OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4737/01**

MATHEMATICS

Decision Mathematics 2

WEDNESDAY 21 MAY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
Insert for Questions 3 and 4
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- There is an **insert** for use in Questions **3** and **4**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **8** printed pages and an insert.

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1 **(a)** Five student teachers have been asked to each shadow an experienced teacher for a day. The student teachers are Amy, Ben, Emily, Frank and Gina, and the experienced teachers are Miss Patel, Mrs Quinn, Mr Roberts, Mr Thomas and Mrs Unwin.

Because of timetabling restrictions, Amy must shadow either Miss Patel or Mr Thomas; Ben must shadow Miss Patel, Mr Roberts or Mr Thomas; Emily must shadow either Mrs Quinn or Mrs Unwin; Frank must shadow either Mr Roberts or Mrs Unwin; and Gina can only shadow Mrs Quinn.

(i) Draw a bipartite graph to represent this information. Put the student teachers (A, B, E, F and G) on the left-hand side and the experienced teachers (P, Q, R, T and U) on the right-hand side. [1]

Initially Amy is asked to shadow Mr Thomas, Ben to shadow Mr Roberts, Emily to shadow Mrs Unwin and Gina to shadow Mrs Quinn.

(ii) Draw a second bipartite graph to show this incomplete matching. [1]

(iii) Construct the shortest possible alternating path from F to P and hence find a complete matching between the student teachers and the experienced teachers. [2]

(iv) Amy would prefer to shadow one of the women (P, Q or U). Find a complete matching that will also satisfy this additional restriction. [1]

[This question continues on the next page.]

(b) Mr Roberts teaches media studies. Some of his class are making a documentary about the five student teachers in the school. He needs to choose a pupil to operate the camera (C), another to be the director (D), a third pupil to be in charge of the lighting (L) and a fourth to be in charge of the sound (S).

Five pupils have volunteered to do these tasks: Harry (H), Iannos (I), Jack (J), Kerry (K) and Nadia (N). Mr Roberts has assessed each pupil for their suitability for each task and has given them a score out of 10. He has then subtracted each score from 10 to give a table on which the Hungarian algorithm can be applied to find the best matching of pupils to tasks.

This table, showing '10 minus score', is given below.

		Task				
		C	D	L	S	X
Pupil	H	1	2	4	4	10
	I	2	4	7	6	10
	J	4	6	5	9	10
	K	3	8	7	7	10
	N	3	7	7	5	10

(i) Explain why the scores needed to be subtracted from 10, and explain the purpose of column X. [2]

(ii) Apply the Hungarian algorithm, **reducing columns first**, to find a minimum cost matching. You must show your working clearly. Which pupil should be given which task, and what is the total **score** resulting from this allocation? [7]

Harry says that he would rather take part in the documentary. This leaves just four pupils (I, J, K and N) to be allocated to the four tasks (C, D, L and S).

(iii) Write out the 4×4 matrix showing '10 minus score' for the pupils I, J, K and N and the tasks (C, D, L and S). Apply the Hungarian algorithm to this reduced matrix to allocate the pupils to the tasks. [4]

2 Rowena and Collette repeatedly play a zero-sum game in which Rowena has a choice of two strategies, P and Q, and Collette has a choice of four strategies, W, X, Y and Z.

The table shows the number of points Rowena scores for each pair of strategies.

		Collette			
		W	X	Y	Z
Rowena	P	2	−3	1	3
	Q	1	2	−1	−4

(i) If Rowena chooses strategy P and Collette chooses strategy W, how many points will Collette score? [1]

(ii) Show that column W is dominated by one of the other columns, and state which column this is. [2]

(iii) Find the play-safe strategy for Rowena and the play-safe strategy for Collette. [3]

Rowena makes a random choice between strategies P and Q, choosing strategy P with probability p and strategy Q with probability $1 - p$.

(iv) Write down and simplify an expression for the expected pay-off for Rowena when Collette chooses strategy X. Write down and simplify the corresponding expressions for when Collette chooses strategy Y and for when she chooses strategy Z. [2]

(v) Using graph paper, draw a graph to show Rowena's expected pay-off against p for each of Collette's choices of strategy. Using your graph, find the optimal value of p for Rowena. Calculate Rowena's minimum expected pay-off if she plays using this value of p. [4]

[This question continues on the next page.]

In a variation of the game, Rowena has a choice of three strategies and Collette has a choice of just two strategies. The table shows the number of points Rowena scores for each pair of strategies.

		Collette	
		Y	Z
Rowena	P	0	3
	Q	–1	–4
	R	2	–2

Rowena intends to make a random choice between strategies P, Q and R, choosing strategy P with probability p_1, strategy Q with probability p_2 and strategy R with probability p_3.

She formulates the following linear programming problem so that she can find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

Maximise $M = m$,

subject to
$$m \leqslant 4p_1 + 3p_2 + 6p_3,$$
$$m \leqslant 7p_1 + 2p_3,$$
$$p_1 + p_2 + p_3 \leqslant 1,$$

and $p_1 \geqslant 0,\ p_2 \geqslant 0,\ p_3 \geqslant 0,\ m \geqslant 0.$

(vi) Show how Rowena obtained the expressions $4p_1 + 3p_2 + 6p_3$ and $7p_1 + 2p_3$. Also explain why m cannot exceed either of these expressions. [3]

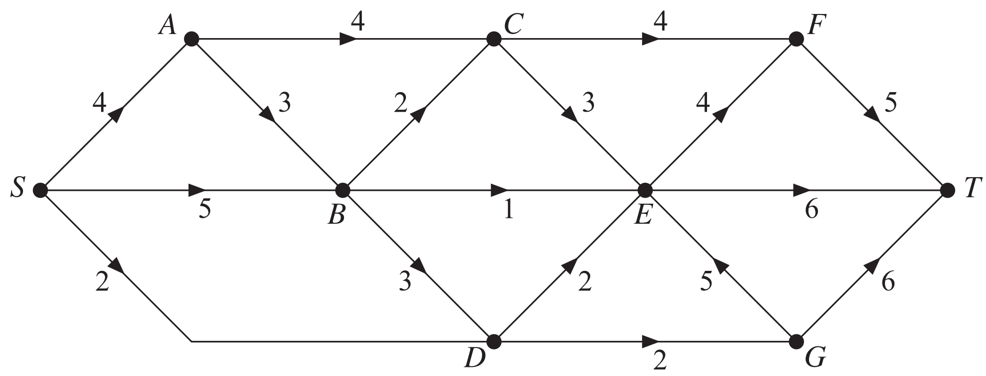
(vii) Explain why the constraint $p_1 + p_2 + p_3 \leqslant 1$ is needed. [1]

Rowena uses the Simplex algorithm to find the optimal values of the probabilities. She finds that the optimal value of p_1 is $\frac{4}{7}$ and the optimal value of p_2 is 0.

(viii) Calculate the optimal value of p_3 and the corresponding minimum expected pay-off for Rowena. [2]

3 **Answer this question on the insert provided.**

The network below represents a system of pipes through which fluid can flow from a source, S, to a sink, T. The weights on the arcs represent pipe capacities (maximum flow rates) in litres per minute.



(i) Calculate the capacity of the cut that separates $\{S, A, B, D, G\}$ from $\{C, E, F, T\}$. [2]

(ii) Explain why the arc GE cannot be full to capacity. [1]

(iii) What is the maximum possible rate of flow through the vertex E? Show such a flow on the diagram in the insert. [3]

The diagram in the insert shows the graph on which the network was formed with arrows for use in the labelling procedure. For each arc, the arrow pointing in the original direction of possible flow is to show how much more could flow (excess capacity) and the arrow pointing against the original direction of flow is to show how much less could flow (potential backflow).

(iv) Label the arrows to show a flow of 4 litres per minute along $SACFT$, a flow of 1 litre per minute along $SBET$ and a flow of 2 litres per minute along $SDGT$. [3]

(v) Apply the labelling procedure to augment the flow using the route $SBDET$. State the amount that flows along this route. Do not obliterate your values from part **(iv)**. [3]

(vi) Further augment the flow by 2 litres per minute using just one route. Leave your values from part **(iv)** and part **(v)** clearly visible. Write down the route that you have used. [3]

(vii) Show your resulting flow on the directed graph in the insert. [1]

(viii) Show that your flow is maximal by finding a cut with capacity equal to the flow. Describe your cut by stating the arcs that it crosses. [2]

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4 Answer part (a) of this question on the insert provided.

(a) The table shows a partially completed dynamic programming tabulation for solving a longest path (maximum path) problem.

Stage	State	Action	Working	Suboptimal maximum
2	0	0	5	
	1	0	4	
	2	0	4	
1	0	0	3 +	
		1	4 +	
	1	1	2 +	
		2	4 +	
	2	1	6 +	
		2	5 +	
0	0	0	4 +	
		1	5 +	
		2	2 +	

On the insert, complete the last two columns of the table. State the length of the longest path and write down its route. [8]

(b) The table below shows the activities involved in a project, their durations and precedences.

Activity	Duration (days)	Immediate predecessors
A	4	–
B	5	–
C	2	–
D	3	*A*
E	4	*A*
F	2	*B*
G	4	*B*
H	6	*C*
I	5	*C*
J	5	*D*
K	4	*E*, *F*, *H*
L	4	*G*, *I*

(i) Draw an activity network to represent the project, using activity on arc. You are advised to make your diagram as large as possible. [2]

(ii) Carry out a forward pass to find the early times for the events. Record these at the vertices on your network. Also calculate and record the late times for the events. Find the minimum completion time for the project and list the critical activities. [6]

It is now realised that activity *K* must follow activity *G*, as well as *E*, *F* and *H*.

(iii) Draw that part of the activity network that changes. This will mean showing the connections between *E*, *F*, *G*, *H*, *I* and *K* and using two dummy activities. [2]

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MATHEMATICS

Decision Mathematics 2

INSERT for Questions 3 and 4

WEDNESDAY 21 MAY 2008

Afternoon

Time: 1 hour 30 minutes

Candidate Forename		Candidate Surname	
Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the boxes above.
- This insert should be used to answer Question **3** and Question **4** part **(a)**.
- Write your answers to Question **3** and Question **4** part **(a)** in the spaces provided in this insert, and attach it to your answer booklet.

This document consists of **4** printed pages.

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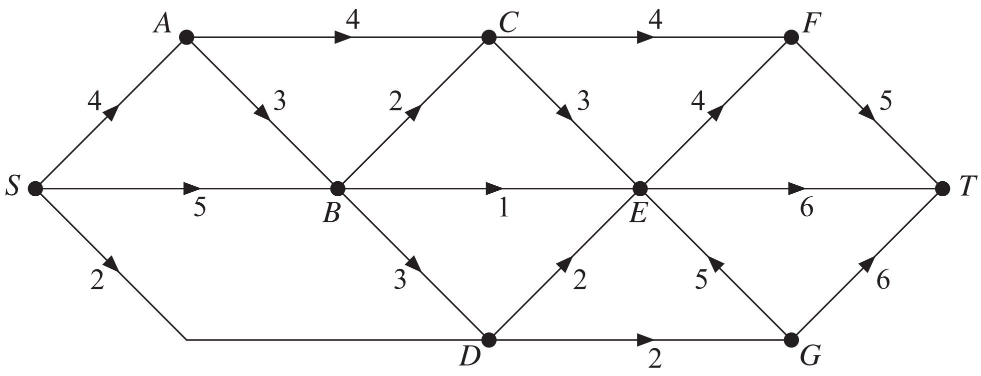


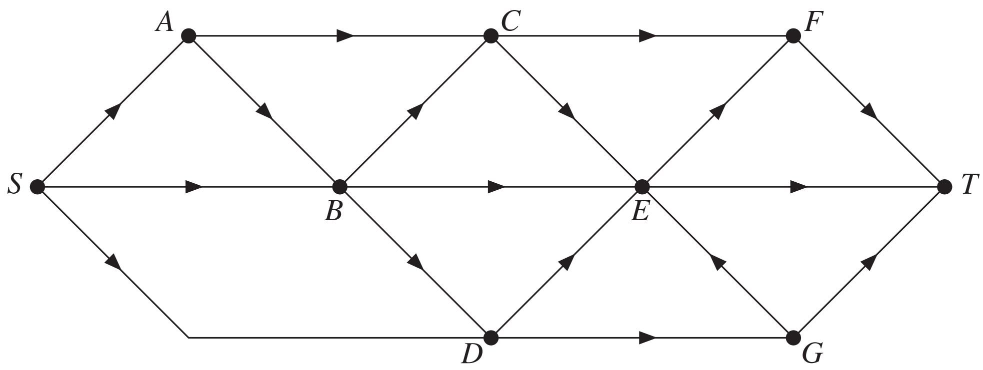
(i) ....

Capacity of cut that separates {S, A, B, D, G} from {C, E, F, T} = ... litres per minute

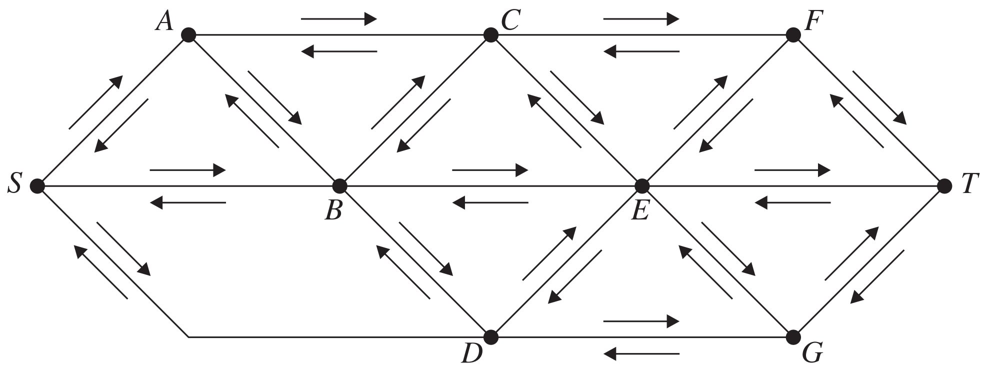
(ii) *GE* cannot be full to capacity since ....

....

(iii) Maximum possible rate of flow through the vertex E = ... litres per minute



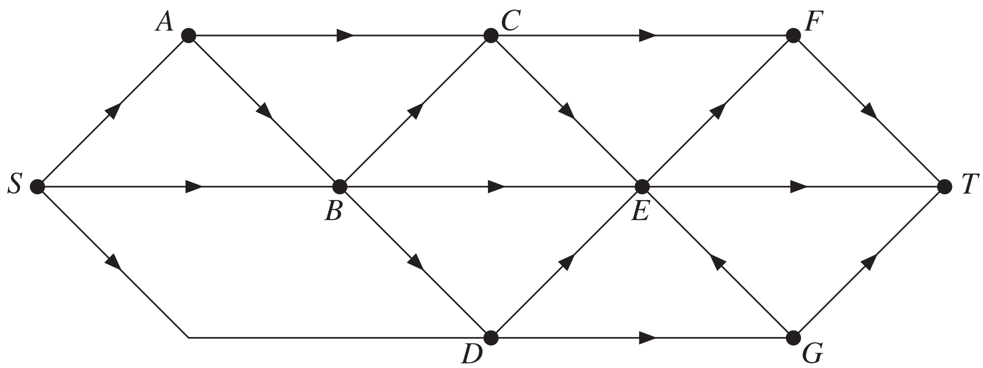
(iv), **(v)** and **(vi)**



(v) Amount that flows along *SBDET* = ... litres per minute

(vi) Route used = ...

(vii)



(viii) Cut through arcs ....

....

4 (a)

Stage	State	Action	Working	Suboptimal maximum
2	0	0	5	
	1	0	4	
	2	0	4	
1	0	0	3 +	
		1	4 +	
	1	1	2 +	
		2	4 +	
	2	1	6 +	
		2	5 +	
0	0	0	4 +	
		1	5 +	
		2	2 +	

Length of longest path = ...

Route = ...

Answer part **(b)** in your answer booklet.

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