}$ or $\frac{8}{3}$	A1	**2**	dependent on 5 marks awarded in part **(i)**
			$\boxed{7}$		

6	(i)	Attempt calculations involving 1.0 and 1.1	M1		using radians
		Obtain -0.57 and 0.76	A1		or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged)
		Refer to sign change (or equiv for rearranged eqn)	A1	3	AG; following correct work only
	(ii)	Obtain correct first iterate	B1		using value x_1 such that $1.0 \le x_1 \le 1.1$
		Carry out iteration process	M1		obtaining at least 3 iterates in all so far
		Obtain at least 3 correct iterates	A1		showing at least 3 dp
		Obtain 1.05083	A1	4	answer required to exactly 5 d.p.
		[$1 \to 1.047198 \to 1.050571 \to 1.050809 \to 1.050826 \to 1.050827$; $1.05 \to 1.050769 \to 1.050823 \to 1.050827 \to 1.050827$; $1.1 \to 1.054268 \to 1.051070 \to 1.050844 \to 1.050829 \to 1.050827$]			
	(iii)	State or imply $\sec^2 2x = 1 + \tan^2 2x$	B1		
		Relate to earlier equation	M1		by halving or doubling answer to **(ii)** or carrying out equivalent iteration process
		Deduce $2x = 1.05083$ and hence 0.525	A1√	3	following their answer to **(ii)**; or greater accuracy
		[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$	B1		
		Use iterative process to obtain 0.525	B1	2	or greater accuracy]
			$\boxed{10}$		
7		Differentiate to obtain $k_1(3x-1)^3$	M1		any constant k_1
		Obtain correct $12(3x-1)^3$	A1		or (unsimplified) equiv
		Substitute 1 to obtain 96	A1		
		Attempt to find x-coordinate of Q	M1		using tangent with $y = 0$ or using gradient
		Obtain $\frac{5}{6}$	A1		or exact equiv
		Integrate to obtain $k_2(3x-1)^5$	M1		any constant k_2
		Obtain correct $\frac{1}{15}(3x-1)^5$	A1		or (unsimplified) equiv
		Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1		
		Attempt to find shaded area by correct process	M1		integral – triangle or equiv
		Obtain ($\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16$ and hence) $\frac{4}{5}$	A1		or equiv
			$\boxed{10}$		
8	(i)	Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$	B1		or equiv
		Attempt to find value of α	M1		condone sin/cos muddles and degrees
		Obtain $\frac{1}{4}\pi$ or 0.785	A1	3	in radians now
	(ii) a	Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$	M1		condone degrees here
		Obtain $\frac{3}{4}\pi$	A1	2	or $\ldots, -\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now
	b	Attempt correct process to find value of $3x - \alpha$	*M1		with attempt at rearranging $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$
		Obtain at least one correct exact value of $3x - \alpha$	A1		$\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
		Attempt at least one positive value of x	M1		dep *M
		Obtain $\frac{1}{36}\pi$	A1	4	
			$\boxed{9}$		

9	**(i)**	Attempt to find x-coord of staty point or complete square	M1	
		Obtain $(\frac{3}{2}, -9)$ or $4(x-\frac{3}{2})^2 - 9$ or -9	A1	or equiv
		State $f(x) \geq -9$	A1 **3**	using any notation; with $\geq$
	(ii)	Make one correct (perhaps general) relevant statement	B1	not 1 –1 , f is many-one, … ; maybe implied if attempt is specific to this f
		Conclude with correct evidence related to this f	B1 **2**	AG; (more or less) correct sketch; correct relevant calculations, …
	(iii)	<u>Either</u>: Attempt to find expression for g^{-1}	*M1	or equiv
		Obtain $\frac{1}{a}(x-b)$	A1	or equiv
		Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1	dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a
		Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$	A1 **4**	AG; necessary detail required; or equiv
		[SC1: first two steps as above, then substitute $a = -1$: max possible M1A1B1]		
		[SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 **2**]		
		<u>Or</u>: State or imply that $y = g^{-1}(x)$ is reflection of $y = g(x)$ in line $y = x$	B1	
		State that line unchanged by this reflection is perpendicular to $y = x$	M2	
		Conclude that a is –1	A1 **4**	
	(iv)	State or imply that $gf(x) = -(4x^2 - 12x) + b$	B1	
		Attempt use of discriminant or relate to range of f	M1	or equiv
		Obtain $64 + 16b < 0$ or $9 + b < 5$	A1	or equiv
		Obtain $b < -4$	A1 **4**	
			$\boxed{13}$	

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