GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Reports on the Units

June 2010

CONTENTS
Advanced GCE Mathematics (7890)
Advanced GCE Pure Mathematics (7891)
Advanced GCE Further Mathematics (7892)
Advanced Subsidiary GCE Mathematics (3890)
Advanced Subsidiary GCE Pure Mathematics (3891)
Advanced Subsidiary GCE Further Mathematics (3892)

REPORTS FOR THE UNITS

Chief Examiner's Report – Pure Mathematics

Recent reports have referred to concerns about many candidates' lack of accuracy in dealing with algebraic expressions and equations. It is expected that candidates will be reasonably fluent and efficient in handling algebra, and in the A2 Core and Further Pure Mathematics units especially, it is hoped that candidates will be sufficiently comfortable with algebraic techniques that they can concentrate on the underlying mathematics. However, as several of the following reports indicate, candidates were often betrayed by their shaky algebraic skills. In the specification for unit 4724, there is an explicit item on simplifying rational expressions. But simple rational expressions can occur naturally in earlier units and candidates would benefit from looking in some detail at the techniques needed when simplifying simple rational expressions and in solving related equations.

In units 4721 and 4722, candidates wrote their solutions in Printed Answer Books and candidates generally handled this well. The corresponding report for the January 2010 examination series contained advice for candidates about using the Printed Answer Books and the General Comments section in the current report for 4722 makes some additional points. As far as possible, candidates should confine each solution to its allotted space. For many candidates the space allotted to a particular question will seem too large, but this is to cater for possible long-winded solutions or to accommodate second attempts. To use a Printed Answer Book effectively, candidates need to approach their solutions with a little care and discipline, features which might also enhance the quality of the mathematics they produce. Two more units – 4723 and 4725 – will use Printed Answer Books in the January 2011 examination series.

4721 Core Mathematics 1

General Comments

This paper proved accessible to almost all candidates and many excellent scripts were seen. Apart from the final part of question 10, most candidates attempted every question and it was pleasing to see an appropriate amount of working shown in most cases.

Graph sketching continues to be a difficult skill for candidates of all abilities, although it is definitely showing improvement; far fewer candidates work out coordinates and plot points now, although too many are still resistant to using a ruler to draw axes. As in previous papers, candidates struggled to solve the quadratic inequality correctly (question 8 part (iv)). The fact that the quadratic expression had 2 rather than 3 terms seemed to increase the difficulty of the question.

Candidates appeared to have time to complete the paper and all marks between 0 and 72 were awarded.

Comments on individual questions

1)   (i)   This proved to be a very simple starter and virtually every candidate answered correctly, the only occasional wrong answer being 0.

      (ii)   The majority of candidates were able to evaluate the expression correctly, although there was a significant number who stopped at $\dfrac{1}{\sqrt{9}}$, earning only the method mark. Others gave the incorrect final answer $\pm\dfrac{1}{3}$. Weaker candidates often squared 9 although most dealt with the negative power correctly.

2)   (i)   As in previous sessions, graph sketching was poor in too many cases. Candidates must be encouraged to use a ruler to draw axes. All but the weakest candidates knew the correct general shape but many drew their curves in incorrect quadrants, with $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$ and $y = -\dfrac{1}{x}$ all seen. There were also cases of $y = -\sqrt{x}$ while the very weakest candidates often sketched the graph of $y = -x^2$.

      (ii)   In general, candidates used their answer to part (i) and translated it vertically, which gained the method mark, although it was translated in the negative direction almost as frequently as in the positive direction. Unfortunately, it was rare to see a horizontal asymptote at $y = 3$ and many candidates lost a mark because the significance of 3 was not indicated in any way on the diagram.

      (iii)   The equation of the stretched curve was given correctly by almost two-thirds of the candidates, but there were many cases of non-standard notation such as $y = 2\left(-\dfrac{1}{x^2}\right)$ or $y = -2\dfrac{1}{x^2}$ seen.

3)   (i)   The method for rationalising a denominator was well known and the majority of candidates scored full marks on this question, displaying their working clearly, although a few 'simplified' their correct answer of $9 - 3\sqrt{5}$ to $3 - \sqrt{5}$.

    (ii)   This question was also done very well. Nearly all candidates realised that they needed to express $\sqrt{18}$ in terms of $\sqrt{2}$ and were able to do so correctly, although some left the answer as $3\sqrt{2} - \sqrt{2}$ and others thought that this expression could be simplified to 3.

4)   (i)   As in previous papers, the expansion of 3 brackets was done efficiently and accurately by the vast majority of candidates, the modal mark being 3. Only a small number of candidates expanded $(x-2)^2$ as $x^2 - 4$ or $x^2 + 4$, in other scripts careless algebra produced $x^2 - 4x + 2$.

    (ii)   The sketch of the curve proved more demanding, with roughly a quarter of the candidates scoring only a single mark, almost always for the correct *y*-intercept. Among the varied incorrect sketches, there were examples of negative cubic curves, quadratics with roots -1 and 2 and lots of cubics with *x*-intercepts at
(-1, 0), (2, 0) and a third unspecified point. However, a large number of candidates drew excellent curves, with the turning point at (2, 0) shown correctly.

5)   In general, candidates showed good understanding of how to approach this question and scored either 4 or 5 marks. The majority recognised that a change of variable was needed to convert the given equation into a quadratic. Most subsequently remembered to square root the solutions of the quadratic to obtain the solutions to the original equation. A pleasing number explicitly stated that $x^2 = -1$ could not be solved to give real roots. Unfortunately, a large proportion forgot the negative square root of $\dfrac{1}{4}$, or stated that $\sqrt{-1} = 1$ or similar. By contrast, there were plenty of candidates who thought that this question was about investigating the value of the discriminant and also those who simply applied the quadratic formula to the quartic equation.

6)   This question was completed very successfully, the modal score being 5/5. However, a significant number of candidates started by rewriting $\dfrac{6}{\sqrt{x}}$ as $6x^{\frac{1}{2}}$ and this error meant that a maximum of 2 method marks were available to them. A disappointingly large number of those who obtained the correct expression for the gradient were unable to evaluate $4^{-\frac{3}{2}}$ correctly.

7)   The solution of two simultaneous equations proved a routine request for most candidates and almost half scored full marks on this question. Most of those who arrived at the correct 3-term quadratic were able to factorise it correctly, many realising that this task could be made easier by first cancelling each term by 3. Efforts to complete the square were pleasingly rare although attempted solutions by the quadratic formula were more common, often petering out or leading to computational errors, usually in the discriminant. Weaker candidates made errors in the earlier stages of substituting, often making errors in squaring $6 - 2x$ or in doubling their squared expression. However, even incorrect algebra often led to a quadratic equation that could be solved so these candidates could still earn 2 or 3 marks out of 6. As in previous sessions, the very weakest candidates 'squared' the linear equation to get $x^2 + 4y^2 - 36 = 0$ but at least these candidates then attempted to eliminate one of the variables, unlike those who rearranged the quadratic expression to equal zero and then equated the 2 expressions. They were unable to make any progress at all and scored no marks.

8) (i)   As noted in previous sessions, most candidates struggle with completing the square when the coefficient of x^2 is greater than 1. Even those who worked out $p = \dfrac{5}{4}$ were often unable to square their p correctly and fewer still understood that they needed to double their value. It was relatively unusual to award all 3 marks for this question, even in good scripts.

   (ii)   Many candidates sensibly used their expression from part (i) to determine the coordinates of the minimum point and were able to earn both follow-through marks. A significant number decided to differentiate instead and reached $5x + 4 = 0$. Surprisingly, this was followed by $x = \dfrac{5}{4}$ almost as often as by $x = -\dfrac{5}{4}$. The candidates who used this method then found it quite difficult to work out the y-coordinate without error.

   (iii)   Only a quarter of candidates scored this mark, with many making no attempt. There were some very strange wrong answers, including many quadratic equations.

   (iv)   Yet again, candidates struggled to solve this quadratic inequality correctly. Of those who treated part (iv) as separate from the earlier parts of the question, the quadratic expression was usually factorised correctly but then either the zero root was omitted or the sign of the non-zero root was incorrect. Candidates who attempted to find the roots by completing the square using their (often incorrect) expression from part (i) fared badly, with many omitting the negative square root. Others used the quadratic formula to find the roots, with mixed success. Even those who found the correct roots then chose the region between their roots or simply replaced the = with > in their answers.

9) (i)   Even those candidates who scored single figure totals for the whole paper usually recorded these 3 marks. There were a few examples of $\dfrac{x_1 - x_2}{2}$ seen. A good number of candidates drew a rough diagram, which also helped in other parts of this question, and is to be encouraged.

   (ii)   Although most candidates used Pythagoras' theorem correctly, there was some confusion as to whether they were finding the radius or the diameter. Some of those who obtained $\sqrt{116}$ then halved it incorrectly, the most common wrong answer being $\sqrt{58}$.

   (iii)   Most candidates knew that $(x + 1)^2 + (y - 3)^2$ was needed but it was quite common to see $(x + 1)^2 - 1 + (y - 3)^2 - 9 = r^2$ or similarly confused working. A disappointingly large number of candidates used r rather than r^2. Equations using a wrong centre, usually (4, 5), were frequently seen. It was relatively rare to award all 3 marks for completely correct working leading to the correct equation.

(iv) Candidates understood how to use the radius to find the gradient of the tangent to the circle, which was encouraging, and many went on to score full marks here. However, the statement $\dfrac{-4}{-10} = -\dfrac{2}{5}$ was surprisingly common. A few candidates attempted to differentiate the equation of the circle most usually resulting in $2x + 2y + 2 - 6$. Solutions with correct use of implicit differentiation were extremely rare but were awarded full marks if correct.

10) (i) Despite being the final question, this was a source of 5 or 6 marks for a large number of candidates who differentiated correctly and then solved the resulting quadratic equation without difficulty. The associated *y*-coordinates proved more of a challenge, although surprisingly it was almost as common to see an error when substituting -2 as when substituting the more difficult fractional value.

(ii) A pleasing number of candidates sketched a graph in answering this question; others simply knew that the solution involved the stationary points although some used their *y*-coordinates from part (i) rather than the *x* values. A commonly seen wrong method was to evaluate $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ at the stationary points and then use these values in an inequality.

(iii) Better candidates completed this in a straightforward way by evaluating the gradient and the *y*-coordinate for the cubic at the point where $x = \dfrac{1}{2}$ and then substituting them into the equation of a line. Some candidates rearranged the equation of the line and then compared the gradients and *y*-coordinates of the line and the curve, which was acceptable. Weaker candidates could not perform the calculations accurately, although many still gained a method mark or two. Candidates should be reminded that, in questions where the answer is provided for them, it is essential to show all steps of their working clearly.

(iv) A large number of candidates made no attempt at this final question and it was very rare to award full marks. The most frequently seen diagram was of a cubic curve with a line intersecting it twice near the minimum point, rather than being tangential. It was also very common to see solutions where candidates differentiated the cubic and then calculated the discriminant for the resulting quadratic and stated '2 roots'. However, there were also some excellent diagrams seen, with or without accompanying working. Some of the best candidates took an algebraic route, solving the two equations simultaneously, and then showing that the resulting cubic factorised to $(2x - 1)^2 (2x + 7)$. Unfortunately, such a rigorous solution could still only gain 2 marks!

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and gave them an opportunity to demonstrate their knowledge. All but the very weakest were able to make an attempt at most questions, but there were also several parts that were sufficiently stretching for more able candidates. Candidates should be able to use mathematical conventions to express themselves clearly, including the use of brackets and a clear indication of where a fraction line is intended to finish. In questions that involve a proof it is essential that enough explanation is provided, both in words and algebra, in order to be convincing or credit cannot be given.

Candidates should ensure that the working throughout the question is done to a sufficient degree of accuracy so as not to compromise the final answer. Premature approximations cost candidates marks on several questions, notably the trapezium rule, the area of the segment and using logarithms to solve an equation. It is also important for candidates to check whether an exact answer is required. Candidates are not always aware of how later parts of a question relate to earlier parts, and should appreciate that 'hence' is used to give a hint as to how to proceed by utilising previous work.

This was the first time that C2 was answered using a Printed Answer Book, and the majority of candidates coped well with this. However some candidates did not confine their solution solely to the dedicated answer space. There were also problems in reading the script where candidates had worked in pencil and then gone over it again, or rubbed out answers and written over the top. Candidates who make two, or more, attempts at a question are advised to make it clear which solution they wish to be marked. Where no choice is made the final solution will be the one that is credited.

Comments on individual questions

1) (i) For candidates who used the factor theorem this proved to be a straightforward first question, with the majority able to find the correct value of a. A few candidates attempted to use long division, but this was rarely accurate. Those who attempted a coefficient matching method fared slightly better, though a number caused further difficulties by electing to use a as a coefficient in their quadratic factor. Whilst it is important that candidates are familiar with a variety of techniques, it is also important that they are aware of which method is the most effective and efficient to use in a given situation.

(ii) Most candidates attempted to evaluate f(-1) but there were a number of sign errors on the second and/or third terms. Once again a significant minority of candidates attempted to use coefficient matching or long division; this was more successful than in part (i) as they were no longer dealing with unknown coefficients.

2) (i) A number of concise and accurate solutions were seen to this question, with the most effective method being to write out a correct expression involving surds and then evaluate this in one go on the calculator. Some candidates showed more steps in their method and this led to some accuracy errors from premature approximation. As always, a disappointing number of candidates were unable to correctly substitute into the rule, despite it being given in the formula book; the two main errors were placing the y-values in the wrong place and omitting the big bracket. A minority of candidates used x-values or

attempted integration first but the main error was not to use the number of strips specified in the question. These strips were rarely of equal width and $h = 3$ was still used in the rule.

(ii)   Most candidates were able to make an observation based on increasing the number of strips or decreasing the width of a strip. A number of candidates tried to determine whether it was an underestimate or overestimate, and some suggested using integration instead. A minority made no attempt at this part of the question.

3) (i)   Most candidates were able to make a good attempt at this question and a pleasing number of fully correct solutions were seen. The main error was applying the power to x only and not the coefficient of $\frac{1}{2}$ as well. The most successful solutions made effective use of brackets, though some candidates did use this approach and then just ignored the brackets. A minority found the required coefficients from Pascal's triangle, usually correctly, rather than using the calculator.

(ii)   The more able candidates could find the three products required. Others attempted a full expansion and then picked out the required terms. A surprising minority found two correct pairs but then used the x^3 term from part (i) rather than using it as a part of a third product, and many candidates just considered one term only.

4) (i)   This was a straightforward question and nearly all candidates wrote down the required three terms.

(ii)   The majority of candidates appreciated the need to sum the terms of an arithmetic sequence, though the values for a and d proved more problematical for some, with one or both of these being used as 1. A few candidates found the 40th term and others attempted to sum the terms of a GP, but a pleasing number of fully correct solutions were seen.

(iii)   This final part of the question proved to be more challenging with a number of candidates unable to find w_3. Many gained one mark for then attempting to solve an equation to find a value for p. A number of candidates did not use $5p + 1$ but found an alternative expression for the nth term of the AP, not always correctly.

5) (i)   The majority of candidates seemed familiar with the sine rule and used it accurately to find the required angle. Some candidates failed to work to the required degree of accuracy and others were unable to rearrange their expression correctly, especially if they used the version with the unknown in the denominator.

(ii) (a)   This question was generally well done, though some candidates clearly obtained the 50° but then failed to show the necessary detail in converting to radians.

(ii) (b)   The majority of candidates knew what was required here and there were many brief and fully accurate solutions. A number of candidates attempted to find the two areas in separate calculations but many then lost the final mark by failing to work to an appropriate degree of accuracy. Most candidates seemed familiar with the formula for the area of a sector, though a few omitted the $\frac{1}{2}$ and others used the angle in degrees rather than radians. The formula for

the area of the triangle was also recalled by most, though some had to resort to finding the base and height of a right-angled triangle.

6) (a) Virtually all candidates made a good attempt at the required integration and then made appropriate use of the given limits, and many fully correct solutions were seen. A number of candidates failed to gain the final mark as they gave a decimal answer rather than the exact area as requested in the question.

(b) This question was less well done. A common error was to have a first term of $2x$ rather than $2y$. Most candidates gained at least one mark for integrating the second term, though some were unable to divide 6 by $\frac{3}{2}$. A few candidates were unable to rewrite the given expression in index form, with an index of $\frac{1}{6}$ being a common error.

(c) Most candidates appreciated the need to rewrite the given fraction in index form, and most gained one mark for doing so. The subsequent integration was usually correct, though an answer of $-2x^{-4}$ was seen on a number of scripts. However, using the top limit of ∞ caused problems for many, with confusion over whether to use 0 or ∞ in the first term. Incorrect working was penalised, even if it resulted in a final answer of 4.

7) (i) Most candidates gained one mark for using a version of $\sin^2 x + \cos^2 x = 1$, though some just quoted it but failed to use it. The most common approach was to start with the left-hand side and replace the expression in the denominator with $\cos^2 x$. It should then have been a simple step to split the fraction into two terms and show the given answer, but some candidates did not show the required detail. When rewriting the denominator, a number of candidates also used the same identity in the numerator and then struggled to make further progress. Some attempts to cancel terms in the fraction showed a lack of confidence and understanding when dealing with algebraic fractions. Whilst this was a fairly straightforward proof, it was obvious that many candidates did not appreciate how to structure a convincing proof showing enough detail at each stage of working. Too many started with both sides and got into a real muddle.

(ii) The majority of candidates gained a mark for stating the equation to be solved, though a number of candidates failed to appreciate the link between this question and the proof in part (i), and wasted time redoing the same work, and others equated just one side of the equation to 0. Most then attempted an appropriate method to solve the quadratic in tan x, and were usually successful. Candidates could then attempt to find at least one solution for x from the roots of their quadratic. Whilst a pleasing number of fully correct solutions were seen, others failed to gain the final mark as they discarded -71.6° as negative rather pursuing it to find further solutions within the given range. A few candidates found the principal angles in radians but then added multiples of 180° and others attempted further angles in the wrong quadrants.

8) (a) This question was well answered, with candidates showing more proficiency in using logarithms to solve equations than we saw in previous years. The first three marks were gained by the majority of the candidates for introducing logarithms and dropping the powers. Some candidates then struggled to make further progress as they were unable to apply the correct order of operations needed to solve the linear equation. The more able candidates worked in terms of logarithms throughout whereas others resorted to decimal approximations quite early on, leading to an inaccurate final answer.

(b)   This part of the question proved to be more challenging for candidates and, whilst a number of fully correct solutions were seen, others failed to gain any credit. A common first step was to incorrectly split the first term into the sum of two logarithms. Other candidates had more of an idea how to proceed but produced a fraction with $\log_x 3$ in the denominator. However a reasonable number of candidates did gain the first mark for correctly combining the two logarithmic terms. It was disappointing that a number then multiplied by 3 to produce $\log_x (5y + 1) = 12$. Candidates who correctly removed $\log_x$ usually went on to gain full marks, though a few used the incorrect order of operations and others gave x in terms of y instead of the requested final form.

9) (i)   To gain the first mark in this question, candidates simply had to link together the terms of the AP and the GP in some way. A number gained this mark, usually for $a + ar + ar^2 = a + a + d + a + 2d$, but it was disappointing that more failed to gain this mark, usually through using $d = r$ from the outset. Having gained the first mark, most candidates then attempted to eliminate a variable, but were let down by poor algebraic skills. Statements such as $ar = a + d$ hence $r = 1 + d$ were common. The most elegant solution was to equate two expressions for d, both in terms of a and r, and rearrange but a number of other solution methods were also possible. In questions where the answer is given it is vital that candidates provide enough working and explanation to ensure that their method is convincing rather than leaving the examiner to guess what is intended.

(ii)   Many candidates struggled to make any progress on this question as they did not appreciate that they were required to solve a cubic equation. Some legitimately identified 1 as a root and could then make a reasonable attempt at finding and dealing with the quadratic factor. Coefficient matching tended to be more successful as some candidates struggled with the lack of an r^2 term if using long division. A number of elegant and fully correct solutions were seen though it was a shame that some candidates gave a decimal answer for the root following correct working. However too many candidates attempted to solve the cubic using the quadratic formula or used other incorrect methods such as $r(r^2 - 2) = -1$, hence $r = -1$ or $r^2 = 2$. Having found the three roots, some candidates chose the wrong one or failed to make any choice. Candidates are expected to show full details of the methods used. Stating the value of the root with no method shown, attracted some credit, but did not get full marks.

(iii)   Most candidates gained the first mark for equating a correct expression for the sum to infinity to the given value, but then struggled to make any further progress. A number of elegant solutions were seen, demonstrating proficiency in manipulating surds. The majority of those who gained full marks in part (ii) went on to gain full marks in this part as well, though a few resorted to decimal evaluation. When several roots had been found in part (ii) it was disappointing to see candidates test them all rather than appreciate that only one was valid in a calculation involving the sum to infinity.

4723 Core Mathematics 3

General Comments

This paper contained several questions of a routine and familiar nature, some parts requiring a little insight and mathematical awareness, and one question which was unstructured and which required candidates to make decisions about the steps to be taken. There were some candidates who struggled to make any progress, even with the straightforward questions such as questions 1, 3, 4, 5(i), 6(i), 6(ii) and 8(i), but most candidates seemed suitably prepared and generally answered these questions accurately. It was pleasing to note the response to the unstructured question 7, where many candidates showed that they understood the problem and were able to chart their way to a correct answer.

However, it must also be recorded that many candidates lacked the understanding and mathematical awareness to make any progress in requests where a little thought was required. Having a comprehensive knowledge of the mathematics involved and the ability to apply this knowledge to the solution of questions of a slightly more challenging nature are vital aspects of an assessment at this level.

Comments on individual questions

1) This opening question enabled many candidates to make a confident and successful start. The product rule was applied accurately in part (i); a few candidates also used the product rule in part (iii) and were usually successful. Most candidates used the quotient rule in part (iii) and this was handled well. Part (ii) was not answered quite as well and the incorrect $\dfrac{1}{3+2x^2}$ was noted on a number of scripts.

2) This question on transformations presented more difficulties to candidates. Many had no problem reaching the correct equation $y = -\ln(x-4)$ in part (i), but missing or misplaced brackets were evident in many attempts. Answers such as $y = \ln(-x-4)$ and $y = e^{x-4}$ were sometimes seen and indicated uncertainty about the effect of the transformation R.

Part (ii) was not done well and it was clear that many candidates did not realise what was required. Some offered attempts which contained no mention of S and T; others suggested meaningless equations such as $y = (\frac{x}{S^3} + T)^3$. Many candidates made errors when dealing with the transformation S and answers involving 3S, S^3 or S^{-3} were common. Many candidates did give the incorrect answer S, S, T but not very many provided the correct T, S, S. Examiners were tolerant of the use of notation such as T+2S which clearly implied the correct sequence of transformations.

3) There were many accurate solutions to this question and many candidates were able to find the two correct angles without any difficulty. Most candidates dealt correctly with $\mathrm{cosec}\,\theta$ but some were less sure when dealing with $\cos 2\theta$; not many candidates immediately replaced $\cos 2\theta$ by $1 - 2\sin^2\theta$. There were many attempts to simplify the equation to the requested form which were needlessly complicated and, inevitably, mistakes were made.

Faced, in part (ii) with $\sin\theta = -\frac{2}{3}$, candidates had no problem obtaining the value $-41.8°$ for θ but not all candidates were able to earn the final mark. Some offered no second value; others offered $138°$ or offered $41.8°$ and $138°$ as well as the correct $-138°$.

4) Part (i) was answered well by many candidates. They provided clear solutions, made appropriate use of logarithm properties and, mindful of the fact that the value of k was given in the question, gave sufficient detail to convince examiners. Other candidates started by assuming the value of 4 for k and proceeded to confirm that the area was $\ln 81$. This approach earned all the credit provided that the values were exact throughout the solution. A minority of candidates tried to show the result without carrying out any integration.

Many candidates also answered part (ii) correctly but other candidates encountered a variety of problems. A few attempted a rotation about the y-axis. Many candidates merely found the volume produced by the rotation of R. Integration of $\dfrac{16\pi}{x^2}$ often involved a natural logarithm. The volume resulting from the rotation of the combined regions R and S could be approached either by integration of $\pi \times 2^2$ or by recognising that the solid involved is a cylinder. Both methods were common but there were also attempts which suggested that candidates thought that a cuboid rather than a cylinder was involved. A significant minority of candidates recorded no marks on this part either because they attempted to integrate $\pi(2 - \frac{4}{x})^2$ or because their attempt involved finding the area of S and using this to claim $\pi(8 - \ln 81)^2$ as the requested volume.

5) There were many successful attempts at the inequality in part (i). The method of squaring both sides of the given inequality was the approach seen from the majority of candidates and, usually, this was concluded accurately and concisely. A few candidates stopped as soon as they had determined the two critical values –4 and $\frac{2}{3}$ and there was a significant number of avoidable slips in the simplification and factorisation. The alternative method of dealing with two linear equations or inequalities was less successful. Sometimes candidates set up as many as four equations or inequalities for solution and, when sign errors occurred, more than two critical values resulted. Candidates using this alternative method were often unsure how to conclude and there was not much evidence of a graph being used to determine the answer. Often candidates solved two linear inequalities, produced $x \le -4$ and $x \le \frac{2}{3}$ and left it at that. The final mark was only given for a correct, unambiguous answer; an answer such as $x \ge -4, \; x \le \frac{2}{3}$ did not earn the mark.

Although a pleasing number of candidates earned the two marks in part (ii) without difficulty, many other candidates revealed considerable doubt about the nature of a modulus function. Some thought that substitution in $(x + 2)^2$ was needed and others carried out a calculation such as $|-4 + 2| = 4 + 2 = 6$.

6) Many candidates carried out the expected calculations in part (i) with no difficulty and, with a brief reference to a change of sign, earned all three marks. But a considerable number of candidates experienced problems. Some seemed not to know what to do despite the explicit item in the specification for this unit, that candidates should be able to 'locate approximately a root of an equation … by searching for a sign-change.' Some candidates used degrees. Calculation of $\tan^2 x$ was carried out incorrectly by many candidates who calculated either $\tan(x^2)$ or $\tan(\tan x)$. Other candidates seemed determined to make matters more involved by rearranging the equation as $x = \tan^2 x - 2$ or as $x = \tan^{-1} \sqrt{2 + x}$ before substituting the two values; a comment rather more subtle than 'sign change' is then needed as a conclusion. There were even a few candidates who started by replacing $\tan^2 x$ by $1 - \dfrac{2 \tan x}{\tan 2x}$.

Part (ii) was answered very well and the vast majority of candidates earned all four marks. Candidates were expected to show an appreciation of the significance of part (i) by commencing the iteration process at a value between 1.0 and 1.1 (inclusive). Candidates duly showed all the iterates as requested and there were only a few instances of candidates going astray with the iteration formula. A number of candidates lost the final mark by giving the answer as 1.05082 instead of the correct 1.05083.

Part (iii) was more challenging. Candidates were expected to use the identity $\sec^2 2x \equiv 1 + \tan^2 2x$ to establish a link with the equation in part (i). A pleasing number did this and duly halved their answer to part (ii) although, for many, the relationship between the two equations led them to double their answer from part (ii). There was some credit for candidates who set up another iteration formula. There were also many doomed, complicated attempts which tried to use various identities to convert $\sec^2 2x - 2x - 3 = 0$ into a solvable equation; the use of the word 'Deduce …' and the mark allocation of 3 should have served as indications that something more concise was needed.

7) Examiners were delighted at the responses to this question. In many cases, all ten marks were earned as candidates moved systematically and confidently through the various steps needed. There were some slips in the differentiation and integration and a few candidates seemed to be finding a volume rather than an area. The main difficulty encountered by candidates was determining the correct limits to use in the integration. Some integrated between the limits $\frac{1}{3}$ and $\frac{5}{6}$ and believed that gave the area of the shaded region. Others attempted evaluating $\int_{\frac{1}{3}}^{1} [(3x-1)^4 - (96x - 80)]\, dx$.

8) The routine request in part (i) was answered competently by the vast majority of candidates and the only error to occur with any frequency was the use of degrees rather than radians.

Part (ii) was more testing and many candidates struggled to make much progress with either part (a) or part (b). Many realised in part (a) that they needed to solve $3\cos x + 3\sin x = 0$ but by no means all could make further progress. Some solved this equation directly and others used part (i) to switch to $3\sqrt{2}\cos(x - \frac{1}{4}\pi) = 0$. Many succeeded in finding one of the possible values of x but incorrect values such as 0 and $\frac{1}{4}\pi$ occurred not infrequently.

Most candidates struggled to record any marks in the final part. Some carelessly thought that the equation involved was $3\cos 3x + 3\sin 3x = \frac{8}{9}\sqrt{6}$ and others effectively treated the equation as $3\text{T}(x) = \frac{8}{9}\sqrt{6}$. Many candidates attempted a solution which did not use the expression from part (i) and made no progress. Candidates who did adopt the appropriate method generally reached the correct $\cos(3x - \frac{1}{4}\pi) = \frac{1}{2}\sqrt{3}$. Most then chose $3x - \frac{1}{4}\pi = \frac{1}{6}\pi$ whereas relatively few candidates showed admirable awareness of the cosine function by choosing the correct $3x - \frac{1}{4}\pi = -\frac{1}{6}\pi$.

9) Most candidates were able to record at least a couple of marks in answering this question, but the knowledge and understanding needed to provide convincing solutions to the four parts of the question were in many cases rather superficial. In part (i), it seemed that many candidates did not know the definition of range and provided answers such as 'all real values' or '$0 \le x \le 3$'. Other candidates, with an understanding of range but not of the nature of the parabola involved, suggested $y \ge -8$, which was the result of calculating f(x) for different integer values of x. Differentiation to find the stationary point of $y = \text{f}(x)$ and completion of the square were methods seen, but the correct conclusion was not always reached.

There was also a disappointing response to part (ii). Many candidates offered spurious reasons such as 'x appears twice in the equation', 'you can't rearrange to get x', 'it cannot be reflected in $y = x$' or 'it is not linear'. Candidates did earn one mark for a correct statement such as 'f is not one-one' but not many candidates provided evidence related to this particular function.

Most candidates had no difficulty in obtaining $\text{g}^{-1}(x)$ but many then struggled to make any sensible progress with $\frac{1}{a}(x - b) \equiv ax + b$. Some immediately substituted $a = -1$ to confirm the result and partial credit was available in this case. Others embarked on convoluted attempts which seldom succeeded. Candidates seemed generally unaware of the nature of an identity; equating the two constant terms is all that is needed to prove the required result. An elegant alternative method for part (iii) was found by some candidates. This exploits the fact that $y = \text{g}^{-1}(x)$ is a reflection of $y = \text{g}(x)$ in the line $y = x$. A straight line which is unchanged by such a reflection must be perpendicular to $y = x$ (or must be $y = x$ itself except that this is excluded by the requirement that b is non-zero). A line which is perpendicular to $y = x$ must have gradient –1, which confirms the value of a.

The majority of candidates gained the first mark of part (iv) for stating the expression for $\mathrm{gf}(x)$ but further progress was limited. Most candidates proceeded to factorise $4x^2 - 12x + 5$ and ended with an answer involving values $\frac{1}{2}$ and $\frac{5}{2}$. A minority of candidates recognised that the determinant of $-4x^2 + 12x + b - 5$ was relevant but not this entire minority realised that this determinant needed to be negative. An alternative approach was adopted, sometimes successfully, by a few candidates. This uses the answer from part (i) to argue that the greatest value of $-4x^2 + 12x + b$ is $9 + b$, leading to $9 + b < 5$ and the correct set of possible values of b.

4724 Core Mathematics 4

General Comments

This paper contained many straight-forward questions, provided candidates just sat and thought for a moment before attempting any solution. It is possible for a candidate who is very untidy in his/her work to obtain a high mark – but it is rare. What is certainly true is that a candidate producing work in a tidy manner is more likely to do well. Presentation of much of the work was shoddy and it felt as if there was little pride associated with it.

Poor algebra continues to be a problem; many instances are mentioned below.
Most candidates appeared to have sufficient time to complete the paper.

Comments on individual questions

1) This was generally well done. Errors included the use of $3x^2$ and $3x^3$ instead of $(3x)^2$ and $(3x)^3$, and the use of $-\frac{5}{3} \cdot -\frac{7}{3}$ and $-\frac{5}{3} \cdot -\frac{7}{3} \cdot -\frac{9}{3}$ but simplification was gratifyingly accurate.

2) This question prompted a disappointing performance, on the whole; a relatively small proportion of candidates scored full marks. There were many careless sign errors and numerous occasions when the quotient rule was applied incorrectly. In the quotient rule, it was not uncommon to see a denominator of $1 - \sin^2 x$ instead of $(1 - \sin x)^2$; another disappointing feature was the sight of $1 - \sin x$ in the numerator towards the end – but the denominator was expanded, so losing any possibility of cancellation. The use of the quotient rule in a question such as this would seem to be obvious but there was not inconsiderable use of the product rule which proved to be much more awkward in the simplification.

3) There were many good solutions to this with the majority producing the correct format. The use of the cover-up rule has diminished over the years and one can understand this in a question with a repeated factor. The normal identity method proved to be most successful, particularly for those substituting three values of x rather than for those equating coefficients.

4) As substitutions go, this was not very difficult but performance in this was disappointing. Most candidates knew they had to differentiate in order to obtain a relationship between du and dx but the algebraic manipulation often produced errors. There were too many candidates who did not realise that the main feature of such a question was to produce a second integral totally concerned with variable u; the integral $\int x^2 \, du$ was not uncommon with a subsequent $\frac{1}{3}x^3$. The manipulation of $u = \sqrt{x+2}$ to find an expression in u for the numerator x^2 proved a fraught exercise, mainly because candidates tended to rush and gave insufficient respect to their work. Likewise the squaring of $(u^2 - 2)$ often produced the incorrect result of $u^4 - 4$ or $u^4 + 4$ although, needless to say, a significant number avoided this and gave the result of the integration as $\dfrac{(u^2 - 2)^3}{3}$. Most changed the limits to values of u but candidates must realise that the mark given for such a change is only valid provided there is no re-substitution at the end.

5) The implicit differentiation was performed very creditably; there were, of course, cases of " $\frac{dy}{dx} = $ " and omissions of " $= 0$ " on the first line but, in general, things settled down and most produced the result $x = -2y$ at the halfway stage. Substitution of this into the given equation proceeded reasonably well but it was amazing how often the results $y = 3, x = -6$ and $y = -3, x = 6$ were summed up as $(3,-6)$ and $(-3,6)$.

6) This was the most successful question from the point of view of candidates and even less able candidates were able to score close to full marks.

   (i) The vast majority knew which vectors to use and the only problem concerned the inability of quite a few to solve the equation $4 + 2a - 6 = 0$.

   (ii)(a) It was pleasing to see so many correct solutions of the three simultaneous equations, although the equation $-7 = 14s$ often produced the solution $s = -2$. A few candidates created their own problems by failing to read the question carefully using $(1,1,0)$ instead of $(0,1,1)$ and/or $(3,-1,0)$ instead of $(3,0,-1)$.

   (ii)(b) This was almost always done in a correct manner.

7 (i) Almost all candidates could quote and use correctly $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ but the differentiation of the two functions proved surprisingly disappointing. The number of candidates who differentiated $\frac{2}{t+3}$ and gave the result as $2\ln(t+3)$ was vastly more than had been expected. The differentiation of $\frac{t+2}{t+1}$ started well but the simplification of the numerator was abysmal; it was often shown as $t + 1 - t + 2$ and simplified to 3, and this was provided the derivatives of $t + 2$ and $t + 1$ were both 1 (it may sound surprising but the number of times these derivatives were t was not inconsiderable). Cancellation of $t + 1$ in both numerator and denominator of $\frac{t+1-(t+2)}{(t+1)^2}$ also occurred frequently. Consequently the number of candidates who progressed correctly to the end of this part was very low.

   (ii) This part was tackled with rather more success, the only real difficulty coming from the algebraic manipulation to eliminate all fractions.

8) (i) This part proved very successful, particularly for the candidates who used straightforward long division. The meaning of the word "remainder" was often not clear – was the remainder 2 or $\frac{2}{x-1}$? This aspect was to be tested in part (ii) and so no penalty was exacted for incorrect use in part (i).

   (ii)(a) Although the separation of variables here was not particularly awkward, surprisingly few were successful. In particular, many did not seem to realise that it was necessary to transform the equation so that dx and dy were in the numerators of any fractions. It was certainly not uncommon to see $\int \frac{y-5}{dy}$.

It had been hoped that part (i) would have been thought of as $\dfrac{x^2 - 5x + 6}{x - 1}$

and that, therefore, its connection with part (ii) would have been recognised.

(ii)(b)   Although relatively few candidates obtained the correct result to part (a), the majority did produce an answer – and, provided their equation contained an arbitrary constant, they were able to gain credit here.

9   There were various elements in part (i); the squaring procedure (if candidates even realised that that was the only way forward), whether or not the cos $2x$ should be changed, whether $(\cos 2x)^2$ should be $\cos^2 2x$ or $\cos 4x^2$ and then the integration of two dissimilar functions $2x \cos 2x$ and $\cos^2 2x$. A number integrated straightaway giving $\dfrac{(x + \cos 2x)^3}{3(1 - 2\sin 2x)}$ and, unfortunately for those candidates, that was the end – although marks were still available in part (ii).

A few changed $x + \cos 2x$ into $x + 2\cos^2 x - 1$ and then squared, often quite accurately; however, the subsequent integration of e.g. $4\cos^4 x$ perhaps ought to have made them wonder if this manipulation had been worthwhile.

Those expanding the integrand, who remembered that there should be a middle term, generally dealt with the integration of $2x \cos 2x$ by parts successfully.

The changing of $\cos^2 2x$ into $\frac{1}{2}(1 + \cos 4x)$ was generally well done and so was its subsequent integration.

However, there were so many places in part (i) where mistakes could intrude, that relatively few scored the full 9 marks – but there were some very good attempts. In part (ii), the vast majority of candidates scored marks provided they could indicate clearly what they should be doing and how it connected to the first part.

4725 Further Pure Mathematics 1

General Comments

This paper proved to be reasonably straightforward for a good proportion of the candidates. Correct solutions to all questions were seen and most candidates showed that they had a sound grasp of a significant amount of the syllabus. There was little evidence that candidates were short of time and most answered the questions sequentially. A significant minority of candidates did produce excessively long solutions to some of the questions.

Comments on individual questions

1) A significant number of candidates did not use Induction, as asked in the question, and used the standard results for $\sum r^2$ and $\sum r$. There was confusion as to the next term in the series, many adding $(k+1)$ rather than $(k+1)(k+2)$. Many candidates did not give a clear conclusion for the induction process.

2) (i) Many candidates did not appreciate that the product **AB** is a 1 x 1 matrix, with 1 x 2 or 2 x 2 matrices often seen. Those who obtained 1 element often omitted the matrix brackets.

   (ii) Most candidates found 4**C** correctly, but many did not obtain a 2 x 2 matrix for **BA**.

3) The most common error was to give $\sum 1 = 1$ not n. Those who had a correct unsimplified expression often expanded completely, rather than factorising straight away. Many missed the final factorisation of $4n^2-1$.

4) (i) This was generally answered well. Errors in finding $a - 2b$ meant that candidates often obtained 5, rather than 5 + 12i.

   (ii) The method was generally well known and most candidates obtained the correct answer.

5) (a) Most candidates obtained the correct answer, but some gave the unit matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

   (b) Both parts were answered well, the main error being the incorrect direction of the rotation in part (ii).

6) (i) Most recognised that (a) was a circle, but frequently it did not pass through the origin. In (b) fewer recognised the perpendicular bisector, or failed to see that it passed through the centre of the circle.

   (ii) Most realised that the inside of the circle was required to be shaded, but fewer recognised that the second inequality led to the right of a vertical line being shaded.

7) Most candidates used the sum and product of the roots approach, although both other solution methods were seen, with varying degrees of success. This question was generally well done, the most common error was giving the final answer as $x^2 - 4kx + 4k$, rather than as an equation, as requested.

8) (i) Most candidates obtained the given answer and showed sufficient working to justify their answer.

    (ii) Most showed how the terms would cancel, but some candidates failed to see that two terms remained at each end of the series.

    (iii) Many did not recognise that the series diverged, and those that did found it difficult to explain clearly why it was diverging.

9) (i) Most candidates knew the correct method for finding the determinant of a 3 x 3 matrix, but many made sign or algebraic slips.

    (ii) (a) Many candidates did not understand that when det **A** = 0, the only method for finding consistency or inconsistency is to start to solve the equations. In this part a pair of inconsistent equations is then found.

        (b) As in part (a), the solution of the equations leads to a repeated equation, which shows the non-uniqueness of the solutions.

        (c) Many solved to find the unique solution, which was not required. A statement that the determinant is non-zero means that there is a unique solution was all that was required.

10) (i) Most candidates produced an algebraic method for this part, but many failed to take account of the restriction on the argument and gave the answers as $\pm(2+i)$.

    (ii) This part was generally answered accurately.

    (iii) Most candidates found $w^2 = 2 \pm 11i$, but then could not connect finding w with the work done in parts (i) and (ii) and the use of the conjugate.

4726 Further Pure Mathematics 2

General Comments

Candidates answered the questions in the order set and were able to pick up marks in each question. The early questions gave most candidates a good way into the paper. There were few poor scripts and candidates in general appeared to have covered all the specification. There was some evidence of a lack of time towards the end of the paper, but this was due to overlong and cumbersome methods and a lack of precision. Candidates were not helped when they did not read questions well enough nor use earlier answers in later parts of the same question. These points are included in some of the comments below relating to individual questions. There were relatively few outstanding scripts although a good number of candidates knew the basic work well enough to gain reasonable marks.

Comments on individual questions

1) This question provided a good start for most candidates, with the majority gaining at least three marks. The most common error was the omission of the factor 2 in $f'(x)$. It was expected that candidates would write down the answers to $f'(x)$ and $g'(x)$ at once, insert $x = \frac{1}{2}$ and solve for p, taking about four lines to do so. It was surprising how long it took some candidates, particularly those who found $f'(x)$ and $g'(x)$ from scratch.

2) Again, time was wasted by many candidates, who used differentiation to derive the Maclaurin series, particularly by those candidates who multiplied out the bracket as a first step. Other candidates expanded both e^{2x} and $\sin x$ beyond the x^2 terms and then used all these terms when multiplying out. Candidates who used the hint in the question and who did not go beyond the x^2 terms gained the expansion in two lines. However, most candidates knew the theory well enough to gain good marks and it was often only careless errors such as $(ax)^2 = ax^2$ which caused marks to be lost.

3) The t-substitution was well-known to most of the candidates. A significant minority derived dx in terms of t and dt, but most candidates sensibly quoted the results for dx and $\sin x$. The correct integral was achieved by the vast majority of candidates. There were some differences in then using $(t - 1)^2$ or $(1 - t)^2$ or in using $\tan(\frac{1}{6}\pi) = \sqrt{3}/3$ or $1/\sqrt{3}$, but most candidates coped with the choice they made. However, the question involved a given answer which therefore had to be obtained carefully. Many candidates lost one or two marks by merely producing this answer with little or no working. Overall this question was much better attempted than similar questions on previous papers.

4) (i) Again, it often took a page of working to produce a, b and c. The values of a and c could be written down at once from the asymptotes, although candidates who divided out also showed a good appreciation of what was required. The value of b generally took more time. However, the majority of candidates scored well on this part of the question.

   (ii) Most candidates produced a reasonable y^2 curve, but many candidates missed important details and lost marks. The crossing of the x-axis at 90º was the point missed most often, but there were careless errors in the crossing-points and the asymptote. Again the candidates did not read the question well enough and coordinates of crossing-points and the equation of the asymptote were seen rarely. In this case, marks were not lost if, for example, $\pm\sqrt{6}$ were marked on the y-axis

and 3 on the x-axis or -1 was labelled somewhere on the only asymptote. A significant number of candidates had other asymptotes such as $x = 3$ or $y = \pm\sqrt{2}$. Some candidates appeared to sketch the y^2 curve from a theoretical point of view rather than looking at the y^2 equation that they had derived.

5) (i) The derivation and use of a fairly basic reduction formula was done well by most candidates. Again, with an answer given, it was important to show its full derivation and not to have any step seen as "obvious".

   (ii) It was surprising how many candidates attempted to write I_3 in terms of I_0 (or I_1) before deriving the basic I_0 (or I_1). Candidates who firstly found I_0 and built up line-by-line via I_1 to I_2 and then to I_3 were generally quicker and more accurate. Again, candidates opting for I_1 rather than I_0 caused themselves some problems in having to do a longer and harder integral. It may well be that such candidates did not appreciate that I_0 was possible, the result of not reading the question with sufficient care.

6) (i) Most candidates gained both marks in this part of the question. Because it was worth only two marks, no marks were lost by not justifying in some way why the positive root in $\cosh y = \pm\sqrt{(1 + \sinh^2 y)}$ was taken or by justifying this wrongly. A number of candidates wrote $\sinh^{-1}(x)$ as a logarithmic function and attempted to find the derivative using this form. Few were successful, but the method was acceptable. Again, with an answer given, it was required to show that $\cosh y = \sqrt{(1 + x^2)}$ and not just to quote it.

   (ii) Most candidates used the result of part (i) to produce dy/dx successfully. The second derivative proved more of a problem. Even those candidates who used the quotient or product rule often made careless errors which caused problems in the latter part of the question. Most candidates put their y, dy/dx and d^2y/dx^2 into the given equation and attempted to cancel terms to get to zero. A good number of candidates were successful. There were very few thoughtful answers such as noting that the use of the quotient rule produced $(x^2 + 1)$ in the denominator. This led to an obvious $(x^2 + 1)d^2y/dx^2$ equation, the right-hand side of which could be rewritten in terms of y and dy/dx at once.
   Some candidates used $\cosh^{-1}y = a\sinh^{-1}x$ with implicit differentiation. However, no candidate considered tidying up. For example, from this rewriting, getting $1/\sqrt{(y^2 - 1)} \, dy/dx = a/\sqrt{(x^2 + 1)}$ and then expressing this as $(x^2 + 1)(dy/dx)^2 = a^2(y^2 - 1)$ before attempting to differentiate again.

7) (i) Most candidates successfully completed this part of the question, although some lost marks by not giving enough iterations to a reasonable degree of accuracy.

   (ii) Most candidates produced a staircase diagram, usually between α and β. Statements then often followed based on this staircase alone, for example that "the staircase always goes towards β". This showed a lack of appreciation as to what could happen. It was expected that candidates would consider each of the regions defined by $x < \alpha$, $\alpha < x < \beta$ and $x > \beta$ and describe the divergence or convergence in each. This rarely occurred. Candidates who discussed staircases in general terms of gradients and did not relate it to the question set gained few marks.

(iii) This part of the question was often completed successfully. A few candidates used the original $y = 2\ln(3x - 2)$ or $\frac{1}{3}(e^{\frac{1}{2}x} + 2)$ for f(x) in the Newton-Raphson part, but most showed a good understanding of the process, even if not always working to an appropriate degree of accuracy.

(iv) This part of the question was answered badly, with many candidates discussing the general case, rather than the specific case given. There was a general lack of appreciation as to possible problems inherent in using Newton-Raphson and as to why the specific value of x was given. Many candidates related it to the diagram given in the question. Even those candidates who showed f '(ln 36) = 0 were not always precise in describing the problem therein, often discussing problems using the formula (which was acceptable if discussed reasonably) rather than the problems of using a tangent at a stationary point.

8) (i) This part of the question was well answered, although it was disappointing to see many candidates unable to cube $\frac{1}{2}(e^x + e^{-x})$ without recourse to brackets. Candidates multiplying out did not always tidy up until the end and, although usually accurate, they lost time in using this method.

(ii) Most candidates realised the connection with part (i) and produced $\cosh 3x = \frac{13}{5}$. A surprising number of candidates could go no further. Those who reverted to the exponential definition were often successful, although a significant minority could not set up the basic quadratic equation in e^{3x}, whilst others stated that $(e^{3x})^2 = e^{9x}$. The candidates who used the logarithmic equivalent often found x quickly. Very few candidates then went on to answer the question set, apparently not appreciating that the question involved an equation in u, not x.

9) (i) Although many candidates gained good marks on this part of the question, it was disappointing to see errors in such basic work. Marks were thrown away by an inability to integrate $(2x+1)^{\frac{1}{2}}$ correctly or using the wrong limits or by splitting the required area into two parts involving negative and positive limits of x. Again time was lost by using integration rather than a basic triangle to find the area to be subtracted.

(ii) This part of the question was answered badly, with many candidates not getting beyond $r\sin\theta = \sqrt{(2r\cos\theta + 1)}$. Even the candidates who squared this rarely spotted that it could be written as a quadratic in r (or $\cos\theta$). There were better methods available, but the basic use of the quadratic formula produced the required answer. Candidates who used $y^2 = 2x+1$ to produce $x^2 + y^2 = x^2 + 2x + 1 = (x+1)^2$ and hence $r = x+1$ are to be commended. Others who replaced the left-hand side as $r^2(1-\cos^2\theta)$ were able to spot a perfect square and to end up in the same place. Surprisingly few candidates started with the polar equation and worked back to the given cartesian equation. Fewer still realised that an explanation of the limits of θ was required.

(iii) This part of the question was often omitted. Candidates who quoted the correct formula, using the given r, for the area in polar form gained a mark, but very few candidates could use the half-angle formula to produce an equivalent answer involving $\operatorname{cosec}^4(\frac{1}{2}\theta)$. The word "deduce" was largely ignored and there were some interesting if nonsensical attempts to integrate $\operatorname{cosec}^4(\frac{1}{2}\theta)$ directly.

4727 Further Pure Mathematics 3

General Comments

This paper was done well by a large proportion of the candidates, who found much that was familiar, but who also managed well with some of the less standard parts. The questions which were answered best were Q 1, 2, 4, and 6; the most demanding parts were Q 5 (ii) and 7 (iii). There did not appear to be any problems with the length of the paper, and nearly all candidates reached the end. There is still a significant number of candidates whose presentation could be considerably improved, which would help both themselves and the examiners. The practice of submitting several attempts at a question, without deleting any or indicating which was to be marked, seemed to be more common this year. In such cases examiners are instructed to mark what appears to be the last full attempt, even if this results in fewer marks than an earlier answer.

Comments on individual questions

1) This was a standard vector problem which was answered well, with more than half the candidates obtaining full marks. After the calculation of the vector perpendicular to both lines, most candidates used Method 1 of the mark scheme. Method 2, using the distance between two parallel planes, was seen from a few centres, and the rather lengthy Method 3 was seen occasionally. In Method 1 most of the errors were numerical, including miscopying of candidates' own figures. An error was made by those who "simplified" the vector between the lines by removing a factor of 2. There were, however, a number who did not know how to use their normal vector to find the required distance.

2) (i) The first part of this group algebra question was done correctly by almost all candidates.

(ii) The more general result, that $r^n a r^n = a$ for all n, allowed candidates to use induction or to devise their own methods of proof. Those who tried induction were generally successful, quoting the starting point from part (i) and using appropriate group algebra to make the step from n to $n + 1$. A suitable form of words was required at the end to show how the proof was completed. In many answers mathematical precision was lacking: for example, it was often unclear what assumption was being made and how it was used in the induction step. Quite a number of candidates produced good answers by other methods. In particular, examiners were impressed by those who started from $r^n a r^n$ and reduced n by stages to reach $rar = a$ quite easily. Some answers mixed up recursive methods with induction, and it was not always clear how the argument progressed. Attempts which used commutativity (except for powers of r) or other incorrect group algebra were not awarded any marks for that part of the answer.

3) (i) In the first part of this question about a fifth root of unity, most answers scored only 3 out of 4 marks, as the argument of w^* was not changed to $\frac{8}{5}\pi$. It was most surprising to find that a number of candidates seemed to be unaware of the notation for the complex conjugate, for they thought that * was some sort of variable.

(ii) The most common diagram for this part consisted of a regular pentagon with its centre at the origin: this was not what was asked for, and it scored no marks. Not many diagrams even had the point *B* in the right place, and some of those that did then went in a spiral shape or continued outwards in the first quadrant. The better candidates, and those who used their calculators sensibly, drew a regular pentagon in the right place. Occasionally the first four points were right, but *E* was not at the origin.

(iii) The intention of this part was to test candidates' knowledge that *w* was a fifth root of unity, and the equation $z^5 - 1 = 0$ or equivalent was expected. In general only the better candidates scored the mark here, with many not offering any answer. Correct alternatives of degree 5 were accepted, provided that an equation and not just an expression was given.

4) (i) Most candidates started by differentiating the given substitution correctly, but those who did not were unable to make any worthwhile progress. Careless removal of brackets sometimes led to an incorrect form for the separable equation, but many were successful up to the integration to logarithmic form. It was then very common, when the removal of the logarithms was attempted, for the arbitrary constant to remain as $+ c$, rather than becoming a multiplicative term. Those who used the tan form, rather than "sec + tan", sometimes ran into trouble when they tried to rearrange to make *y* the subject, which in any case was not required. Very occasionally a valid solution of the equation in *z* and *x* was seen using the integrating factor method.

(ii) Most answers scored the mark for putting the given values into their general solution, even if it was wrong. But quite a number gave only the value of their constant, without stating the solution as requested.

5) (i) The first part of this complex number question was often done well, with very little working being needed. It was necessary to see the series expressed in exponential form before the sum of the infinite G.P. was used, and answers which omitted this were not awarded the first two marks. Most then wrote down the sum easily and multiplied top and bottom by 2 to obtain the given answer. Many candidates were careless about the detail of showing that it was an infinite series, giving just the first three or four terms without ". . .", but this was not penalised. However, a significant minority of candidates did not realise how to find the sum of the exponential series, and a few found the sum of only 4 or *n* terms.

(ii) Although most candidates realised that they had to find the real and imaginary parts of the expression in part (i), few used the correct complex conjugate and therefore failed to obtain the expressions for *C* and *S*. Typical attempts often used $\dfrac{2 + e^{-i\theta}}{2 + e^{-i\theta}}$ or $\dfrac{2 + e^{i\theta}}{2 + e^{i\theta}}$. Some of those who used the correct $\dfrac{2 - e^{-i\theta}}{2 - e^{-i\theta}}$ were careless about brackets when they reverted to cos and sin; if this gave *C* correctly the error was not penalised. The use of an incorrect complex conjugate should have meant that equating real and imaginary parts was impossible, but many such attempts arrived at the given expression for *C* by completely invalid working which scored no marks. Many of those who did find *C* correctly did not find *S*, either omitting it, or including an i or a 4 on the top line, or just giving the numerator of the fraction.

6) (i)   This question was found to be straightforward, and many answers scored full marks. The most common errors in finding the complementary function were to give the roots of the auxiliary equation as $-1 \pm 8\mathrm{i}$, to write the C.F. as $\mathrm{e}^{-1}(\ldots)$, to leave the C.F. in complex form, or to omit the constants. Careless algebra sometimes led to an incorrect particular integral.

   (ii)   A correct linear equation was often given, with follow-through being allowed provided it was relevant. The essential part of the reason for the approximation to a linear function was that $\mathrm{e}^{-x} \to 0$, but it was good to see a few candidates stating also that the trigonometrical part of the solution was bounded. Those who had left their solution in complex form often had difficulty in dealing with the large value of *x*. Quite a number of answers omitted the reason altogether.

7) (i)   The equation of the plane was often found correctly, although a few gave only the cartesian form. Errors in the calculation of the vector product were sometimes seen.

   (ii)   All five methods shown in the mark scheme for finding the distance between the line and the plane were used by candidates. The simplest was probably Method 2, although here a sign error was sometimes made with the term 23. Method 5, using the distance between parallel planes, was seen from some centres. For those who were not really sure what to do, an attempt at something like Method 3 was often tried. Typical mistakes here included the use of a vector product, an assumption that the perpendicular from the line *l* to the plane passed through the point (1, 3, 5), and an attempt to use the formula for the distance between two skew lines.

   (iii)   Correct answers for the equation of the reflected line were seen only from the best candidates. Those who had used Method 1 in part (ii) were the most successful, as they needed only to double the value of *k* that they had previously found. From others, many answers scored only the mark for giving the correct direction of the line, but quite a number also gained a method mark for an attempt to double the distance found in part (ii). The main problem here was that terms involving $\sqrt{21}$ remained in the answer, or that a doubled length was multiplied by the wrong vector. An implication that the reflected line passed through the point (7, 3, 0) was seen quite frequently.

8) (i)   The majority of answers gave the subgroup {*A, D*}, but {*A, E*} and {*A, F*} are also correct. A few misread the question and gave instead *D, E* and *F* as the elements of order 2.

   (ii)   Many candidates scored the mark for a correct statement derived from Lagrange's theorem about 5 not being a factor 6, or that elements can be only of order 1, 2, 3 or 6. But not many also stated that *A* was the identity: this may have been obvious to candidates, but it was an essential part of the answer.

(iii)   Many answers found *BE* and *EB* correctly, but did not explicitly show how the closure property was satisfied: it was necessary to state that the resulting elements *D* and *F* were in the set *M*. In some answers it was evident that there was confusion between the terms closure and commutativity. As a clear request to find *BE* and *EB* was made, the method mark was not awarded until the terms in ω had been simplified using $\omega^3 = 1$, and a minority of candidates did not do this. For the award of the accuracy marks final answers of *D* and *F*, or the equivalent matrices, had to be reached. A few answers clearly multiplied *B* and *E* in opposite orders, perhaps because the products were associated with matrix transformations, and others assumed that *EB* was the same as *BE*.

(iv)   Although correct answers were often seen for the inverses, it was surprising that so many set up and solved equations from $\begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and similarly for the inverse of *E*, rather than using the standard procedure for 2×2 matrices. As in part (iii) answers had to be simplified to *C* and *E* or the actual matrices. A small number of candidates found the inverses by writing the inverses of the non-zero elements: this "worked" for the inverse of *B*, but not for *E*, and credit was not given for either if it was clear that this had been done.

(v)   Most candidates tried to show their knowledge of isomorphism in a sensible way, but sufficient detail was not always given. The most popular approaches were by orders of the elements or by consideration of the self-inverse elements. A fair amount of work had to be done to ensure that the elements of the two groups did have different orders or that there were different numbers of self-inverse elements. Some claimed that *N* was cyclic, usually justifying this by its having 2 or 5 as a generator, but it was less obvious that *M* was not cyclic. It was surprising that very few used the non-commutativity in group *M* which had been implied by the answer to part (iii); a statement that group *N* was commutative was then all that was needed to establish that the groups were not isomorphic. A few correctly used the non-commutativity of *M* as their justification for *M* not being cyclic.

Chief Examiner's Report - Mechanics

General Comments

Across all four modules, much good work was seen. In general candidates were well prepared, and able to demonstrate the knowledge and understanding acquired in their study of Mechanics.

All modules require candidates to apply mathematics learnt earlier, and some difficulty was encountered with this. In M2 and M3 degrees were used where radians were appropriate, and in M4 there was confusion about radians and revolutions. In M1 and M3 marks were lost by many candidates who disregarded the relevance of the arbitrary constant in integration.

However the most general problem seemed to be the accuracy for a numerical final answer. In a branch of mathematics tied to physical reality, the values for numerical answers should convey appropriate physical information, most often by a number quoted to three significant figures. Many candidates incorrectly regard an initial zero as a significant figure, but this would lead to a distance of 433 m being marked wrong if written as 430 m, but right if written as 0.43 km. Values which are recurring decimals may be left in recurring form, provided this is apparent in the script.

Numerical values which are expressed as fractions should have positive integer numerator and denominator, and the denominator as the larger number. Surd values do not usually convey an indication of magnitude and should be followed by their decimal equivalent. Candidates should also be aware that their answers can only be correct to three significant figures if earlier working is more accurate than this.

Examiners take care to ensure that no candidate is over-penalised by this aspect of their work. However, candidates should be aware of the rubric, and know how to apply it.

The comments above apply to answers which are numeric. An answer which is an algebraic expression is usually treated differently as it is not intended to convey a numerical fact but a general relationship. In such cases, surds and "top heavy" fractions are acceptable.

4728 Mechanics 1

General Comments

Candidates were in general well prepared for the demands of the paper, with very high marks being scored on the first four questions. Indeed it seemed often the case that marks were more likely lost through misunderstanding how to give three significant figure answers than any other cause. The later questions, where less guidance was given on how to proceed with a solution, caused difficulty through a weakness in appreciating how to apply their knowledge, rather than a lack of knowledge itself.

This was the first of the mechanics suite of papers to be marked online. This is a positive step in the maintaining of the level of service to candidates. Students must be warned that over-writing an area from which work has been erased gives the text a speckled background against which work may be impossible to read.

Comments on individual questions

1)   (i)   Most candidates began the paper with a good solution and examples of a positive t value arising from $5 + 1.2t = 0$ were rare. The only common error was to truncate the decimal value of time to 4.16 s.

     (ii)   The distance calculation can be done independently of (i), so there was no need for a candidate making an error in (i) to lose any mark in (ii). All the standard *suvat* formulae for *s* were seen, and used with equal success.

    (iii)   It was relatively common for a candidate to use a negative term (almost always for *F*) in the equation for μ. This would lead to a maximum mark of 2/4. Weaker candidates were unsettled to need to create a Newton's Second Law equation in which there was a single force terms, and tended to improvise a second.

2)   (i)   Very few candidates encountered significant difficulty with this question. Treating momentum as a scalar quantity or one which incorporates *g* were rarely seen errors. The most common mistake was regarding 0.43 s as being correct to 3 significant figures.

     (ii)   Though a minority of scripts showed a distance calculation based on the values found in (i), nearly all candidates began correctly by finding the velocity of *Q* after the collision. In a number of scripts this was needlessly rounded to 0.57, which led to an inaccurate answer of 2.01 m for the separation of *P* and *Q* 3 s after their collision.

            A minority of candidates did not appreciate in which directions the particles would be moving after the collision, creating a situation in which *P* and *Q* have apparently passed through each other. There were also several instances of a solution with *P* and *Q* moving with the same speed, a question in the June 2008 paper.

            One unexpected feature of the distance calculation was the significant proportion of candidates not having the particles travelling at *constant* speeds after their collision.

3) (i)   Though some candidates tried to use an obtuse angle to find the appropriate component of the 5 N force, most selected a calculation with an acute angle.

   (ii)   Again the use of an obtuse angle was seen, and, without a printed answer as a guide, errors were often made, most often through the use of the "other" trigonometric ratio. Candidates were more confident when first finding an acute angle.

   (iii) Almost all scripts showed a valid and accurate method for the magnitude of the resultant force. The bearing was frequently not attempted by candidates who had found a preliminary acute angle value associated with the direction of the resultant. Any candidate who used 3 significant figure values for both the answer in (ii) and the resultant (eg $\sin\theta = 7.67/10.1$) could not obtain the bearing to a sufficient degree of accuracy.

4) (i)   Candidates correctly interpreted the phrase "at instantaneous rest" and restricted their answer to the positive value of t.

   (ii)   Though a small minority of candidates integrated, or attempted a constant acceleration formula, nearly all candidates tackled the item with complete success.

   (iii) There was a minority of candidates who used a constant acceleration formula. Candidates who integrated usually did so accurately. Common errors were a failure to address the issue of the arbitrary constant, and to use $t = 5$ as the instant of greatest distance. An extreme example of a rounding error was to see $0.2/3$ approximated to 0.07.

5) (i)   The majority of candidates correctly used the "area of a triangle" approach. Those who used $s = (u + v)t/2$ formula with $u = 0$ and $v = (-)3$ were treated as using a concept inappropriate to the context of the question. No penalty however was levied in (ii) or (iii) when their value of $s = 30$ m was used.

   (ii)   Candidates were most successful when using the simple idea of extracting the distance travelled at constant speed and finding the "base time" for an "area" of 18 m. One common misapprehension was taking $t = 20$ as the time when the lorry left the weighbridge.

   (iii) Many correct solutions were seen for this part of the question, with solutions based on distances (with $v^2 = u^2 + 2as$) and times ($v = u + at$) equally popular. A small minority of candidates attempted to use equations for the graph lines, but this method – though theoretically possible – failed in practice.

6) (i)(a) Many candidates adopted the approach of setting up individual Newton's Second law equations for the two objects and were usually successful. A smaller number adopted the wrong notion that $T = 0.55g$ N, and then used this value of T in an equation of motion for the block. A further minority attempted to form an equation of motion for a fictitious 1.4 kg object. Neither of these last two methods could gain full credit.

          Though this topic (connected particles) almost always involves both acceleration and tension, some candidates lost a mark by giving the value of only one, usually the acceleration.

   (i)(b) Very few candidates knew a valid method to employ on this part of the question.

   (ii)   The last part of the question had less explicit structure than usual. The descent of P with the string taut, and the continued motion of B after the string had gone slack offered distinct challenges.

It was common for the descent of P to be assigned $v = 0$, from which the descent time of P was found. This value of t combined with a from (i) "gave" the the speed of B at the instant P hit the ground. The internal inconsistency of this analysis did not occur to candidates.

The free motion of B required a new acceleration (-4.9 ms^{-2}) but in many scripts the value of a from (i) continued in use.

7) (i)   Many scripts showed candidates familiar with this style of question, and full marks were obtained by many. Where partial credit was given, the initial error arose more often in finding the normal reaction rather than friction. When awarding marks for calculating the coefficient of friction, it was imperative candidates substituted a value of the former less than 12 N, and for the latter a value greater than 4 N. In this way they were required to show an understanding of how the forces interact.

(ii)   The new configuration of the problem changes both the normal reaction and the required frictional force, but many candidates understood the nature of this change clearly.

(iii)   Establishing the correct forces on the upper portion of the block was done successfully by many candidates, who usually found the mass of the block though some persisted with the use of weight in their Newton's Second Law equation. It was pleasing to see many instances of the acceleration of the upper part of the block correct.

When analysing the behaviour of the lower part of the block, the effect of the frictional force exerted on it by the upper portion was usually ignored, so missing an application of another of Newton's Laws of Motion. Where this force was included, it was usually given the same direction as the frictional force on the lower face of the 3 N block; a few candidates did however add its effect to the 4 N force. Candidates who did reach this stage in their work realised that the maximum frictional force the horizontal surface could exert on lower part of the block exceeded the sum of the other two forces, so that the block would not accelerate (as distinct from being accelerated by a frictional force).

4729 Mechanics 2

General Comments

The many good scripts seen demonstrated the high level of skill and mathematical understanding of the majority of candidates, only a small percentage of the candidature appeared to be totally unprepared. Candidates should pay attention to the precise requirements of questions and to the use of clear diagrams with components of forces shown where appropriate. As is often the case, poor or non-existent diagrams frequently led to misunderstanding, particularly with Q4 and Q5.
Where the question paper contained given answers, candidates were expected to refer clearly to the underlying physical principles involved, and carry out algebraic and arithmetic manipulations without ambiguity.

Comments on individual questions

1)   This question was usually well answered. Only a minority of candidates attempted to use $v^2 = u^2 + 2as$ with $u = 7$.

2)   (i)   Most candidates used the correct formula to find the centre of mass. The majority of errors arose from using the correct angle, but measured in degrees rather than radians.

       (ii)   Most candidates knew how to find the centre of mass of a composite object. However there was much confusion regarding from which point the centre of mass was being measured. Invariably, measuring from the centre of the circle proved the least successful method. This required the displacement of the centres of the masses to have opposite sign, but examiners usually saw these displacements with the same sign. The most successful method seen was to measure from the circumference of the circle.

3)   (i)   Although well answered by many candidates, there were many instances of using the 60 m s^{-1} instead of the 80 m s^{-1} in finding the constant of proportionality for the air resistance.

       (ii)   This part of the question required the use of three forces in a Newton's 2nd Law equation. It was quite usual to see at least one of these forces omitted, in particular the component of weight or the air resistance found in part (i).

4)   (i)   Use of moments was the method required to successfully answer this part. Examiners reported that there are candidates for whom resolving is a method used with no regard for the force exerted at the hinge.

       (ii)   Candidates were quite successful in calculating the vertical and horizontal components of the required force using their value of T from (i). Although most could find a magnitude of these two components and could calculate a related angle, it was often not clear as to the exact direction of this force. Often examiners saw $58.8°$ to the horizontal or $31.2°$ to the vertical without any indication as to the precise direction. A diagram showing the resultant force would have been useful to aid clarity.

5) (i)   This was the least successfully answered question on the paper. Candidates would be well served by drawing a full force diagram which also included the acceleration and its direction. Many examples were seen of resolving horizontally without the acceleration being included or being in the wrong direction.

   (ii)   This part depended upon the understanding that the normal reaction was 0 for the greatest value of ω. It was common to see attempts which implied the tension in the string was zero.

6) (i)   Many correct solutions were seen to this question with only occasionally candidates giving u in terms of v rather than v in terms of u.

   (ii)   Well answered by the majority.

   (iii)   There were many good solutions to this part seen. However a significant number did not fully understand that a loss or $\frac{3}{4}$ of the kinetic energy meant that there is $\frac{1}{4}$ of the original kinetic energy remaining.

   (iv)   The majority knew that a momentum equation and a restitution equation were required in the solution of this question. However there were a significant number of errors in setting up these equations. Candidates are reminded that a diagram helps them and the examiner understand what they are trying to do. There were instances were the velocities of *A* and *B* were inconsistent between their equations. The directions of motion of the particles would have been clearer with the aid of a diagram or mention of directions relative to the wall.

7) (i)   There were two methods to arrive at the given answer both of which were equally successful in being applied by candidates. A common mistake in the energy approach was to include the gain in potential energy twice. Another significant error was to work with a very crude degree of accuracy. For example, in using the Newton's 2nd Law approach, working with 3 significant figures gave a deceleration of 9.15 m s^{-2} instead of 9.14. This leads to a speed of 5.43 m s^{-1} and not the given answer.

   (ii)   There were many possible approaches in the solution of this question of which examples of all were seen. The most common was to find how far the particle was horizontally from B at the instant that it was at the same level as BC. However the common errors were to see an incorrect angle of projection from B, and calculating the horizontal distance from B at the instant that it was at the same vertical level as B.

4730 Mechanics 3

General Comments

Most candidates seemed well prepared for examination and there were very few with marks fewer than 20. Questions 1, 2 and 3 were very well attempted and almost all candidates made some mark worthy attempt at each of the four later questions.

Among questions 4, 5, 6 and 7 none seemed conspicuously easy or conspicuously difficult to candidates. This generalisation did not apply at the individual level; for candidates who scored a mark close to 60 the majority of the 12 or so marks not scored were those of one particular question. This feature invites the assumption that preparation for examination may have been less thorough in relation to one topic than to others, but not the same topic for all such candidates.

Some candidates seemed to take a cavalier attitude to accuracy; this may arise because the rubric invites answers to a degree of accuracy different from 3 significant figures where this is 'clearly appropriate'. Where answers are given in the question paper candidates should assume, in the absence of an indication to the contrary, that these are exact. Thus when attempting to find the factor 3 in Q4(i), the expression for the tension in Q7(ii) and the value of V^2 in Q7(iv) exact arithmetic is necessary throughout.

Q4(i) and Q7(ii) are examples of structured questions in which an intermediate answer, an expression for $v(t)$ in Q4(i) and an expression for v^2 in Q7(ii), is required en route to a particular goal (factor 3 in Q4(i) and an expression for T in Q7(ii)). In these circumstances a mark is always available in the mark scheme for the required intermediate answer, which should be shown explicitly.

Comments on individual questions

1) Most candidates opted to use the cosine rule, in the triangle of momentum and impulse method, and were generally successful. However a significant minority of candidates found the value of the angle which is supplementary to α (151.9º), in the belief that this is the actual value of α.

   Some candidates found the value of a different angle of the correct triangle, often 53.1 º.

2) Part (i) was very well attempted although some candidates assumed the force at B to act upwards on BC as well as on AB, with the consequence of a wrong answer for the vertical component at C.

   Some candidates had problems with the relevant trigonometric ratios in part (ii), with triangle CMB, where M is the mid-point of AB, having sides in ratios 2: 1: $\sqrt{5}$ arising in some cases.

3) The 2 marks in part (i) were both scored in almost all cases, the exceptions being a few candidates whose answer was 2 for both u and v.

   Candidates scoring full marks in part (ii) usually defined symbols for post-impact quantities in a clear sketch showing the components of velocity in both relevant directions and for both spheres. This was especially useful if the components normal to the line of centres were shown numerically (2 and 0) at the outset. Candidates who applied the principle of conservation of linear momentum and NEL in the direction of the line of

centres usually wrote correct equations and obtained correct answers.

Candidates who used the principle of conservation of energy instead of NEL were more vulnerable to error, firstly because energy is a scalar quantity and secondly because of the need to use logic in decision making when deciding which of 2 solutions to a quadratic equation is the relevant one.

4) Because part (i) refers exclusively to $v(t)$ for $t \geq 12$, a considerable minority of candidates did not see any relevance to the expression for the force applying for $0 \leq t \leq 12$ and ignored it. In most such case candidates used $v = 0$ when $t = 0$ to obtain zero as the constant of integration so that values of $v(12)$ and $v(24)$ were found by the presumably bemused candidate to be -1.92 and +1.92 respectively.

A pleasingly large proportion of candidates who were successful in part (i) scored full marks in part (ii).

5) Full marks were usually scored in part (i) but few candidates scored more than 1 mark in part (ii). The most common wrong answers in part (ii) were 22.9m, 7.62 and 16.05. The first of these answers is calculated from $5.99 \times 16 \div (4\pi/3)$, implicitly assuming that P_2's speed is constant. The second of these answers is P_2's displacement to the left from its starting point, thus ignoring the first complete cycle. The third of these answers derives from using 1.5×5.99 in degrees instead of radians.

In part (ii) candidates differentiating $x(t)$ for the two cases were more successful than those using $v^2 = n^2(a^2 - x^2)$. With both methods using degrees instead of radians was common.

6) A common wrong answer in part (i) was 8000 N, obtained by implicitly assuming the acceleration of the jumper is zero at the lowest point, so that $W = T$.

Although in part (ii) most candidates were aware that the maximum speed occurs where $W = T$, a significant minority thought it occurs where the string ceases to be slack or at the lowest point.

In part (iii) many candidates used $ma = T - W$, thus finding the acceleration of the jumper upwards. The answer $a = 88.2$ ms^{-2} did not score the final mark unless followed by 'deceleration = 88.2 ms^{-2}', thus implying an awareness that the deceleration on arrival at the lowest point is equal to the acceleration on departure. Most candidates however, espoused the spirit of 'during the downward motion' in the question and used $ma = W - T$ obtaining the acceptable answer $a = -88.2$ ms^{-2}.

Surprisingly at this level, there were a few cases of confusion between mass and weight in this question.

7) A significant minority of candidates obtained answers of 9.70 and 0.505 in part (i) by assuming implicitly that P is released from rest and its speed is increased by 6 ms^{-1} immediately before impact with Q.

In part (ii) it seemed that candidates were executing a well understood and well rehearsed procedure. Candidates generally were less certain in part (iii), although there were many correct answers.

Very few candidates were able to show $V^2 = 0.8575$ but many were able to obtain the required greatest height by using the given value of V^2.

4731 Mechanics 4

General Comments

The majority of candidates demonstrated a sound understanding of most of the topics being examined, and there was a lot of confident, correct and well presented work. The paper was found to be slightly more accessible than usual, and the marks were generally high: almost half the candidates scored 60 marks or more (out of 72), and only about 10% scored fewer than 30 marks. The question on relative velocity was found to be the most difficult one.

Comments on individual questions

1) This question, on constant angular deceleration, was answered well, with about two thirds of the candidates scoring full marks. In part (ii), many candidates took two, or even three, steps to obtain the answer, which can be found in just one step. In part (iii), very many found the angle (2000 radians) correctly but did not convert this into revolutions.

2) The methods for finding the centre of mass of a lamina were well understood, and just over half the candidates scored full marks. Most carried out the integrations (including the integration by parts) correctly to find the *x*-coordinate. The *y*-coordinate was sometimes omitted entirely, and a factor $\frac{1}{2}$ was quite often missing. Those who attempted to use strips parallel to the *x*-axis were rarely successful.

3) This question, on angular momentum and kinetic energy, was answered very well indeed, with about 85% of the candidates scoring full marks.

4) About 30% of the candidates scored full marks on this relative velocity question, but this was the only question on which a significant number (about 15% of the candidates) scored no marks at all. Most candidates could not solve the closest approach problem in part (i). Finding the relative velocity in part (ii) was straightforward for those who had the correct velocity triangle in part (i), but it could also be found using the given answer. The techniques required in parts (ii) and (iii) were quite well understood by those who attempted them, and many candidates answered these correctly, even if they had not been able to answer part (i).

5) In part (i), most candidates could derive the moment of inertia of the rod correctly, although the limits of integration caused confusion for some. In part (ii), the use of the work-energy principle to find the angular velocity was generally well understood (although there were a few attempts to use constant acceleration formulae). However, sign errors in the equation were quite common, the change in potential energy was often wrong, and one and a half revolutions was sometimes converted to $\frac{3}{2}\pi$ radians instead of 3π radians. About 40% of the candidates scored full marks on this question.

6) This question, on the energy approach to equilibrium and small oscillations, was quite well answered, and about 40% of the candidates scored full marks. In part (i) most candidates demonstrated the position of stable equilibrium correctly. Some used very complicated methods (for example involving $R\sin(\theta+\alpha)$) to show that $\theta=0$ is a solution of $3\cos\theta+4\sin\theta-3=0$, and some stopped after showing that $V'=0$ and did not consider stability. In part (ii), the kinetic energy was usually found correctly, although some explanations, especially those stating or implying that Q has moment of inertia $3ma^2$, were not convincing. In part (iii), there were some careless errors in differentiating the energy equation (such as losing a factor 2 or missing $\dot{\theta}$ from one of the terms), but the small angle approximations were usually applied correctly to obtain an SHM equation.

7) Most candidates answered parts (i) and (ii) correctly. To find the force acting at the axis in part (iii), most candidates formed appropriate equations of motion, but these often contained sign errors, particularly in the acceleration perpendicular to GA, and some thought that GA made an angle of $45°$ with the vertical. Some preferred to work with horizontal and vertical components, which made the working much more complicated but was sometimes completed successfully. About one third of the candidates scored full marks on this question.

Chief Examiner's Report – Statistics

General Comments

It is pleasing to report that the standard of work in all statistics units continues to be high, despite the increase in numbers taking these units. Nevertheless there are specific points to be noted. Once again the attention of Centres is drawn to the advisability of teaching how to use the formula booklet properly.

Many Centres have noted the requirement for conclusions to hypothesis tests to be given not only in context (mentioning what it is that the random variable is representing in the real-life scenario described) but also with some acknowledgement of the uncertainty involved. Thus "No increase in the mean time" does not score full marks, compared with "there is insufficient evidence of an increase in the mean time". But it should also be noted that "There is significant evidence that there has been no increase in the mean time" is incorrect; evidence never establishes that the null hypothesis is correct. "There is insufficient evidence that there has been an increase in the mean time" is to be preferred. Examiners have noticed that many candidates give the conclusion to one hypothesis test question carefully and then lose marks by failing to do so in another question, suggesting that it is laziness rather than lack of knowledge that is losing them marks.

Many candidates write poor mathematical grammar, for instance with wrong or meaningless use of the = sign. Incorrect mathematical syntax is usually a sign of muddled thinking.

4732 Probability & Statistics 1

General Comments

Because the paper was marked on line this year, candidates were required to answer in the answer book with spaces allocated for each part-question. It was pleasing to note that very few candidates answered questions in the wrong space. Centres should note that if candidates run out of space for a particular answer, they should ask for extra sheets which should then be attached at the back of the answer book. Then they should indicate in the normal space for the particular question that there is work on additional sheets.

Many candidates showed a reasonable understanding of a good proportion of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. There were several questions that required an interpretation to be given in words, and these were generally answered fairly well.

This year again, very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases, a mark was lost through an incorrectly rounded answer without any previous answer having being shown.

There were no questions that made a significant call upon candidates' knowledge of Pure Mathematics. However, questions 6, 7 and 8 required some clear, logical thinking and many candidates found these difficult.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 2(ii), 3(ii) and 4(for binomial tables). However, as usual a few candidates appeared to be unaware of the existence of MF1. Other candidates tried to use the given formulae, but clearly did not understand how to do so properly (eg Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ in question 2(ii)). In question 3(ii) a few candidates quoted their own (usually incorrect) formulae for b, rather than using the one in MF1. Some thought that, eg, $S_{xy} = \Sigma xy$. Some candidates used the less convenient version, $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1, but most of these completely misunderstood this formula, interpreting it as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$. Some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Others did not know how to use the tables to handle P($X = 2$).

Responses to question 4 gave evidence that many students (understandably!) prefer to use the binomial formula rather than the tables. In this particular question there was no problem, but questions are sometimes set for which the use of the formula would be extremely long, giving plenty of scope for minor errors, whereas use of the tables would be almost instantaneous.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were

taught to use exclusively the versions given in MF1 (except in the case of *b*, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on individual questions

1) (i) Many candidates answered this part correctly, with a few giving 585 or 590 divided by 55 or some other number. Some tried to answer without even calculating a French IQR.

   (ii) This part was well answered. Some candidates stated that the curve became "straight", which was not accepted.

   (iii) Most candidates answered this part correctly. A few found $55 - 0 = 55$ or $60 - 14.5 = 45.5$. Others left their answer as, for example '14.5 to 55'.

   (iv) Most candidates understood what was required. A few found the number of students scoring 26 or less (150), but then instead of doubling this number, read off at 27 (180) and added these two.

   (v) Most candidates made a good attempt at the inter-quartile range, but some read the quartiles inaccurately. A few used a total frequency of 700, despite having answered part (i) correctly. The comments were generally acceptable although a few referred to a greater "range" of marks which is wrong. Some candidates thought that IQR measures how well students did.

2) (i) Confusion between the meaning of $r_s = 0$ and $r_s = -1$ was shown by a common incorrect answer of "$r_s = 0$ because the ranks are opposite". A common inadequate answer was "$r_s = -1$ because the judges disagreed totally". Some candidates wrote about "perfect negative correlation" which did not score the mark. A few used the formula to work out the value of r_s. Some gave the value but no reason.

   (ii) This part was well answered. A few used the table from part (i). As usual, a few used an incorrect version of the formula $\frac{1-6\Sigma d^2}{n(n^2-1)}$.

   (iii) Most candidates understood what was required. Various methods were used to find the probability, which was generally correct. Many candidates listed all the possible orders instead of just finding 3!. A common incorrect attempt was $(\frac{1}{3})^3$.

3) (i) The first mark was awarded for dealing with the case where one of the variables is independent or controlled. Many candidates gained this mark for answers such as "If *x* is controlled, use *y* on *x*." But there was evidence of considerable confusion in many answers. For example: "If *x* controls *y*, use *y* on *x*", "If *x* is dependent, use *y* on *x*", "If *x* values are known and they are the dependent factor, use *y* on *x*." A common inadequate attempt to gain the first mark was "It depends on which variable is controlled." Some candidates stated that "It depends on whether the correlation (or the gradient) is positive or negative". Some thought that if *x* is controlled, the *x* on *y* line is appropriate.

   The second mark (which few candidates gained) was awarded for describing what to do if neither variable is controlled. There were two common inadequate attempts to gain this mark. "If you want to estimate *y* from *x*, use the *y* on *x* line and vice versa" (which omits the essential phrase "If neither variable is controlled ...") and "If you want to minimise the sum of squares in the vertical direction, use the *y* on *x* line."

   (ii)(a) Most answers were good, although the usual errors were seen, such as those mentioned above. Some of these incorrect methods led to the correct answer, but

did not score marks. Some candidates found *r* instead of *b*. Some used an incorrect method to find *a*: $14.4 - 0.008 \times 1800$. A few ignored the given answer for *b*.

(ii)(b) Most candidates answered this part correctly. Some substituted $x = 2.5$ instead of $y = 2.5$.

(ii)(c) Most candidates answered this part correctly. Some substituted $y = -50$ instead of $x = -50$.

(ii)(d) Most candidates mentioned extrapolation, thus gaining a mark. Many, however, failed to gain the other mark. Some candidates ignored the request for a comment on the "meaning". Some stated "The extension has decreased." Others stated that a negative result was impossible or that a negative "length" was impossible. "Negative extension" was a common, inadequate answer. Correct answers such as "contraction" or "shrunk" or "length decreased" were seen, however.

Some candidates unsuccessfully mixed the two answers together with "The estimate is unreliable because a negative value is impossible."

4) (i)(a) This part was almost always answered correctly. Some used the formula. A few candidates gave
$1 - 0.1040 = 0.8960$

(i)(b) A few candidates subtracted the wrong pair of values from the table, usually $P(X = 3) - P(X = 2)$. Some candidates treated the values in the table as if they were individual, rather than cumulative, probabilities. A muddled understanding of the tables was shown by the following (not infrequent) method: $0.2991 - 0.1040 - 0.0173$. Many candidates used the formula rather than tables, generally successfully.

(ii)(a) Candidates answered this part well, on the whole. A few used the values from part (i) instead of part (ii). Some used 0.88 instead of 0.78.

(ii)(b) This part was also well answered. Some candidates attempted to use the general formulae Σxp and $\Sigma x^2 p - \mu^2$. These all failed to obtain the correct answers. Strangely, some candidates found $\text{Var}(X)$ correctly, but could not find $E(X)$.

5) (i) A variety of methods were employed, mostly correct.

(ii) Some candidates appeared not to know what is meant by a probability distribution.

For the probabilities, a variety of methods were employed, mostly correct. Some candidates listed pairs of numbers correctly (either 6 or 12 pairs) and derived the probabilities by counting. Others attempted this method but had only 9 pairs and thus obtained some incorrect probabilities. A few candidates used incorrect values for *X*, often 1, 2 and 3. Some used a "with replacement" method, which yields *X* values of 2, 3, 4, 5 and 6. A few candidates tried to apply the geometric distribution. Some candidates did not understand what was required and only gave the values of *X*. Some of these then produced probabilities in the next part but marks were not credited retrospectively. The value of $\frac{1}{12}$ for $P(X = 3)$ was frequently seen and some candidates changed the given probabilities to ensure their answers added up to 1.

(iii) This part was well answered. A few candidates made the usual errors of $(\Sigma xp)/4$ and $(\Sigma x^2 p)/4 - \mu^2$. Most candidates, wisely, did not attempt to use $\Sigma(x - \bar{x})^2 p$. Some attempted to use the formulae for a binomial distribution, usually $B(4, \frac{1}{3})$ or a geometric distribution such as $\text{Geo}(\frac{1}{4})$.

6) (i)   The mean was found correctly by many candidates, but in the variance calculation there was much confusion about whether to use 9 or 10, and what needs to be multiplied or divided by 9 or 10. Many did not appreciate that finding Σx^2 required working backwards from the formula for variance. Some thought that $\Sigma x^2 = (\Sigma x)^2$. Many did not appreciate that 3^2 needed to be added to the original value of Σx^2. Some candidates tried to use formulae involving probabilities. These generally scored no marks. Some candidates tried to use $\frac{\Sigma(x-\bar{x})^2}{n}$ but without success.

7) (i)   Many correct answers were seen. Some candidates added combinations instead of multiplying. Others used permutations.

    (ii)   This part can be answered with no working, but many candidates gave long and complicated methods, such as $1 - \frac{^3C_2}{^4C_2}$ or $\frac{^3C_1 \times ^6C_3 \times ^5C_4}{^4C_2 \times ^6C_3 \times ^5C_4}$ (which are correct) and
$\frac{^4C_1 \times ^6C_3 \times ^5C_4}{^4C_2 \times ^6C_3 \times ^5C_4}$ (which is incorrect). Others listed all the possibilities, often
correctly. Some other incorrect methods were $\frac{1}{4} \times \frac{3}{3}$, $\frac{1}{4} \times \frac{1}{3} \times 2$, $\frac{^4C_1}{^4C_4}$ and $\frac{^4C_1}{^4C_2}$.

    (iii)   Some candidates correctly found the number of combinations including boiled rice without potatoes, but did not find the number without boiled rice. Those who attempted the complement method often found $^4C_1 \times ^6C_3 \times ^4C_3$ instead of $^3C_1 \times ^6C_3 \times ^4C_3$. A few candidates attempted to use probability rather than combinations, but most were unsuccessful.

8) (i)(a) This part was well answered. Some incorrect answers were $0.7^4 \times 0.3$ and 0.7^5.

    (i)(b) $1 - 0.7^3 \times 0.3$ and similar expressions were common. Of more merit, though incorrect, was an answer of $1 - 0.7^4$. Some candidates used the long method and were frequently successful, although some added or omitted a term.

    (ii)(a) Most candidates tried to deal with all five people together, using methods which included $^5C_2$ or $0.7^3 \times 0.3^2$. The latter was often seen without explanation, and was taken to be unworthy of any marks. However, partly correct methods such as $0.3 \times 0.7^3 \times 0.3$ and $0.3 \times 0.7 \times 0.7 \times 0.7 \times 0.3$ earned 2 marks. Some other commonly seen incorrect methods were $0.09(1 - 0.09)^4$ and $0.3 \times 0.7^4 \times 2$.

    (ii)(b) This part was too difficult for many candidates. Amongst those who had some understanding, many found only P(0 in 5) or only P(1 in 5) rather than both. Other incorrect attempts were $0.7^5 \div 2$, $0.7^5 \times 2$, $1 - 0.7^4$, $1 - ^5C_2(0.7)^3(0.3)^2$, $1 - ^5C_1(0.7)^4(0.3)$ and $1 - $ (ii)(a).
A few attempted $(1 - (0.3^2 + 2 \times 0.3^2 \times 0.7 + 3 \times 0.3^2 \times 0.7^2 + 4 \times 0.3^2 \times 0.7^3)$, but most omitted either the coefficients or one term. A very small number used the very simple and correct method of reading $P(X \leq 1)$ directly from the table for B(5, 0.3).

4733 Probability & Statistics 2

General comments

The general standard on this paper was high, with many candidates scoring at least 60 out of 72, and it is pleasing to see that most Centres have responded to previous comments about, for instance, hypothesis tests. Few candidates now fail to give their conclusions to hypothesis tests in a form that acknowledges the uncertainty involved (the wording "there is evidence that" is recommended). However, conclusions stating that "there is significant evidence that the null hypothesis is correct" are in fact logically wrong and should be avoided.

It is stressed that candidates who make structural errors in carrying out significance tests (for instance calculating $P(\leq 11)$ when what is needed is $P(\geq 11)$, or, in the normal distribution, omitting the $\sqrt{n}$ factor or confusing the roles of sample mean and population mean) are liable to lose a lot of marks.

There was much poor mathematical notation, for example the absence of dx in integrals, confusion between z and $\Phi(z)$, and brackets in general.

There was some evidence that confusion between population mean μ and sample mean (often $\bar{x}$ or, as in question 5 here, $\bar{t}$) is widespread. The distinction is central to S2.

Comments on particular questions

1   (i)   For most this was a simple start to the examination. Few failed to use tables appropriately in (a), and only a handful attempted to use tables in (b).

       (ii)   Answers that merely regurgitated slogans ("the disease must occur randomly, independently, singly and at constant average rate") scored no marks unless there was an attempt to explain what these conditions meant in this context. In fact most candidates did make such an attempt. Good answers were "it is infectious so contracting the disease does not occur independently" and "it does not occur at constant average rate as it is more likely at some times of the year" or "when a lot of people have it, it is more likely that others catch it". As usual, the "singly" condition is irrelevant here, and "randomly" should be taken for granted (once again it is emphasised that "randomly" is not a synonym for "independently"). It is not correct to say that some people are more likely to catch the disease than others; that is a binomial condition, not a Poisson one, and in any case it is irrelevant in this context.

           It is pointed out that a scattergun approach to answering this sort of question, where a candidate writes down everything he or she can think of, will generally not score full marks.

2   (i)   The calculation was usually correct.

       (ii)   Few candidates scored either mark here. There are two possible correct answers:
- Use of the binomial condition requires that each choice of student is independent of every other choice, but sampling without replacement negates this; *or*
- it makes little difference as the population is large.

           Many candidates said that sampling without replacement causes "the probability" to change from one trial to another, but this is wrong as it confuses a conditional probability with an absolute ("prior") one. In the present case, even without replacement the (prior) probability that, say, the third student picked is a science student is 0.35, as it is for the first, second, …, tenth. This is a familiar fact from S1 and can easily be demonstrated using

a tree diagram, assuming an appropriate total population size (say 100). Attention has been drawn before to the confusion that exists for many candidates between "trials are independent" and "each trial has the same probability of success", caused by too much emphasis on the example of drawing counters out of a bag.

In fact the binomial distribution applies only to sampling *with* replacement. Strictly, the proper method of calculating probabilities when sampling *without* replacement is the method using ^nC_r from S1. Again suppose the population is of size 100, of whom 35 are studying science subjects. Consider the probability that a sample of 10 students consists of exactly two who are studying science subjects.

- Case 1 (with replacement. Binomial): $^{10}C_2\ 0.35^2\ 0.65^8 = 0.1757$.
- Case 2 (without replacement. ^nC_r): $^{35}C_2 \times\ ^{65}C_8\ /\ ^{100}C_{10} = 0.1735$.

The difference is small, though not non-existent. The bigger the population, the smaller the difference; for a population of size 1000 the second probability is 0.1755. In real life, repeats are usually not allowed, but use of the binomial distribution remains appropriate provided the population is large enough. (There is a technical name for the ^nC_r method; it is called the *hypergeometric distribution*.)

3 (i) This was often well answered. Few failed to find a *z*-value by using the tables back to front and very few instances of incorrect signs were seen.

(ii) This was found rather harder. Many had no difficulty in identifying the distribution as B(90, 0.2) and using a normal approximation, though as always the continuity correction was often wrong or omitted. However, some attempted to use a Poisson approximation, which is not valid, and some misunderstood the question.

4 This routine example of a hypothesis test using the binomial distribution was perhaps better done than in some previous sessions, although too many candidates still attempt to find $P(\leq 11)$ or $P(> 11)$ or $P(= 11)$ instead of $P(\geq 11)$. Such candidates lost a lot of marks. Hypotheses and conclusions were often stated well. On this occasion a normal approximation was valid, but certainly required more effort, not least because it needed a continuity correction.

5 Some (including some otherwise weaker candidates) did all three parts of this question confidently and accurately. Others showed important misunderstandings. The question asked for the critical region in terms of the sample mean $\bar{t}$, but this standard notation seemed to confuse many candidates who gave answers involving both $\bar{t}$ and a critical value *c*, for example $c = \bar{t} + 2.6$. It was hard to pinpoint the misunderstanding here, but they seemed to be confusing the sample mean with the population mean. Others found the critical value correctly but could not remember which side of it the critical region was.

Some candidates could not interpret the wording of the question correctly and attempted to find a left-hand tail such as < 27.4.

Many also failed to realise that the same critical value, 32.6, found in part (i)(a) had to be used in the calculations in (i)(b) and (ii) as well. In consequence, many standardisations involving 30 or, in (ii), 35 were seen. The main point in this question is that anything involving the probability of Type II errors requires use of the critical region.

6 (a) Well done, apart from those who used the wrong, or no, continuity correction.

(b)(i)   Too many attempted to answer this in terms of Poisson modelling conditions ("not a constant average rate", etc). The conditions for *approximations* involve parameter values, and all that was needed here was to say that np ($= 196$) is too high.

(ii)   Some answered this well. Others tried to use a normal approximation (not valid as nq is too small) or simply failed to see what was needed. Some correctly identified Po(4) but then reverted to B(200, 0.98) or equivalent. Many found P(< 6) instead of P(≥ 6).

7   A standard example of a hypothesis test using a large sample drawn from a population with unknown variance. Many had learnt this well, but those who omitted the $\sqrt{300}$ factor or confused the roles of 56.8 and 56.95 lost a lot of marks. As in previous Reports, it is emphasised that the value of μ used throughout, both in stating the hypotheses and in doing the calculation, has to be the hypothesised value (here 56.8) and never the sample mean (here 56.95).

8   (i)   Many candidates got this whole question completely right and made it look easy. But others were confused by the absence of an explicit upper limit for the interval and instead of using the correct limits of 1 and ∞ used instead either 0 and 1, or 1 only (in some fudged way). There were many mistakes in integrating x^{-a} and in handling the signs.

(ii)   Surprisingly many forgot to subtract μ^2, or subtracted μ. It was again clear that those who tried to work out the variance using a single formula (one integral minus the square of another one) usually found the expressions too complicated to get right. This method is not recommended. In this example some made life hard for themselves by working throughout in terms of k, or a, and substituting only at the end.

(iii)   The correct answer was seen pleasingly often, though many candidates failed to realise that $\dfrac{a-1}{1-a} = -1$, which either brought them to a halt or, at best, involved them in a lot more algebra. Some fell back on trial and improvement, usually successfully, though logarithms give the right answer quickly.

4734 Probability & Statistics 3

General Comments

The overall standard was high and more candidates were able to answer more of the questions successfully than last year.

Use of a graphical calculator in carrying out significance tests seemed more widespread than hitherto but it was not always stated which test was being applied. Examiners expect sufficient indication of what the calculator is finding.

Although test procedures are often familiar, candidates often give their hypotheses too briefly. In a test of association it is unacceptable to state Ho: no association, H: association without indicating what are associated. In t-tests and z-tests hypotheses should be given symbolically in terms of population parameters. Test conclusions should not be assertive and should indicate uncertainty. If Ho is not rejected then "there is insufficient evidence at the 5% significance level that anxiety is reduced..." is what is expected by Examiners. Some candidates in this situation stated that there is evidence that the level is not reduced. Statisticians look for evidence to reject Ho. Many candidates are aware of these points.

Some centres encourage candidates to draw a curve indicating a critical region and a critical value and this is fine providing the critical value is consistent with the region. This was not always the case.

Some candidates still work with 10SF for most of a calculation and then round to 3SF, which is a waste of time. Others round everything to 3SF which often leads to inaccuracy in later work.

Comments on individual questions

1) This proved to be an easy start to the paper, the mean being usually found correctly but there were some errors in finding the required probability. Many thought $P(\geq 2)=1-P(\leq 2)$. Most were aware that the flaws on a wire needed to be independent of other flaws.

2) There were high scores on this and only a minority used a *z*-test rather than a *t*-test.

3) It was encouraging to find so many who recognised the relevant chi-squared test and realised that Yates' correction was needed. There were, however, many errors in the application of the correction. Part (ii) was easily deduced. Some used a test for difference in proportions but this was not given credit unless it was fully correct.

4) (i) Most candidates are now familiar with the calculation involved with a confidence interval, and this proved to be straightforward.

   (ii) This was not always understood, but many could point to the uncertainty of the number of intervals that contain the mean.

   (iii) Following on from (ii), the relevance of B(4,0.9) should have been apparent but many found difficulty with the calculation.

5) (i) It was expected that this part might discriminate, but in fact, it was found to be straightforward.

(ii)   The goodness of fit test was usually carried out well although many forgot to combine the last two cells.

6)   Some lost a mark by not stating their validity conditions in context. The data should have made it easy to calculate the pooled estimate of variance, but many made heavy weather of it. A significant number misinterpreted the sample means as sample sums. Most candidates were aware that the sample sizes were too small for the Central Limit Theorem to apply in (ii).

7)   Those that could find the variance correctly found little difficulty with this exercise on the sum of normal rvs, but part (ii) was found to be more difficult, as was intended. Two cases were needed, one when the unknown was male and one when the unknown was female. Some used an "average person" but this earned little.

8)   (i)   The main problem was finding the cdf of S. It was recognised that integration was required, but limits were often missing. The procedure for finding the change of variable is becoming better known. The answer was given for the pdf so it could not just be written down without details.

(ii)   Most of the marks could be obtained from the pdf of $g(x)$ although the range was needed.

4735 Probability & Statistics 4

General Comments

There was an entry of similar size to 2009 with candidates coming (mostly) from the same centres. The general performance on the paper was significantly higher, which indicates that the paper was not as challenging.

Questions with the lowest average scores were Q8 (involving P(L|M)), Q3 (Wilcoxon Rank Sum Test) and Q4 (involving the mgf of a chi-square variable).

Comments on individual questions

1) This was found to be very straightforward in both parts, and high scores abounded.

2) It was hoped that the way in which the pgf was expressed would help in the analysis, but some preferred to write it as e^{4t^2-4} . The first part was was usually well done and both methods were seen in the calculation of P(*X*=2).

3) The theory behind the calculation of P(*R*<=17) was known by many but in (ii) few understood the phrase "exact significance level".

4) (i) The handling of the mgf of the chi-squared(*n*) variable was often good when finding E(*Y*) and Var(*Y*). Expansion and differentiation were used equally often.

    (ii) Although most candidates could obtain the mgf of *S* not all recognised the relation with the chi-squared distribution. Of those that did, the number of degrees of freedom was not always stated.

    (iii) Most knew how to use a normal approximation to obtain the required probability, but several candidates applied a spurious continuity correction.

5) Validity assumptions should be given in the context of the question, and several candidates lost a mark because of this in part (i). The normal approximation was recognised as being appropriate and was mostly carried out accurately. Many candidates were unaware that both *P* and *Q* can be approximated by a normal distribution with mean and variance as given in the Formula List, so it was unnecessary to calculate both and use the smaller as test statistic.
Very few candidates did not know what to do if the Wilcoxon test could not be applied. Some, however, referred to it as a Binomial Test.

6) No hints were given to this question but it caused very little difficulty. The last part, involving the independence of *N* and *R* required careful choice of *N* and *R* if a particular case were used. These cases should be used only to show that the variables are not independent.

7) (i) The correct integral for finding $E(X^n)$ was usually found but candidates were not always careful when substituting the limits. E(*X*) was sometimes found independently and not "deduced" as instructed.

    (ii) Many knew how to obtain the variances, which followed from (i).

    (iii) There were some perfect solutions but many fell down in the algebra associated with finding the estimator T_2.

(iv)   Finding $\text{Var}(T_2)$ proved to be straightforward for those who were successful earlier.

8   50% of the candidates scored full marks but, overall, the average score was lowest on the paper.

Chief Examiner's Report – Decision Mathematics

This was the first session at which Decision Mathematics 1 was marked on screen. There were few problems with the use of the printed answer booklets although centres and candidates would be well advised to refer to the Instructions to Candidates printed on the front of the question paper. In particular centres and candidates should note that the answer to each question must be written in the space provided in the Printed Answer Book. Additional paper may be used if necessary, but must be labelled clearly, including the question and part number. Work that had been erased can sometimes still be seen after scanning, so candidates should either make it very obvious when work has been deleted or they should make a new start. This is particularly the case for diagrams.

Candidates often struggle with the questions asking for an explanation, frequently they just repeat the information given in the question. If an argument is required for a general case then describing a specific example will not usually be adequate, whereas if a specific case is being asked about then it will be necessary to refer to features of this instance.

Candidates also need to be careful that they have read the question carefully before launching into an answer to a completely different question (for example, using bubble sort when shuttle sort was asked for) or only giving part of the answer (for example, giving the optimal point but not the optimal value of the objective).

4736 Decision Mathematics 1

General Comments

Many candidates found this paper difficult and some were not able to finish the last part of the last question. However, good candidates performed well and answered the more challenging questions thoughtfully.

Candidates need to write their answers in the correct spaces in the answer book, and if additional sheets are used these should be labelled with the question part number.

Comments on individual questions

1) (i)(a)   This should have been a simple starter, and most of the candidates were able to carry out the first pass successfully. Some showed every individual comparison and swap, which wasted a little time but led to the same eventual result.

                The number of comparisons and swaps used should have been written down rather than just using tallies. Some candidates counted the number of values included in the comparisons instead of counting the actual comparisons.

                Some candidates used shuttle sort instead of bubble sort, or sorted into increasing order rather than decreasing order, or started from the right-hand end of the list instead of the left-hand end.

    (b)   Most candidates were able to write down the correct list, a few gave the first two passes in part (i)(a) but usually these candidates copied out the list for the end of the second pass again.

    (c)   For this list of ten numbers a total of nine passes would have been needed, and hence a further seven passes would have been needed.

    (ii)   Some candidates only showed the part of each list where the shuttling had taken place and a few did not record the results of the first pass.

    (iii)   Some candidates just repeated the information given in the question, and a few claimed that shuttle sort is always more efficient than bubble sort. Several candidates thought that more comparisons were needed with bubble sort, because 'you need a pass with no swaps before you can stop'; however this was not relevant here because bubble sort would not have terminated early.

                There were two aspects to the argument. The first was that although bubble sort can terminate early it would not for this particular list, because the largest value started at the right-hand end of the list and could only move up one place in each pass. The second was that in each pass of shuttle sort we only need to shuttle up as far as the point where no swap is made, for example the value 42 will be compared and swapped with 87 and 75 and will be compared with 31, but then the next pass starts.

    (iv)   Several correct answers were seen, but some candidates just scaled linearly to give a time of 100 seconds and others used a quartic function instead of a quadratic. When an algorithm is stated as having quadratic order, the run time may be regarded as being proportional to the square of the size of the problem. In this case the size of the problem was scaled by a factor of five so the run time should have been scaled by a factor of twenty five to give 500 seconds.

2) (i)   Candidates need to be careful with questions like this and answer the question that has been asked and not some other question. The question asked why no such graph was possible, not why such a graph could not be simply connected. In any graph the sum of the vertex orders must be even, but here the sum would have been 9.

    (iii)   Several candidates drew an appropriate graph that was neither simple nor connected and had the required vertex orders. Those who did not do this, usually either gave a graph that was not simple but was connected or they counted loops from a vertex to itself as only adding 1 to the vertex order instead of 2.

    (iii)   Some candidates seemed to be think that a simple graph must be a tree and some used the term 'connected' to meant 'complete'. It was possible to draw a connected graph with the given vertex orders, but such a graph could never be simple. With only four vertices available the vertex of order 4 would either have to connect to one of the other vertices twice or connect in a loop to itself, or possibly both of these.

    (iv)(a)   Some candidates did not appreciate that the graph in part (iv) had five vertices.

           There could not be exactly three vertices of order 4 because then for the graph to be simply connected the other two vertices would both have order at least 3, and no more than 4. For the graph to also be Eulerian, the only possibility is that it has five vertices of order 4.

    (b)   Most candidates were able to draw at least some appropriate diagrams, but the majority missed at least one of the four structures.

3) (i)   Candidates often omitted the boundary $x = 0$, or misread the scales on the graph. Some candidates gave the inequalities that excluded the feasible region rather than those that defined it.

    (ii)   The optimal point was at $(0, 7)$, some candidates assumed that the vertex $(4.2, 4.2)$ was the optimal point, or claimed $(7, 0)$ instead of $(0, 7)$. Several candidates calculated the value of P_1 at each vertex, but then did not identify the values of x and y at the optimal point or did not clearly associate the optimal value of P_1 with this point.

    (iii)   Many candidates omitted this part. Those who did attempt it usually found the new optimal vertex, although it was not always given as coordinates, but fewer gave a general expression for P_k.

    (iv)   Some candidates wrote their answers to this part in the space for part (iii), most of these did copy their answer into the appropriate space. Many candidates gave no response to this part.

4) (i)   Usually Dijkstra's algorithm was applied well, although some candidates made arithmetic errors and several crossed out their working values. In dealing with the double arc between F and G only the shorter arc needed to be considered – if the formulation had been given as a matrix the longer arc would not have been used. Some candidates applied the algorithm correctly but did not write down the route of the shortest path.

    (ii)   Most candidates were able to identify that this was a route inspection (or Chinese postman) problem.

    (iii)   Many candidates achieved full marks in this part, those who did not had usually omitted some of the working. Because the arcs AB and EF must be travelled twice, this leaves C, D, F and G to be paired. The shortest routes between pairs

of these needed to be found: $CD = 0.3$, $FG = 0.45$; $CF = 0.5$, $DG = 0.65$; $CG = 0.95$, $DF = 0.2$. These then needed to be paired and the minimum pairing ($CD + FG = 0.75$) identified and its weight added to $3.7 + 0.3 + 0.5$ to give a final distance of 5.25 km.

(iv)   Many candidates used nearest neighbour to get *BDFGC* but forgot to close the cycle.

(v)   Most candidates were able to find the minimum spanning tree for {*B, C, D, F*} although a few used nearest neighbour. The question had asked candidates to draw the tree and give the order in which vertices were added to the tree. To find a lower bound for the travelling salesman route on what was now a complete network it was then necessary to add on the two least weight arcs from *G*, according to the matrix.

5)   (i)   Most candidates were able to extract the constraints $600x+800y+500z \leq 5000$ and $120x+80y+120z \leq 800$, these could then be simplified to give $6x+8y+5z \leq 50$ and $3x+2y+3z \leq 20$, some candidates also simplified the given constraint.

(ii)   Any reasonable constraints were then followed through to the setting up of the initial simplex tableau. Some candidates only used the two constraints that they had found, which considerably simplified the problem.

Most candidates remembered to include a *P*-column, and only a minority had sign errors in the objective row. The candidates with a reasonable tableau then needed to choose a pivot entry form the *z*-column by finding the positive entry for which the ratio of the value in the final column divided by the value in the *z*-column was minimised. Some candidates had sign errors leading to negative entries in the final column or positive entries in the objective row for the *z*-column, and often these led to an inappropriate pivot choice.

Having chosen the pivot, candidates needed to divide the pivot row by the value of the pivot entry and update the other rows using operations of the form: *old row ± multiple of new pivot row*, where the multiples are chosen to give 0 entries in the pivot column. Candidates also needed to show the method used, for example by writing 'r1+120pr' alongside the appropriate row.

The tableau achieved needed to have the correct structure with non-negative values in the final column and the value of the objective having not decreased. There should then have been a negative entry remaining in the objective row, enabling a second iteration to be performed.

Several candidates achieved a correct final tableau and were able to read off the resulting values of *x*, *y* and *z*, although not all of them put the interpretation into context.

(iii)   The candidates who attempted this final iteration were usually successful, some realised that once they had achieved the pivot row and a non-negative objective row they only needed to state that the objective row was non-negative. Some candidates thought that the objective row was positive and then they needed to give the entire tableau to get the marks.

4737 Decision Mathematics 2

General Comments

The candidates for this paper were, in general, well prepared and were able to show what they knew. Candidates should be reminded to read the questions carefully as several dropped marks for not answering exactly what had been asked.

Comments on individual questions

1) (i)   Nearly all the candidates were able to draw the bipartite graph correctly.

    (ii)   The majority of candidates drew a second bipartite graph correctly showing the incomplete matching. Only the new matching was required, candidates' answers could not always be determined when they had drawn the arcs in the matching using pen and the arcs not in the matching using pencil, or had used arrows to indicate which arcs were to be included. With the transition towards on-screen marking it will be even more important that candidates only show the information requested in a question like this.

    (iii)   Several candidates wrote down the shortest alternating path correctly and went on to find the matching. Some gave a longer alternating path, which led to the wrong matching here, although they often recovered in (iv).
Both the alternating path and the complete matching needed to be written down, perhaps using the initial letters to identify the suspects and weapons, in both cases referring to a diagram was not enough.

    (iv)   Most candidates were able to find a complete matching, although sometimes an arc was duplicated from the answer to part (iii).

2) (i)   Many candidates achieved full marks or were within one mark of full marks for this question. Those who slipped up often had forgotten to convert from a minimisation problem to a maximisation problem.

    Candidates' numerical work was noticeably more accurate than in previous sessions.

    A minority of candidates only reduced rows before starting to augment. A tiny number of candidates were not able to carry out the augmenting operations correctly, the most common error in this case being to reduce uncrossed values by, say, 2 but only increase the values crossed through twice by 1.

    There were two matchings that resulted; most candidates only gave one of these.

    (ii)   The answers here were allowed on follow through from the candidates' matchings in part (i). Despite being told that Dr Silverbirch was not the burglar, some candidates still decided that he should be suspected.

3) (i)   Many good answers, although several candidates omitted the 'action' column, or used the wrong values in the 'state' or 'action' columns. Some candidates used (stage; state) labels instead of just giving the state values in the state column, and several made errors in the action values. The action value is the state of the vertex (in the next stage) that the arc joins to, this enables the solution to be traced back through the table without needing to refer to a network.

Several candidates worked forwards through their network, instead of working backwards, this was only given partial credit.

(ii) Only the better candidates were able to give a coherent description of how the route is traced from the table without recourse to the network diagram.
The key ideas were that you need to start at the bottom of the table (stage 0, state 0) and see which action led to the minimum value of 17. This action label then gives the state label for the stage above. So (0; 0) uses action 0, which means that the next vertex is (1; 0). Repeating this method, (1; 0) uses action 1, which means that the next vertex is (2; 1); then (2; 1) uses action 0, so the next vertex is (3; 0); and finally (3; 0) uses action 0, so the final vertex is (4; 0).

4) (i) Most candidates were able to explain what a 'zero-sum' game is.

(ii) Because the table shows the number of points that Euan scored for each combination of strategies, the number of points won by Wai Mai with this particular combination of strategies was the negative of the entry in the table.

(iii) Most candidates realised that this was about dominance, and the majority realised that it was dominance between Y and Z. Some were able to convincingly explain why Z is dominated by Y.

(iv) Although most candidates produced the reduced table, there were several who did not show the row minimum values and the column maximum values and then the selection of the maximin for rows and the minimax for columns. Other candidates showed all this but did not write down the play-safe strategies as being D for Euan and Y for Wai Mai.
Most candidates understood the idea of a stable or unstable game.

(v) Some candidates did not make the change to the table, so their game was still stable, others changed the wrong entry. Several candidates wrote down the expected scores from Euan's perspective rather than from Wai Mai's.

(vi) Many candidates got quite a lot of credit for correct methods applied to incorrect equations. The ones who lost out most heavily were those who had ignored the instruction to use graph paper.

5) (i) Many correct answers, most candidates had identified the cut correctly.
Amongst the incorrect answers the most common errors were minor arithmetic slips or trying to include the capacity of 23, or −23, for the arc AB (which flows across the cut the wrong way). The minimum that can flow through this arc across the cut from S to T is 0.

(ii) Many good explanations, some slightly confused explanations. Most candidates realised that the capacity of CE was too small for SC or BC to be full. Some candidates thought that they had to say why SC and BC could not both be full, rather than why neither of them could be full.

(iii) Some candidates used a labelling procedure type of diagram here instead of just showing a flow. A flow is shown by giving a weight to just a forward facing arrow on each arc. Blanks were interpreted as either being empty of full to capacity, provided they were used consistently.

(iv) The candidates who understood about the labelling procedure were usually able to give good answers to this part, except that some of them did not read the part that told them the direction of flow in arc HG had been reversed, and then they were not able to progress any further.

54

(v) Despite being instructed not to augment the labels from part (iv), some candidates not only augmented but completely obliterated their answers to part (iv). A few candidates seemed to think that because the insert had given two rows for the answer they needed to find two routes. If the extra 2 litres per second could be sent along one route then that was all that was needed.

(vi) The candidates who actually shows their final flow, by giving a weight to just a forward facing arrows on each arc, were also usually able to explain why the flow was then maximal. Often this was done by identifying the cut $X = \{S, C\}$ and $Y = \{A, B, D, E, F, G, H, T\}$, although there were other ways of showing it.

6) (i) The majority of the candidates could complete the precedence table without error. A few candidates struggled to deal with the dummy activities.

(ii) Few candidates were able to give good explanations of why the two dummy activities were needed. The dummy between events [2] and [3] was a precedence dummy and was needed because D depended on A only but C, E and F all depended on both A and B. The dummy between events [4] and [5] was needed because otherwise C and F would have shared a common start at [4] and also shared a common finish at [5]; the dummy is needed to distinguish between the two routes.

(iii) The forward and backward passes were generally completed accurately.

(iv) As usual, candidates found the algebraic variations testing. Many realised that $9+x$ was involved, but often that was as far as they got.

(v) Some candidates were able to see that the critical value was $x = 17$, even when they could not put their thoughts down in part (v).

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored