OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Simplify $\dfrac{20 - 5x}{6x^2 - 24x}$. **[3]**

2 Find $\displaystyle\int x \sec^2 x \, dx$. **[4]**

3 **(i)** Expand $(1 + 2x)^{\frac{1}{2}}$ as a series in ascending powers of x, up to and including the term in x^3. **[3]**

(ii) Hence find the expansion of $\dfrac{(1 + 2x)^{\frac{1}{2}}}{(1 + x)^3}$ as a series in ascending powers of x, up to and including the term in x^3. **[5]**

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. **[1]**

4 Find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} (1 + \sin x)^2 \, dx$. **[6]**

5 **(i)** Show that the substitution $u = \sqrt{x}$ transforms $\displaystyle\int \frac{1}{x(1 + \sqrt{x})} \, dx$ to $\displaystyle\int \frac{2}{u(1 + u)} \, du$. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_1^9 \frac{1}{x(1 + \sqrt{x})} \, dx$. **[5]**

6 A curve has parametric equations

$$x = t^2 - 6t + 4, \quad y = t - 3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

7 **(i)** Show that the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ meets the line passing through $(9, 7, 5)$ and $(7, 8, 2)$, and find the point of intersection of these lines. **[6]**

(ii) Find the acute angle between these lines. **[4]**

8 The equation of a curve is $x^3 + y^3 = 6xy$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[4]**

(ii) Show that the point $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ lies on the curve and that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ at this point. **[4]**

(iii) The point (a, a), where $a > 0$, lies on the curve. Find the value of a and the gradient of the curve at this point. **[4]**

9 A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $160 - \theta$, where θ °C is the temperature of the liquid at that time.

(i) Write down a differential equation connecting θ and t. **[2]**

When the liquid was placed in the oven, its temperature was 20 °C and 5 minutes later its temperature had risen to 65 °C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. **[9]**