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**ADVANCED GCE**

# MATHEMATICS

Probability & Statistics 4

**4735**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**
- Scientific or graphical calculator

**Thursday 24 June 2010**
**Morning**

**Duration:** 1 hour 30 minutes

* 4 7 3 5 *

**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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**1** For the variables $A$ and $B$, it is given that $\text{Var}(A) = 9$, $\text{Var}(B) = 6$ and $\text{Var}(2A - 3B) = 18$.

**(i)** Find $\text{Cov}(A, B)$. **[3]**

**(ii)** State with a reason whether $A$ and $B$ are independent. **[1]**

**2** The probability generating function of the discrete random variable $X$ is $\frac{e^{4t^2}}{e^4}$. Find

**(i)** $E(X)$, **[3]**

**(ii)** $P(X = 2)$. **[3]**

**3** $X_1$ and $X_2$ are continuous random variables. Random samples of 5 observations of $X_1$ and 6 observations of $X_2$ are taken. No two observations are equal. The 11 observations are ranked, lowest first, and the sum of the ranks of the observations of $X_1$ is denoted by $R$.

**(i)** Assuming that all rankings are equally likely, show that $P(R \leqslant 17) = \frac{2}{231}$. **[5]**

The marks of 5 randomly chosen students from School $A$ and 6 randomly chosen students from School $B$, who took the same examination, achieving different marks, were ranked. The rankings are shown in the table.

| Rank | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| School | *A* | *A* | *A* | *B* | *A* | *A* | *B* | *B* | *B* | *B* | *B* |

**(ii)** For a Wilcoxon rank-sum test, obtain the exact smallest significance level for which there is evidence of a difference in performance at the two schools. **[2]**

**4** The moment generating function of a continuous random variable $Y$, which has a $\chi^2$ distribution with $n$ degrees of freedom, is $(1 - 2t)^{-\frac{1}{2}n}$, where $0 \leqslant t < \frac{1}{2}$.

**(i)** Find $E(Y)$ and $\text{Var}(Y)$. **[5]**

For the case $n = 1$, the sum of 60 independent observations of $Y$ is denoted by $S$.

**(ii)** Write down the moment generating function of $S$ and hence identify the distribution of $S$. **[2]**

**(iii)** Use a normal approximation to estimate $P(S \geqslant 70)$. **[3]**

**5** In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries $x$, in thousands of pounds, of 50 randomly chosen employees were obtained.

**(i)** The values $|x - 19.5|$ were calculated and ranked. No two values of $x$ were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of $(x - 19.5)$ was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. **[10]**

**(ii)** If the assumption you stated in part **(i)** does not hold, what test could have been used? **[1]**

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**6** Nuts and raisins occur in randomly chosen squares of a particular brand of chocolate. The numbers of nuts and raisins are denoted by $N$ and $R$ respectively and the joint probability distribution of $N$ and $R$ is given by

$$\mathrm{f}(n, r) = \begin{cases} c(n + 2r) & n = 0, 1, 2 \text{ and } r = 0, 1, 2, \\ 0 & \text{otherwise,} \end{cases}$$

where $c$ is a constant.

**(i)** Find the value of $c$. **[3]**

**(ii)** Find the probability that there is exactly one nut in a randomly chosen square. **[2]**

**(iii)** Find the probability that the total number of nuts and raisins in a randomly chosen square is more than 2. **[2]**

**(iv)** For squares in which there are 2 raisins, find the mean number of nuts. **[4]**

**(v)** Determine whether $N$ and $R$ are independent. **[2]**

**7** The continuous random variable $X$ has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{x}{2\theta^2} & 0 \leqslant x \leqslant 2\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta$ is an unknown positive constant.

**(i)** Find $\mathrm{E}(X^n)$, where $n \neq -2$, and hence write down the value of $\mathrm{E}(X)$. **[3]**

**(ii)** Find

**(a)** $\mathrm{Var}(X)$,

**(b)** $\mathrm{Var}(X^2)$.

**[4]**

**(iii)** Find $\mathrm{E}(X_1 + X_2 + X_3)$ and $\mathrm{E}(X_1^2 + X_2^2 + X_3^2)$, where $X_1$, $X_2$ and $X_3$ are independent observations of $X$. Hence construct unbiased estimators, $T_1$ and $T_2$, of $\theta$ and $\mathrm{Var}(X)$ respectively, which are based on $X_1$, $X_2$ and $X_3$. **[6]**

**(iv)** Find $\mathrm{Var}(T_2)$. **[2]**

**8** For the events $L$ and $M$, $\mathrm{P}(L \mid M) = 0.2$, $\mathrm{P}(M \mid L) = 0.4$ and $\mathrm{P}(M) = 0.6$.

**(i)** Find $\mathrm{P}(L)$ and $\mathrm{P}(L' \cup M')$. **[3]**

**(ii)** Given that, for the event $N$, $\mathrm{P}\big(N \mid (L \cap M)\big) = 0.3$, find $\mathrm{P}(L' \cup M' \cup N')$. **[3]**

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