OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4733

Probability & Statistics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** The number of inhabitants of a village who are selected for jury service in the course of a 10-year period is a random variable with the distribution Po(4.2).

(a) Find the probability that in the course of a 10-year period, at least 7 inhabitants are selected for jury service. **[2]**

(b) Find the probability that in 1 year, exactly 2 inhabitants are selected for jury service. **[3]**

(ii) Explain why the number of inhabitants of the village who contract influenza in 1 year can probably not be well modelled by a Poisson distribution. **[2]**

2 A university has a large number of students, of whom 35% are studying science subjects. A sample of 10 students is obtained by listing all the students, giving each a serial number and selecting by using random numbers.

(i) Find the probability that fewer than 3 of the sample are studying science subjects. **[3]**

(ii) It is required that, in selecting the sample, the same student is not selected twice. Explain whether this requirement invalidates your calculation in part **(i)**. **[2]**

3 Tennis balls are dropped from a standard height, and the height of bounce, H cm, is measured. H is a random variable with the distribution $N(40, \sigma^2)$. It is given that $P(H < 32) = 0.2$.

(i) Find the value of σ. **[3]**

(ii) 90 tennis balls are selected at random. Use an appropriate approximation to find the probability that more than 19 have $H < 32$. **[6]**

4 The proportion of commuters in a town who travel to work by train is 0.4. Following the opening of a new station car park, a random sample of 16 commuters is obtained, and 11 of these travel to work by train. Test at the 1% significance level whether there is evidence of an increase in the proportion of commuters in this town who travel to work by train. **[7]**

5 The time T seconds needed for a computer to be ready to use, from the moment it is switched on, is a normally distributed random variable with standard deviation 5 seconds. The specification of the computer says that the population mean time should be not more than 30 seconds.

(i) A test is carried out, at the 5% significance level, of whether the specification is being met, using the mean $\bar{t}$ of a random sample of 10 times.

(a) Find the critical region for the test, in terms of $\bar{t}$. **[4]**

(b) Given that the population mean time is in fact 35 seconds, find the probability that the test results in a Type II error. **[3]**

(ii) Because of system degradation and memory load, the population mean time μ seconds increases with the number of months of use, m. A formula for μ in terms of m is $\mu = 20 + 0.6m$. Use this formula to find the value of m for which the probability that the test results in rejection of the null hypothesis is 0.5. **[4]**

6 **(a)** The random variable D has the distribution Po(24). Use a suitable approximation to find $P(D > 30)$. **[5]**

(b) An experiment consists of 200 trials. For each trial, the probability that the result is a success is 0.98, independent of all other trials. The total number of successes is denoted by E.

(i) Explain why the distribution of E cannot be well approximated by a Poisson distribution. **[1]**

(ii) By considering the number of failures, use an appropriate Poisson approximation to find $\mathrm{P}(E \leqslant 194)$. **[4]**

7 A machine is designed to make paper with mean thickness 56.80 micrometres. The thicknesses, x micrometres, of a random sample of 300 sheets are summarised by

$$n = 300, \qquad \Sigma x = 17\,085.0, \qquad \Sigma x^2 = 973\,847.0.$$

Test, at the 10% significance level, whether the machine is producing paper of the designed thickness. **[11]**

8 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} kx^{-a} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants and a is greater than 1.

(i) Show that $k = a - 1$. **[3]**

(ii) Find the variance of X in the case $a = 4$. **[5]**

(iii) It is given that $\mathrm{P}(X < 2) = 0.9$. Find the value of a, correct to 3 significant figures. **[4]**