GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

January 2007

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REPORTS FOR THE UNITS

Chief Examiner's Report – Pure Mathematics

These seven units (4721 – 4727) seemed to work well at this examination session, each providing an appropriate combination of straightforward and challenging questions. The AS papers contained a greater proportion of requests which should have been familiar to candidates and those with a firm grasp of the topics together with secure algebraic skills were able to record good marks. The A2 papers necessarily contained some more demanding questions and candidates met requests designed to require them to think and devise strategies for solution.

Failure to use brackets correctly remained a concern in several units. Candidates are familiar with the process of removing brackets although many do not carry out such expansions accurately. A skill which perhaps needs greater emphasis is the insertion of brackets to produce accurate algebraic statements. The failure by many candidates to produce accurate algebra in their solutions was responsible for subsequent errors in their solutions. Solutions to the following, taken from the current set of units, required brackets to be inserted in locations where they were absent from the original question.

- Expand $(1+4x)^7$.
- Given $y=\dfrac{2x+1}{3x-1}$, find $\dfrac{dy}{dx}$.
- Find $\displaystyle\int \frac{1}{3x+2}\,dx$.
- Use the substitution $u=2x-5$ to find $\displaystyle\int_{\frac{5}{2}}^{3}(4x-8)(2x-5)^7\,dx$.

This session was the first in which Graph Paper was removed from the list of Additional Materials to be available automatically to candidates in the examinations. This change was made to encourage candidates to produce sketch graphs in the Answer Booklet rather than on separate sheets of graph paper. Examiners reported that the change had a beneficial effect; instead of laboriously plotting points, many candidates were able to concentrate on producing a careful sketch which showed the key features of the curve involved.

4721: Core Mathematics 1

General Comments

This paper was tackled well by the vast majority of candidates. Most were able to demonstrate good mastery of the key techniques needed at this level such as the manipulation of surds, simple differentiation and factorisation of quadratic expressions. Candidates are clearly becoming more confident about calculating with fractions and negative values, although there were still plenty of errors seen.

In general, candidates used the most appropriate method for each question, although occasionally the quadratic formula was employed to solve a straightforward quadratic equation with integer roots.

Nearly all candidates worked through the paper sequentially and attempted every question, although a few failed to finish Q10. There was evidence that many had sufficient time to check and correct their answers, often resulting in a significant improvement to the final mark.

Presentation was generally good, although some candidates decided to write their answers to the entire paper in 3 pages or less or to use 2 columns per page. This makes it difficult for examiners to annotate working and should be discouraged when preparing candidates.

Comments on Individual Questions

1) This question proved a very accessible starter for almost all candidates and most scored the full 3 marks. Candidates who knew how to rationalise the denominator usually produced a correct answer, although a few then divided by 5 and gave their final answer as $2+\sqrt{3}$, which meant that they lost the final mark. Weaker candidates started by multiplying by $\sqrt{3}$ or $2-\sqrt{3}$ and were unable to gain any marks at all.

2) Part (i) was answered correctly by almost every candidate, the only incorrect answers seen being 0 or 6 in a handful of scripts.

   Part (ii) proved to be one of the most difficult questions on the paper although many candidates picked up one mark, usually for knowing that 2^{-1} was equivalent to $\frac{1}{2}$. Some candidates tried to work out 32^4, while others failed to progress because they did not know that the fifth root of 32 was 2. There was a significant number of candidates who started by combining the numbers 2 and 32, leading to answers like $64^{-\frac{1}{5}}$ or $64^{-\frac{4}{5}}$. A few left their final answer as 2^3 rather than evaluating it as 8.

3) The linear inequality in part (i) was handled well by most, although there were rather too many instances of 39 divided by 3 giving 12! It was also worrying to see a few candidates systematically reversing the direction of the inequality sign on each line of working. These candidates appear to have little understanding of the algebra involved.

   As in previous papers, the quadratic inequality proved more challenging. The vast majority of solutions finished either with the incomplete solution $x > 4$, or the incorrect statement $x > \pm 4$. Candidates who rearranged the original inequality to $5x^2 - 80 > 0$ and factorised into 2 brackets were much more likely to reach a fully correct solution, as were candidates who sketched a graph. As in part (i), some surprisingly poor arithmetic was seen; 80 divided by 5 too frequently resulting in 20.

4) This question proved quite demanding with many candidates unable to make a start. Many weaker candidates stated that they were multiplying each term by 3 and wrote $x^2 + 3x - 30 = 0$, while others cubed each term giving $x^2 + 3x - 1000 = 0$. However, many candidates did arrive at the values 2 and –5, although it was sometimes difficult to be sure of their thinking. The substitution $x = x^{1/3}$ was accepted although it would be better if candidates were encouraged to use a different variable when making a substitution. Having obtained 2 and –5, candidates often tried to find a cube root rather than a cube. Some of those who cubed 2 stated that -5 could not be cubed, a strange misconception. There was a small number of candidates who spotted that 8 was a solution and showed working to support this. This earned them some marks.

5) In part (i), it was very pleasing to see that most candidates eschewed the use of graph paper and produced a good sketch in their answer booklet. Nearly everyone recognised that the transformation was a reflection, although many reflected in the wrong axis, with a few using the line $x = 1$. Other errors included reflecting the point (4, 2) to (4, –1) or making the portion of the graph for $-1 \le x \le 1$ curve the wrong way or become straight. However, there were very many good sketches, a marked improvement on previous sessions.
Part (ii) was usually correct, the most common wrong answers being (3 ,1), (3, 3), (1, $\frac{1}{3}$) or (1, 4).

In part (iii), as in previous papers, a mark was often lost, even by the most able candidates, because of a failure to use the word 'translation'. Despite this, most candidates were able to give the direction and magnitude of the translation correctly.

6) Almost all candidates scored at least 2 marks in part (i) but far fewer managed to gain full marks due to a lack of rigour with brackets. The most common wrong values for *c* were 4 and 44, although –32 was also seen in many scripts. A few candidates made the question easier by dividing the original expression by 2 at the outset, meaning that they could only score 2 out of the possible 4 marks.

The responses to parts (ii) and (iii) were much more varied. Whilst the very strong candidates were able to use their (usually correct) answer to part (i) and write down the required equations, the connection between the 3 parts was not apparent to most. Many candidates differentiated to find the coordinates of the minimum point but were still unable to answer part (ii). They sometimes gained the mark for part (iii) by substituting the coordinates of their point into the equation of a straight line with gradient zero. While this obviously produces the correct answer, it often took almost a whole page of working and was worth only a single mark.

7) This question was the most successfully answered question on the paper. Even the weakest candidates scored well. However, it was disappointing to see so much careless notation, many answers starting with '$y =$' .

Part (i) was very rarely wrong.

Part (ii) proved the most challenging expression to differentiate, although most candidates were able to handle it successfully. A lack of understanding of negative indices was evident in expressions such as $(2x^2)^{-2}$ or $2(x^2)^{-2}$. A power of $\frac{1}{2}$ was also occasionally seen. The very weakest candidates were likely to give $\dfrac{2}{2x}$ as their final answer.

Part (iii) was efficiently done by the vast majority and nearly all candidates gained all 4 marks. There were a few slips (such as $7x^2$ or a sign error) in expanding the brackets but these were extremely rare.

8) There was the full range of responses to this question, which many candidates found the most demanding part of the paper. A good number of candidates, not always those with high final scores, produced concise and fully correct solutions. Others failed to start the question, possibly because they were unfamiliar with the term 'stationary points', although they had demonstrated a perfect grasp of differentiation in the previous question.

In part (i), much incorrect working was seen. Some candidates attempted to find the roots of the cubic, while others changed the question and worked with $-y$ throughout. A few attempted to work out coordinates and plot the graph, stating that the stationary points were at $x = -3$ and $x = 0$, but showing no evidence for this. Of those candidates who differentiated correctly and then set $\frac{dy}{dx} = 0$, many were unable to factorise correctly because of the $-3x^2$ term. Some sensibly factorised 3 out of the expression but then claimed that the equation had 3 roots, $x = 3$ or $x = 0$ appearing as the extra solution. The final mark in this part (for correct y-values) was often forfeited, either because there was no attempt to find these or because of errors when substituting into the original equation.

There was further evidence of muddled thinking in part (ii). Many candidates wrote $\frac{d^2y}{dx^2} = 0$, although they often then ignored this and continued correctly. Others used the second derivative of $-y$ and hence reversed the maximum and minimum points. Still others substituted their x-values from part (i) into $\frac{dy}{dx}$, (often not obtaining zero), instead of $\frac{d^2y}{dx^2}$.

It was clear from part (iii) that candidates' understanding of increasing functions is much improved, although a significant minority left this part out. Of those that made an attempt, nearly all realised that the solution was related to the x-values of the turning points that they had found previously, with many fully correct inequalities seen.

9) Candidates tended to score well on this question.

Part (i) proved very straightforward, although a fair number of candidates used the gradient of the line AB, instead of the given line.

Part (ii) was also very well done. It was pleasing to note that almost all candidates not only knew the correct formula for finding the length of a line but also simplified the resulting surd with confidence. There was some careless arithmetic seen, the total of 81 + 9 being given as 100.

The majority of candidates were able to find the midpoint of AB correctly in part (iii). Most also knew how to use the equation of a straight line but a significant minority used a gradient of $-\frac{1}{4}$ (confusing the gradient of AB with the gradient used in part (i)). Of those that correctly identified the required gradient as $-\frac{1}{3}$, the final mark was very often lost because of an inability to rearrange their equation into the correct form, with the fractions and the negative signs causing difficulties for some.

10) Despite being the final question, this was quite straightforward and most candidates gained at least half the available marks.

Some very poor algebra was seen in part (i) and large numbers of candidates could not identify the centre nor work out the radius. There were various errors in the completed square expressions with $1^2 = 2$ seen too often. Even if the expression was correct, the centre was often given as (1, –2) or (2, –4). Some candidates added 1^2 and 2^2, leading to a radius of $\sqrt{3}$. Other common mistakes included giving the radius as $\sqrt{8}$ or as 13. If this incorrect value was used in subsequent parts of the question, many marks were lost. Candidates from some centres preferred to use the expression $\sqrt{g^2 + f^2 - c}$ but there were still many incorrect answers, as wrong values were used for g and f (usually 2 and –4) or errors were made in dealing with the negative signs.

Part (ii) seemed to confuse the weakest candidates who often did not attempt it, but there were many good solutions. Obviously, it was safer for candidates to use the circle equation given in the question rather than their (possibly incorrect) rearrangement of it. Most candidates who reached a correct quadratic equation solved it efficiently although it was strange to find so many candidates stating that $k < 0 \;\therefore\; k = 5$.

The majority of candidates did part (iii) extremely well. They understood that they had to solve the equations simultaneously and the algebraic manipulation was of a better standard than in previous sessions. There were pleasingly few instances of $x^2 + y^2 = 36$ being used. Only a few candidates could not eliminate a variable and most were able to expand $(6-x)^2$ or $(6-y)^2$ correctly. Those who made an error with the sign of the squared term lost all subsequent marks as they produced a linear rather than a quadratic equation to solve. Candidates who used trial and improvement were fortunate in this particular question as both intersection points had integer coordinates.

4722: Core Mathematics 2

General Comments

This paper was accessible to the vast majority of candidates, and overall the standard was very good. It allowed candidates to display their knowledge to maximum advantage, but there were still aspects to challenge the most able. There were also a number of straightforward questions where weaker candidates who had mastered routine concepts could gain marks.

As on previous papers, only the most able candidates could manipulate logarithms accurately, though candidates are becoming more proficient in using logarithms to solve exponential equations. A number of candidates struggled in using Core 1 methods accurately, particularly when asked to manipulate surds or solve quadratic equations.

Whilst the majority of candidates seemed familiar with the concepts in this module, it is important that they select the most appropriate method for answering a question. Using the trapezium rule when integration is required, or vice versa, will not gain any credit, and neither will trial and improvement when a more stringent method has been requested. In other cases candidates need to consider which method is the most efficient – when finding the remainder in Q8, it is expected that the remainder theorem will be used. Other methods could gain full credit but also wasted a lot of time.

Generally, more candidates seem to be showing their methods in detail allowing examiners to give any credit due. However, this paper contained several parts where candidates were asked to prove a given statement. It is essential that sufficient detail is provided to convince the examiner that the principle has been fully understood. There are some candidates who approximate values throughout their working; this leads to an inaccurate final answer that will be penalised. It is becoming much more common to see sketch graphs appearing in the answer booklet rather than on graph paper. This approach is to be encouraged, but some candidates do need to take more care in their presentation – a sloppy graph, hastily sketched in pen with no ruler used, may not fully convey the detail required. Candidates must also ensure that they clearly state the coordinates of any requested points.

Comments on Individual Questions

1) This was a straightforward first question, and most candidates scored well. Those who quoted the formula for the n^{th} term of an AP were generally successful, though there was a surprising number of misreads of 12 for 20. Some candidates used a more informal method, and this usually resulted in the equivalent of $a + nd$ being used. The sum formula was nearly always quoted and used correctly.

2) (i) This was generally well done, though some candidates ignored the request for 3 significant figures and left their answer as a multiple of π. A number simply divided 46 by 180 and made no attempt to include a factor of π.

(ii) Most candidates could correctly quote the correct formula and attempt to use it. However, as in part (ii), using θ as 46^o was a fairly common error.

(iii) Most candidates used their attempt at the angle in radians in the correct formula, though the factor of ½ was sometimes omitted. However, despite having converted the angle to radians, some candidates chose to ignore that and instead worked in degrees and calculated the relevant fraction of the area of the circle.

3) (i) This question was very well answered, even by the weaker candidates. A few attempted to use the Core 3 method for integrating $(ax + b)^n$, but this was rarely successful. A number omitted the constant of integration, which was not penalised in this part of the question, but made further progress in part (ii) impossible.

(ii) Many candidates chose to ignore the work done in part (i), and assumed that it was a straight line that they were being asked to find the equation of, and hence employed variants of $y = mx + c$. This is a standard Core 2 question, yet many candidates still struggled to understand what was required of them. Some failed to gain the final mark by not putting their expression in the form of an equation (i.e. equating it to y). A number of correct solutions were seen, but generally produced by more able candidates.

4) (i) Most candidates could quote and apply the correct formula for the area of a triangle. However, the significance of the word 'exact' escaped many and decimal answers were much more common. Of those who did use the surd form for sin 60°, very few could then manipulate the surds to arrive at the correct answer. Some simply failed to write $\sqrt{2} \times \sqrt{3}$ as $\sqrt{6}$, but most errors were much more fundamental such as $20\sqrt{2} \times \frac{1}{2}\sqrt{3}$ becoming $40\sqrt{3}$, or worse still. A few tried to use more longwinded methods involving the perpendicular height, but these were rarely successful.

(ii) This was generally well answered, with the majority of candidates able to apply the cosine rule confidently. Squaring $5\sqrt{2}$ caused problems for some, and a surprising number simply evaluated AC^2, forgetting to take a square root as a final step. There were the usual errors in evaluating the formula, and a minority of candidates could not even quote the correct formula, which is disappointing as it is given in the Formulae Booklet.

5) (a)(i) Logarithms continue to be the topic that candidates find most challenging, and both parts (i) and (ii) were very poorly done. In part (i), a number of correct solutions were seen, but these were in the minority with $\log_3(3x + 7)$ being the most common error. Of those who did obtain the correct expression, methods often contained an error, most usually the appearance of $\log_3 x$ in the denominator.

(ii) This was poorly done by all but the most able candidates. To make any progress candidates had to appreciate that 2 can be written as $\log_3 9$, which very few did. Even those who did mange this failed to gain any further credit as they simply removed $\log_3$ from each of the two terms in the equation given in the question rather than using the expression from part (i).

(b) Whilst most candidates seemed familiar with the trapezium rule, a surprisingly large number could not apply it accurately. The most common mistake was in finding the value of h, even when the correct number of strips had been used. A number of candidates attempted to integrate the expression before using the trapezium rule, an approach that gains no credit. Other common errors included using $\ln x$ not $\log_{10} x$, using the wrong number of strips and omitting crucial brackets in the formula. However, it was pleasing to see very few candidates using x values as has happened in the past. Some candidates failed to correctly evaluate their expression, either through inefficient use of a calculator or through prematurely rounding values.

6) (i) Most candidates could make a good attempt at the binomial expansion, but algebraic insecurity led to a number of errors. Most candidates could obtain the first two terms of 1 and $28x$, and then make an attempt at the next two terms. These were nearly always a product of the correct binomial coefficient and an attempt at squaring $4x$ but a lack of brackets led to errors, with $84x^2$ and $140x^3$ being the most common mistakes. Occasionally candidates found the terms in descending not ascending powers, and only a few attempted to expand the brackets.

(ii) This part of the question was poorly done, with many candidates failing to appreciate that the product of two terms was required. Whilst $28a$ was usually seen, this was often in isolation with no other term involved. Candidates seem able to successfully attempt standard questions on the binomial expansion, but then struggle to make progress on less routine requests.

7) (i)(a) The graph sketching was generally good, and there were fewer candidates who felt the need to plot curves on graph paper. However, some attempts were so poor that they failed to correctly show the general shape of the graph. Common errors included uneven scales on the axes, and graphs that did not have maximum and minimum points the same distance from the x-axis. Most could attempt a graph of the correct shape, though some drew $\cos 2x$ rather than $2\cos x$. When sketching a graph, candidates need to pay attention to the behaviour of the curve at the extremities of the range – some of the curves seen showed no sign of levelling off at 360°. Some candidates failed to clearly identify the coordinates of any points of intersection as requested. Only partially labelling the axes and leaving examiners to infer these values did not get credit.

(b) This question was generally well done, but too often candidates failed to divide by 2 before employing inverse trigonometric functions. A number of candidates struggled to find a correct second solution, and others had solutions in all four quadrants.

(ii) Most candidates attempted to convert the equation to one in $\tan x$, and this was generally done well, though some used the fraction in the identity the wrong way around. Others attempted to square both sides and use the Pythagorean identity but this creates extra solutions which even the most able candidates failed to discard. However, the weaker candidates struggled to make progress with this question with errors such as $\cos x = 1 - \sin x$ being common. Some candidates did not read the question carefully and failed to give a second solution in the correct range.

8) (i) Using the remainder theorem is the most efficient method in this question, but far too many candidates embarked on a more longwinded method often containing errors. Having obtained –25, some candidates then went on to conclude that the remainder was 25.

(ii) Candidates seem more willing to use the factor theorem than the remainder theorem. Most applied it accurately, though candidates must ensure that the attempt is concluded appropriately. Some longer methods were still seen, though in this case the quotient could then be used in part (iii).

(iii) Whilst some candidates found the quadratic factor through inspection or coefficient matching, the majority used long division and it was pleasing that this was usually done correctly. Having obtained the quadratic factor, most then attempted to find its roots, though there were a number of candidates who did not use the quadratic formula correctly. Giving these two roots in an exact form caused problems for many, either because they did not appreciate the significance of the request and just gave decimal solutions or, more commonly, because their manipulation of surds was weak. A number forgot to state the final root, namely $x = 3$. However, a number of pleasingly accurate solutions were seen and even the weaker candidates gained some credit here.

9) (i) This was very well answered, with most candidates attempting to use the correct formula for the n^{th} term of a GP, with r as 1.02. The weaker candidates could gain credit by calculating all the intermediate terms, though it was disappointing to see some at this level calculating 2% of an amount, adding it on and laboriously doing this four times (often with an ensuing lack of accuracy).

(ii) Some good attempts at this question were seen, but a number of candidates seemed unsure of what was actually required – solving the inequality and then substituting the value back in was a common error. Of those who attempted the required method, most could attempt an expression for the sum of n terms, relate it to 39 and attempt to rearrange. However there were very few who gained full marks, as they failed to prove the given inequality convincingly. The most common error was failing to reverse the inequality sign when necessary. This would have been avoided had candidates chosen the more appropriate formula for the sum, given that the ratio was greater than 1. Some initially ignored the inequality sign, inserting it on the last line only, if at all.

(iii) Whilst many candidates find manipulating logarithms difficult, they are becoming increasing adept at using logarithms to solve exponential equations. Most introduced logarithms on both sides, then dropped the power and attempted to solve for N. Whilst the occasional error was seen, most candidates obtained 21.144 and could conclude accordingly, though some left their final answer as an inequality with either a decimal or integer value. A few candidates offered a solution involving a logarithm to the base 1.02; this gained no credit unless they indicated how they intended to evaluate the expression. Some candidates failed to use the given inequality and instead attempted to solve some other equation. The question specified the method to be used; those who employed trial and improvement gained no credit.

10) (i) When candidates are asked to demonstrate a given value or expression, such as showing that the given point is on the curve, examiners expect to see a rigorous proof. Simply inserting the given coordinates into the equation is not enough; there must be an attempt to evaluate or solve depending on the method used. A number of candidates showed insufficient working to be convincing, whilst others showed sufficient working to be unconvincing in their method.

(ii) It was pleasing to see how many excellent attempts there were at this last question on this paper, with a number of candidates gaining half marks or more. Virtually all candidates attempted integration, with most producing a term containing $x^{1/2}$, though the coefficient was not always correct, and in other cases the 1 disappeared. The majority then attempted to use the required limits, though there were the usual muddles with which order to use the limits in, and whether to add or subtract. In some cases, limits other than a and 9 were used. It was disappointing that those candidates who had successfully got this far could not then correctly evaluate their integral; both numerical errors and sign errors were common. Most candidates then equated their definite integral to 4, though some used 4^2. Solving the resulting disguised quadratic proved too difficult for most. The most common error was to square both sides, but other attempts involved logarithms. However, those who did attempt the correct method were usually successful, providing elegant and concise solutions.

4723: Core Mathematics 3

General Comments

Examiners were pleased to note that there were very few candidates recording low marks for this examination. It was certainly possible for candidates of only modest ability to record some marks from all the questions. There was encouraging work associated with the calculus of expressions of the form $(ax+b)^n$ – as in Qs 4 and 6 – and the presumably familiar requests in the first six questions led to much sound work although a tentative grasp of basic algebraic skills betrayed some candidates.

Qs 7, 8 and 9 contained elements which were particularly challenging and the small group of candidates recording full marks on the paper showed commendable mathematical ability.

Calculators are becoming more powerful and some can now deal effectively with exact values such as surds. Candidates need to be aware of the conventions used in questions relating to the use of calculators. In Q2, no specified method was indicated and, accordingly, solutions which just consisted of the correct exact values were credited. But, in Q5(ii), the 'Hence …' is crucial and any attempt which consisted just of the correct answers received no marks.

Comments on Individual Questions

1) The vast majority of candidates recognised the relevance of the quotient rule to this question but the general standard of algebraic skills shown was disappointing. The absence of essential brackets, incorrect expansion of brackets and errors with signs were failings evident on many scripts. Most candidates attempted the equation of the tangent appropriately but further elementary errors occurred as attempts were made to reach the required form.

2) It was surprising how many candidates struggled with part (i). The main difficulty was in finding the value of $\cos\theta$ and many were unable to find any way of doing so, often leaving their answer for $\cot\theta$ as $\dfrac{13\cos\theta}{12}$. The identity $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ was used successfully by some candidates. There was greater success with part (ii) and candidates generally used an appropriate identity without difficulty. Doubts about basic understanding of trigonometry were raised for a minority of candidates when statements such as $\cos 2\theta = \frac{10}{13}$ and $\cos 2\theta = \cos^2 \frac{5}{13} - \sin^2 \frac{12}{13}$ were noted.

3) This question was answered very well and many candidates earned all seven marks. For others, the sketches in part (a) were the problem; often the curve was similar to $y = x^2$ or $y = x^4$ and the straight line passed through the origin. Some candidates confused matters by superimposing an attempt at the graph of $y = x^5 + bx - a$. A brief indication of how the graphs showed exactly one root was required and most candidates did provide this.

   The iteration in part (b) was carried out well although a few candidates used unusual starting values, among the most bizarre of which were –53, 0.01 and 6.348. Efficient use of calculators was evident and, as requested, candidates generally provided sufficient detail. The answer was required correct to precisely 3 decimal places and candidates usually did this although, in a number of cases, a sequence such as 2, 2.17791, 2.17473, 2.17479, 2.17479 was concluded with the incorrect answer 2.174.

4) The differentiation in part (i) was done very well and all four marks were earned without difficulty in most cases. In part (ii), the necessity to use $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ was usually evident but evaluating $6(4t+9)^{-\frac{1}{2}} e^{\frac{1}{2}x+1}$ for the given value proved awkward because many candidates could not deal with the x in the expression. Some substituted only $t = 4$, leaving an expression still involving x, whilst others substituted 4 for both t and x. Many candidates did obtain the correct answer and it was pleasing that many had a clear grasp of the notation used and the distinctions between the various derivatives involved.

5) This was a straightforward routine question for many candidates who obtained the two correct answers in part (ii) with ease. In part (i), signs were the main problem with $\tan\alpha = -\frac{1}{4}$ being a common step. Some insisted that α was –14° whilst others discreetly dropped the minus sign. Candidates had no difficulty finding the angle 46.9° in part (ii) but many did not adopt an appropriate strategy for finding the remaining angle. Some merely claimed –46.9° as the second answer whilst others suggested a value such as –133.1°. On a number of scripts the promising $\theta + 14.04° = \pm 60.98°$ was immediately followed by $\theta = \pm 46.9°$.

6) Both parts of this question were answered well by many candidates; they dealt successfully with the powers involved, retained exactness throughout and applied the appropriate logarithm property in part (ii). For other candidates, integrals such as $2(3x+2)^{\frac{1}{2}}$ and $\frac{2}{9}(3x+2)^{\frac{3}{2}}$ were frequent errors in part (i) and too many resorted to decimal approximations when substituting the limits.

In part (ii), the vast majority started with a correct statement such as $\pi\int[(3x+2)^{-\frac{1}{2}}]^2\,dx$ but some could not convert this to a form ready for integration. For some recognising the involvement of a natural logarithm, the factor $\frac{1}{3}$ was missing. Poor notation was evident on a number of scripts, the integral being given as $\frac{1}{3}\pi\ln 3x+2$ or the final answer being presented as the ambiguous $\frac{1}{3}\ln 4\pi$ or $\frac{2}{3}\ln 2\pi$.

7) This was a more challenging question although most candidates earned some marks.

There were many correct answers to part (i); common errors involved a stretch by factor $\frac{1}{2}$, sometimes in the y-direction, and a translation in the negative x-direction. Most candidates knew the general shape of the logarithm curve although many sketches showed a curve, a large part of which appeared parallel to the x-axis. For this part of the question no indication of the asymptote was required.

Both marks were available in part (iii) for an attempt which correctly dealt with the curve from part (ii) but many attempts were unconvincing. Often the curvature of the reflected part of the curve was wrong and, sometimes, the axis of reflection seemed to be the line $y = x$, or the reflection produced a curve symmetrical about a line parallel to the y-axis.

Correct answers to part (iv) were rare and, far too often, candidates adopted an algebraic approach which quickly led them into aimless complications. Use of the curve transformations specified earlier in the question leads readily to the answer. The relevant part of the $y = \ln x$ curve is $0 < x \le 1$. The translation maps this to $a < x \le 1 + a$ and the stretch then maps this to $2a < x \le 2(1+a)$. Alternatively, it can be argued that the relevant part of the curve $y = \ln(\frac{1}{2}x - a)$ is given by $0 < \frac{1}{2}x - a \le 1$, and the required result follows.

8) The majority of candidates realised that the product rule was needed in part (i) but the derivative of e^{-x^2} was often incorrect, errors including $-2e^{-x^2}$, e^{-x^2} and $-x^2e^{-x^2}$. The equation $8x^7e^{-x^2} - 2x^9e^{-x^2} = 0$ proved too daunting for some although factorisation of the left-hand side readily led to the required result. Indeed many candidates produced a careful and thorough solution, also referring to the stationary points at *P* and *O*. Confirmation of the *x*-coordinate of *Q* by substitution into the derivative was accepted as a method but full detail was needed.

Many candidates answered part (ii) well, showing commendable care and precision with the calculations needed for Simpson's rule. A common mistake was to take the *y*-value when $x = 0$ as 1. The major difficulty though concerned evaluation of e^{-x^2}; many candidates, presumably as the result of careless calculator work, effectively found values of $x^8e^{(-x)^2}$. They did not seem concerned at obtaining an answer 2492.5 for the area of *A*.

In answering part (iii), a few candidates apparently looked at the diagram and judged that an answer three times the area of *A* would do as an approximation. An appreciation of the symmetry of the curve is needed but there were frequent mistakes such as taking the length of *PQ* to be 2 or subtracting only the area of *A*, rather than twice this, from the area of the relevant rectangle.

9) The vast majority of candidates gained at least some marks but it was rare for full marks to be awarded; only candidates with a firm grasp of functions, inverse trigonometrical functions and quadratic inequalities were able to answer part (iii) correctly.

Part (i) was answered correctly by many candidates; even those who provided incorrect answers such as $-1 < y < 1$ or $y > 4$ clearly knew what is meant by the term range. To earn the first two marks in part (ii), candidates had to show evidence that radians were used; the common answer $4 - 2 \times 0.01745^2 = 3.999$ earned one mark as did the insufficiently detailed $4 - 2(2\sin 0.5)^2 = 2.16$. Explaining why fg(0.5) is not defined needed precise terminology. Loose reference, for example, to 3.5 being outside the range of values was not accepted; for any acceptable answer, clear reference to the domain of f was needed.

In part (iii), many candidates correctly identified $f^{-1}g(x)$ as $\sin^{-1}(2 - x^2)$ and some then appreciated that, for $f^{-1}g(x)$ to be valid, $-1 \le 2 - x^2 \le 1$. It was common for only two of the four critical values of *x* to be obtained – either 1 and $\sqrt{3}$ or –1 and 1. Not many candidates obtained all four values and then the conclusion tended to be an answer such as $x \le \pm 1, x \ge \pm\sqrt{3}$. Very few candidates demonstrated the mathematical awareness, perhaps aided by a calculator sketch of the curve $y = \sin^{-1}(2 - x^2)$, to produce the correct answer $x < -\sqrt{3}, -1 < x < 1, x > \sqrt{3}$.

4724: Core Mathematics 4

General Comments

As in June 2006, there was a wide range of marks extending over the complete spectrum. Many candidates showed clarity of thought with concise presentation and it was a joy to see their work; for others there had been little preparation and it was clear that they had little idea what they were doing. Fortunately the latter were in a minority but it was not insignificant.

As in many mathematics papers, some of the answers were given in the questions; the answers produced by candidates in these parts will always be very closely scrutinised and a higher degree of explanation will be required.

Candidates did not appear to have a problem with the length of the paper except, possibly, in a few cases where inordinate amounts of time were spent on Q4 and/or Q8(iv).

Comments on Individual questions

1) This gave the vast majority of candidates a good start and there were but a few instances of a lack of knowledge of such solutions. Errors mainly occurred in the denominator; there was some mis-reading of $x^2 - 4$ for $x^2 - 4x$ (though candidates ought to have realised they had made an error when there was no progress possible after the initial factorisation) and $x^2 - 4x$ was sometimes factorised as $(x-2)(x+2)$.

2) Most realised that the technique of integration by parts was necessary here; as might be expected, some candidates thought that the integral of $\ln x$ was $\frac{1}{x}$ and so produced an incorrect split but the majority worked competently.

3) In part (i), many produced $\overrightarrow{AB}$ but then failed to evaluate its modulus. There were very many mistakes in part (ii), angle AOB being found as often as the required OAB; this was not treated as misreading but as careless transcription. Of those trying to evaluate the correct angle, only a few realised they should work with $\overrightarrow{AO}.\overrightarrow{AB}$ (or $\overrightarrow{OA}.\overrightarrow{BA}$) rather than $\overrightarrow{OA}.\overrightarrow{AB}$ and consequently an angle of 137° was seen much more often than the required 43°; those candidates producing 137° and 'deducing' that the correct amount should be 43° were not credited with the final mark as there was no indication that the angle is acute rather than obtuse.

4) In general the technique was well known; the statement $\frac{\mathrm{d}u}{\mathrm{d}x} = 2$ was generally used as $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{2}$ and the limits were usually converted. It was not uncommon to see $4x - 8$ being transcribed into $2u - 2$ though the given answer frequently caused this to be corrected. Another error involved $\int (4x-8)u^7$ being integrated as it stood, which shows no understanding at all. Candidates often failed to simplify their work as they made progress and $(2u+2)u^7\,\frac{1}{2}\mathrm{d}u$ was sometimes not converted to $u^8 + u^7$ whilst others changed it to $u^7(u+1)$ and then used parts.

5) The binomial expansion was well done; there were the usual expected errors but candidates seemed to take more care than usual. Part (ii) was equally well despatched except by those candidates who thought that $1-3\left(x+x^3\right)$ could be factorised.

6) There were few errors in part (i) apart from the use of the wrong identity $2x+1 \equiv A\left(x-3\right)^2 + B\left(x-3\right)$. In part (ii), the integral of $\dfrac{A}{x-3}$ was usually sound but that of $\dfrac{B}{\left(x-3\right)^2}$ was also sometimes thought to be a logarithm.

7) This produced excellent solutions. The usual mistakes rarely occurred: "$\dfrac{\mathrm{d}y}{\mathrm{d}x} =$" at the beginning appeared hardly at all and, when $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4x+y}{x+2y}$ was equated to 0, $x+2y=0$ was almost never seen. The equations $x^2=1$ or $y^2=16$ were generally produced and the multiplicity of solutions, caused by solving either of these equations with the equation of the curve, was rarely noted.

8) The results being given in the first three parts, working was scrutinised carefully. In part (ii), an intermediate step showing factorisation was expected when the expression $\dfrac{2\left(p-q\right)}{p^2-q^2}$ was simplified to $\dfrac{2}{p+q}$. In part (iii), most used the results of parts (i) and (ii); very few found the equation of the normal at *P* and then substituted ($2q^2, 4q$).

Part (iv) was not a disaster but it was nearly so for two reasons. Firstly, hardly anyone realised the significance of the result in part (iii); that it could thought of as "(parameter at start of normal)^2 + (parameter at start)(parameter at end) + 2 = 0" was only occasionally seen. Most worked out the equation of the normal at (8, 8) and found its intersection with the curve; a not difficult piece of work but time-consuming. Secondly, many candidates read the question too quickly and thought there was just a single normal at *R* meeting the curve at *S*; relatively few, using either method, performed the operation a second time.

The question was clear but sensible, clear thought took a back seat here.

9) This question prompted another very good set of solutions. Most candidates separated the variables and, of those, the majority knew what to do with the $\cos^2\left(2x\right)$ and did it accurately. A significant number omitted the constant at the end of part (i); how they thought they were going to find the particular solution was not obvious but they made an attempt.

10) On the whole, this was also done well with just niggling omissions. Equations were asked for in parts (i) and (ii) but rarely was "**r** =" seen. It was interesting to see the usual answer in part (ii) being (**r** =) (**I** + 2**j** – **k**) + *t*(**I** + 2**j** – **k**) rather than just (**r**) = *t*(**I** + 2**j** – **k**) but, of course, it was fully accepted.

4725: Further Pure Mathematics 1

General Comments

Most candidates made an attempt at all of the questions and there was no evidence that candidates were under any time pressure. The majority of candidates worked sequentially through the paper and correct solutions to all questions were seen.

A good proportion of the candidates were able to score high marks, while a small, but significant, minority were only able to score well on a few of the questions. Many candidates did not take sufficient care when producing sketches in Qs 4, 5 and 9.

In general the presentation of work from candidates was of a high standard.

Comments on Individual Questions

1) This proved to be a straightforward starter question, with the majority of candidates scoring full marks, although errors in solving the linear equation $4 + a = 1$ in part (i) and $2a - 3 = 7$ in part (ii) were seen.

2) Some candidates did not know the method for finding the square root of the given complex number and either found the square root of both real and imaginary parts, or squared the given complex number. Those who knew the method were usually very accurate in their algebra, with the omission of finding the second square roots being the most common error.

3) Some candidates were unable to expand $r(r-1)(r+1)$ correctly, while a significant number of candidates expanded a correct expression for the required sum, before trying to factorise, rather than using the obvious common factors in their unsimplified expression and so were unable to find a fully factorised answer.

4) A good number of candidates were able to score full marks on this question. The most common error was to have the centre of the circles in either the first or second quadrant. Too many candidates did not take sufficient care to show that the circle in (i) passes through the origin, while the other circle required in (ii) touches both axes. Some candidates produced sketches containing a circle and a straight line, or even two straight lines.

5) Part (i) was usually verified satisfactorily. However, the number of errors in solving the quadratic equation was quite large, $-1 \pm 12\,\mathrm{i}$ and $-1 \pm 6\,\mathrm{i}$ being seen too frequently. In the sketch required in (iii) the root $z = 2$ was often omitted, while the omission of scales from one or both of the axes was a frequent error.

6) Most candidates could show the given result in part (i).

   A significant number of candidates thought that the induction hypothesis in (ii) was $u_k = 2k + 4$, instead of u_k is divisible by 2, while many candidates did not try to establish that u_{k+1} was divisible by 2. Once again a large number of candidates did not give a satisfactory explanation of the induction conclusion.

7) Parts (i) and (ii) were generally answered correctly

   Most candidates were able to find the sum and product of the new roots, but many failed to give their answer as a quadratic **equation**, and omitted "= 0" from their solution.

8) This question proved to be quite demanding. Some candidates seemed very unsure of the factorial notation and were unable to establish the given result in part (i).

Part (ii) was generally answered well, with the cancelling of the terms clearly demonstrated.

The explanation of the divergence of the series in part (iii) was often poorly attempted, many candidates stating that the "the terms get larger" or similar expressions when a more detailed explanation was required.

9) Many candidates did not indicate clearly the scales on one or both axes, or did not show the calculations for the images of the vertices of the unit square.

The rotation R was usually described fully, but the direction of rotation was often incorrect or omitted.

Finding that S was a stretch often proved quite difficult, many candidates thinking that it was an enlargement, while others thought that it was a stretch with the x-axis, rather that the y-axis, invariant.

10) A good proportion of candidates worked concisely and with total accuracy, which was most pleasing. The most common errors occurred in finding the cofactors or in the evaluation of the determinant, which was often omitted at some stage of the solution. Those candidates who tried to solve the equations in (ii) by algebraic methods, rather than by 0using their inverse matrix, usually made an error in one or more of the required values.

A small, but significant minority, assigned a numeric value to a, and if this occurred at an early stage, were penalised quite heavily.

4726: Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible, answering the questions in the order set. The majority of candidates also completed the paper, although a small number appeared rushed at the end. Problems of timing were usually a result of a poor choice of method or of a lack of precision in algebraic manipulation. A surprising number of candidates wasted time by not reading the question well enough. Candidates continue to lose marks in questions such as Q3(i) where an answer to be shown is given. Candidates should be encouraged to ensure that all relevant steps are shown to derive such an answer.

The early questions successfully gave candidates the chance to pick up marks, with many candidates gaining full marks on Qs1 and 2. However, candidates in general did not appear to be prepared for the whole range of questions. No single question proved to be a problem for a large number of candidates, but some questions were problematical for candidates from some centres. Nevertheless, there were a number of outstanding scripts with original solutions.

Comments on Individual Questions

1) This question was successfully completed by the majority of candidates. Those who opted to complete part (ii) by using $\ln(3+x) = \ln 3 + \ln(1+\frac{1}{3}x)$ and the standard Maclaurin expansion of $\ln(1+x)$ were not penalised.

2) Again, this question was successfully completed by most candidates. Marks were lost by candidates who merely worked out f(0.8) and f(0.9) without indicating or stating that a change in sign had taken place. The Newton-Raphson process was done very accurately. However, a surprising number of candidates failed to read the question and continued with further approximations to find the root correct to 3 decimal places. Such candidates did not lose any marks, but they wasted time in so doing.

3) (i) This was considered to be a number-crunching question by many candidates. It was expected that some consideration would be given to the inequality between the area under the curve and the area of the rectangles, and a comment as brief as "A< area of rectangles" was acceptable. The answer to be derived was given, so that a value to at least 3 decimal places was required to show that the 1.71 had been arrived at. It was fortunate that the answer rounded up to the required value, and no marks were lost just by so doing.

(ii) A sketch was not necessary, but one helped to ascertain the correct areas and inequality. Many candidates lost marks by claiming that A>1.276 meant that A>1.28 followed. Others merely produced the 1.28 without explanation.

4) (i) The sketches were generally known, although candidates showed imprecision at the approaches to asymptotes.

(ii) The substitution proved difficult for many candidates, with the omission of $du = e^x dx$ being surprisingly common. It was pleasing to see that many candidates used the Formulae Booklet to write down the answer to the integral, but others were equally successful using partial fractions methods, albeit with some time loss. Marks were lost by using x instead of u when quoting the answer, or by not replacing x for u in the final answer. No marks were lost by omitting the constant of integration.

5) (i) The method of integration by parts was well known. However, there were many minus signs floating about and this led to some candidates glossing over details to reach the answer. Others were not precise in the use of limits. The best solutions clearly showed the derivation of $(\frac{1}{2}\pi)^n$ and the use of 0 as a limit, with clear brackets to show the number of minus signs involved.

(ii) Candidates could usually derive I_4. The use of the formula to produce I_0 was sometimes seen, whilst other candidates wasted time by integrating to find I_2.

6) (i) It was expected that candidates would just write down the equations of the asymptotes by inspection. Those who did gained full marks. Others reverting to partial fraction methods (often without dividing out) wasted time. The vertical asymptotes were often found accurately (though some missed the ±), but the horizontal one proved harder. The given diagram clearly indicates a horizontal asymptote, so perhaps candidates did not make full use of what was given to them.

(ii) With three of five marks available to candidates giving asymptotes and crossing points as requested, it was surprising how many candidates gave a rough sketch with no detail, presumably from calculators. Because of this, full marks were rare. Again, the horizontal asymptotes proved most difficult, and candidates were not always precise in showing the manner in which the curve crossed the x-axis.

7) (i) A few candidates used a substitution of $t^2 = u$ (say) which simplified the partial fraction. Others were less precise, and full marks were only given for complete methods. Even so, the majority scored a minimum of two marks. A number of candidates used values of t to produce the constants. This wasted time compared to candidates who equated coefficients.

(ii) In using the t-substitution, it is expected that candidates will quote the results for $\sin x$, $\cos x$ and dx in terms of t. There is no need to derive these results. The algebraic manipulation proved difficult for some candidates, but a substantial majority produced the expression in part (i) and the correct answer.

8) (i) The definition was usually given accurately, although some candidates had obviously learnt a definition in terms of e^{2x} only. Such candidates gained the mark.

(ii) This is a standard piece of work which appeared to be new to many candidates. It generally resulted in either zero or three marks. Candidates should expect to prove standard results, even if they are given in the Formulae Booklet.

(iii) Candidates who could not do part (ii) recovered in this part by reverting to basic hyperbolic definitions and producing a quadratic equation in e^x. This manipulation was often long and resulted in more time loss. Those who used part (ii) produced the same result much quicker.

(iv) This part largely involved the use of log rules. These were not applied with sufficient accuracy by many candidates. In particular the equation in the form ln A = ln B, or equivalent, was often not arrived at. Those who spotted that the left-hand side was equivalent to $\frac{1}{2}\ln(1 - x^2)$ were most successful.

9) (i) The best solutions gave a table of values of θ against r so that initial and final values could be seen to give an impression of scale. In particular, it was expected that r should be clearly increasing. Candidates who produced more of the curve than was requested were not penalised.

(ii) This part showed that candidates had an awareness of what was in the Formulae Booklet, with the majority scoring over half marks. Again it was expected that candidates would quote results for the integrals of $\sec^2\theta$ and $\sec\theta\ \tan\theta$, with the challenge being to integrate $\tan^2\theta$. It was pleasing that so many candidates were up for the challenge, with minor errors in the formula in terms of $\sec^2\theta$ gaining some credit.

(iii) The most successful candidates eliminated θ first and then r. Any correct cartesian equation gained full marks. There was no need to make y the subject of the equation, nor to simplify it greatly. Many errors were seen, especially in attempting to square difficult expressions, but these were not penalised if an earlier correct version was seen. The neat $y = (x-1)\sqrt{(x^2+y^2)}$ was quickly produced by the better candidates.

4727: Further Pure Mathematics 3

General Comments

The entry for this session was small because most centres use the whole of Year 13 to prepare their candidates for this unit. The standard and presentation of most candidates' answers was good, and none of the questions proved to be too demanding, although the algebraic manipulation in Q4 was not always done successfully. Q7 appeared to take longer for some candidates to write out than had been expected, and as a result attempts at the last two parts of Q8 were not always seen.

Comments on Individual Questions

1) This question was answered well, with full marks often being obtained.

   (i) Most answers showed that the table was not closed, by stating that the product of any one of 3, 5 and 7 with itself gave 1. A smaller number gave the absence of an identity as a reason, which is clear if the operation table is written out.

   (ii) This was almost always correct, although occasionally 9 was stated which was not accepted as it should have been reduced modulo 8.

   (iii) Most candidates gave a correct reason for the two groups not being isomorphic, usually by making appropriate statements about the orders of the elements. Alternatively, some used the cyclic and non-cyclic properties of the groups.

2) This was a standard piece of work which nearly all candidates answered correctly. The more popular approach was to use the vector product to obtain the direction of the line of intersection, and then to find a point on the line by putting x, y or z equal to 0. The alternative method of solving the cartesian equations of the planes in terms of a parameter was straightforward. The latter method was only marginally longer in this case.

3) (i) This was usually done correctly, with the quadratic equation formula being used first to obtain the roots in cartesian form, and then finding the values of r and θ. Occasionally an incorrect second value of θ was given.

   (ii) The first mark, for writing 6^{-3} in some form, was usually gained. Most candidates realised that an easy application of de Moivre's theorem was then needed, but a considerable number of answers stopped at expressions such as $\frac{1}{216}(\cos\pi - \mathrm{i}\sin\pi)$ or $\frac{1}{216}\mathrm{e}^{\mathrm{i}\pi}$. Students studying complex numbers at this level should surely be familiar with the result $\mathrm{e}^{\mathrm{i}\pi} = -1$. A few candidates thought that Z^{-3} was the same as $\sqrt[3]{Z}$.

4) (i) The expression $y = xz$ was usually differentiated correctly and the subsequent substitution was then carried out correctly.

(ii) This part was a good test of candidates' ability. Almost all earned the first mark, for separating the variables and writing integrals, and many then integrated both sides to logarithmic forms, often correctly. But it was the second accuracy mark, for exponentiating both sides and dealing with the arbitrary constant, which caused some problems. In general, those who had written the arbitrary constant as $+\ln c$ were successful, while those who wrote $+k$ were not. The substitution back to variables x and y was usually carried out at an appropriate stage, but only the best candidates scored the final mark which was essentially for a complete solution with all stages worked accurately.

5) This question was answered well by the majority of candidates, many scoring full marks.

(i) Most answers were correct.

(ii) Justification of the orders of pq and pq^2 being 3 was done well. The key steps were to use the orders of each of p and q, and to use the commutativity of the group.

(iii) Subgroups of order 3 only were stated by almost all candidates.

(iv) Many answers gave both subgroups correctly, often without any working. The work done in part (ii) had, of course, indicated which elements needed to be considered. The inclusion of the request to simplify the elements was to avoid the use of powers of p and q beyond 2, but some alternatives were allowed since both groups could clearly be written neatly in the form $\{e, r, r^2\}$.

6) The first order differential equation in this question was intended to be solved by the C.F. and P.I. method which is perhaps more familiar as a method for second order linear equations, but examiners were surprised to find that a substantial number of candidates, perhaps the majority, used the integrating factor method. Of course, both methods are possible in this case, and could score full credit, but the C.F. and P.I. method was a little shorter.

(i) The request for the complementary function was a pointer towards the intended method of solution, and those who used an integrating factor obtained the mark only if they explicitly stated the correct C.F. somewhere in their answer. In fact, many such answers either ignored this part altogether, or gave the integrating factor e^{3x} as if it were the C.F.

(ii) Both methods of solution were usually followed through correctly, although as the use of an integrating factor required integration by parts there were some sign and factor errors. Those who found a particular integral were usually accurate in their work. Both methods benefited from a follow-through mark at the end for the general solution.

(iii) The normal procedure for finding the solution with these given conditions is to differentiate the general solution, but in this case an alternative approach was used by some candidates, namely to use the original differential equation with the two conditions to find y, and then to find the value of the arbitrary constant.

(iv) Surprisingly, this part was not well answered. When x is large and positive, the solution approaches the straight line which is the linear part of the solution just found. But it was quite common for the constant term to be omitted from this, giving a line parallel to the correct one. Perhaps candidates thought that the extra $\frac{1}{9}$ was not important, but the fact is that the solution curve becomes arbitrarily close to $y = \frac{2}{3}x + \frac{1}{9}$, not to $y = \frac{2}{3}x$. Another common incorrect answer was that y goes to infinity; those who gave this answer did not appreciate the idea of an asymptote.

7) Although this was quite a long question to work through, it was done very well indeed, with many candidates obtaining at least 11 of the 13 marks. As usual with vector problems of the type tested in this specification, different methods were available to candidates, and any correct method could earn full marks. Nevertheless it is worth pointing out that some methods are shorter than others, and if a particular approach is not required then candidates should try to use a shorter method, so that they do not get lost in unnecessary algebra. In particular, when the equation of a plane has to be found and no special form is specified, it is usually quicker and more reliable to use the vector product method than to use the two-parameter form. The former requires just one vector product and one substitution, whereas the latter often entails using the three linear equations and some elimination. In this question parts (i) and (iv) were done better using the vector product method. It may well be that candidates who used the longer methods were those who found themselves short of time in Q8. But it was pleasing to find that the majority of candidates were able to work through a long question accurately.

(i) After writing down the equation of AG, it was necessary to find the equation of the plane BCD. As mentioned above, the working is shorter using a vector product than using the parametric form. In both cases, a general point on AG was substituted into the equation of the plane to find the point of intersection, and nearly all answers showed the given result without error.

(ii) The ratio of 3 : 4 or an equivalent value was found correctly by most candidates.

(iii) The correct position vector for P was found correctly in nearly all cases.

(iv) The equation of the plane ABD had to be found, and the same remarks about method apply as in part (i). It was a straightforward matter to check that the position vector of P satisfied the equation, or that the three linear equations were consistent when the coordinates of P were substituted, but again the latter method took longer.

8) A number of candidates did not progress beyond part (ii) of this final question. Similar problems have been set before, although they have not always been done well, and it is likely that such candidates were short of time, possibly because of the length of their solutions to Q7.

(i) This was a standard piece of bookwork, which was done quite well, although the fact that the answer was not given may have contributed to there being fewer completely correct answers than expected. There is quite a lot to write in obtaining trigonometrical identities by de Moivre's theorem, and examiners found fewer candidates than usual who used an abbreviated notation: it is perfectly acceptable to use c and s for $\cos\theta$ and $\sin\theta$, without explanation, and the use of this notation is also less likely to lead to errors.

(ii) For those who had done part (i) correctly, it was straightforward to invert their answer and to replace $\tan\theta$ by $\dfrac{1}{\cot\theta}$. Some others gained the mark by returning to an earlier version of $\tan\theta$ in terms of $\cos\theta$ and $\sin\theta$.

(iii) The three stages in this part were to identify x with $\cot^2\theta$ in the identity of part (ii), to put $\cot 4\theta$ equal to 0, and to obtain or verify $\theta = \frac{1}{8}\pi$. Most candidates who reached this part gained at least one of the three marks.

(iv) This was not an easy last part, and it required verification, at least, of the other solution to the quadratic equation of part (iii). Then the sum of the roots of the quadratic was used and the final step used the identity linking $\cot^2\theta$ with $\operatorname{cosec}^2\theta$. There were a few correct answers, some of which included variations such as solving the quadratic in surd form to obtain the sum of the roots.

4728: Mechanics 1

General Comments

The standards achieved by candidates were generally high, and few poor scripts were seen. Each question posed some challenges, and these were successfully met in the majority of solutions seen. The difficulties encountered in individual questions are identified below. The only general problem was confusion caused by the use of F to mean variously:

- the sum of several forces,
- a force of magnitude *ma*,
- a specifically frictional force,
- any individual force.

The trend towards more work being done on calculators continues, and is welcomed. However, candidates must appreciate that marks are awarded only for what appears in the answer booklets. A lack of detail in the expressions evaluated, or equations solved, necessarily restricts the marks that can be gained. In particular, quoting a standard formula and the values to be substituted is regarded as an incomplete method.

A less welcome trend is the compressing of all answers into one or two pages of the answer booklet. Annotating such scripts with the marks awarded at each stage of a solution cannot be done clearly. Still worse are those scripts where earlier working has been deleted and over-written, as the solutions, though clear to the candidate, may be impossible to interpret.

These strictures apply to relatively few candidates. The large majority of candidates present both their methods of solution and details of their calculations clearly.

Comments on Individual Questions

1) (i) This was extremely well answered, and almost all candidates gained full marks. A minority of scripts contained solutions based on the evaluation of *ma* and its subsequent use as a force in an apparent statics approach. In such scripts, sign errors were common, and candidates experienced difficulty in gaining full marks.

(ii) Both main ways of solving this part required choices to be made of which forces to include or omit. Those candidates who used an equation of motion for the combined car and trailer tended either to omit the resistance on the trailer, or include the tension in the tow-bar. Conversely, when the car alone was considered, the tension in the tow-bar was ignored.

2) Almost all candidates gained full marks on this question. The most common problems seen concerned the appropriate use of calculators.

(i) The invitation to show the y component of the resultant to be zero can be restated as a request to show that $15\sin\alpha + 11\sin\beta - 13 = 0$. Without clear evidence of the steps taken in the enumeration of the left hand side of this equation, full marks could not be awarded.

(ii) In contrast where no printed result was given, $15\cos\alpha - 11\cos\beta = 7.8$ was regarded as sufficient evidence both of the method employed and the accuracy of its use. A small minority of candidates marred their solution by combining their 7.8 N force with the 13 N force.

(iii) This was correctly answered in nearly scripts. Candidates who evaluated angles α and β were given all the marks in part (i) even though the sum of the forces could be only approximately zero. However, in part (iii) it could not be demonstrated that the resultant force was precisely along the positive x-axis, and the final mark was accordingly withheld.

3) A small but significant number of candidates did not appreciate the statics nature of the question. In such cases, the approach taken was similar to that for two particles attached to the ends of a string which passes over a pulley, both portions of the string being vertical. Though some credit could be awarded for such an analysis, the complexity of having the block and particle accelerate made significant progress impossible. Scripts in which limiting equilibrium was understood very often saw the award of all the marks in part (i), though part (ii) might be marred by the inclusion of too many or too few forces.

4) (i) It was pleasing to see nearly all candidates handle this part successfully.

(ii) Part (ii) began, in most cases, with the correct evaluation of the total momentum before the second collision. Two distinct errors were noted: the use of a speed of 1 ms^{-1} for particle M (a value obtained as the answer to part (i)), and the deliberate omission of the minus sign of one momentum, giving the answer 1.55 $kgms^{-1}$.

Making further progress in part (ii)(a) required candidates to explain that 0.85 $kgms^{-1}$ was the *total* momentum *after* the collision *shared* between the two particles M and N. Though candidates were expected to comment on the directions possible for the motion of the particles after the collision, the scripts were not required to contain mention of the impossible situation of one particle passing through the other. Part (ii)(b) was answered correctly by most candidates, including some who added momentums in part (ii)(a).

5) Candidates showed competence in handling variable acceleration, with accurate integration, then changing successfully to the correct use of constant acceleration formulae in part (iii). Where candidates continued to use integration in part (iii), a failure to evaluate constants of integration was the main error. A minority of candidates used constant acceleration formulae throughout, but could gain some marks in part (iii). The solution of part (iii) starts with candidates using the expression for v found in part (i) to calculate the velocity of the particle when $t = 4$. It was not uncommon for the formula $s = vt – at^2/2$ to then be used (rather than the correct $s = ut + at^2/2$).

6) Very many scripts showed that candidates understood the geometric significance of (t, v) diagrams, and used ideas of slope and area appropriately. A minority, however, used constant acceleration formulae throughout.

(i) Use of the formula $s = (u+v)t/2$ was quite common, with the incorrect assumption that when $t = 25$ the maximum velocity occurs. Clearly from the graph, $v = 0$ when $t = 25$. A smaller number of candidates treated the initial motion as symmetric, so that the maximum velocity was reached after 12.5s when the particle had travelled 4m. Though both approaches had some merit, and give the correct value for v, neither was regarded as a correct solution.

(ii) This was entirely correct in almost all scripts.

(iii) Many scripts contained a correct approach to part (iii), though in a few cases the area under the graph was equated to 40m rather than 32m. Frequently candidates subtracted the distance covered while the hoist accelerated, and correctly answered the question by considering only the constant speed and deceleration portions of the graph.

(iv) This was well answered by a most candidates, with the final correct value of $0.04ms^{-2}$ often seen.

7) Parts (i) and (iii) of this question were tackled well by most candidates, weak understanding being seen most often in part (ii). Many scripts contained a correct calculation of acceleration either up or down the plane, but not in the other direction. Often the outcome of these calculations was to have equal accelerations in both directions. As noted in previous sessions, relatively few candidates were content to have two negative forces (friction and the component of weight) in an equation of motion for a particle moving up a slope.

4729: Mechanics 2

General Comments

A large number of able candidates were entered for this paper. There were only a few candidates who were incorrectly entered and showed minimal understanding. Generally, candidates who lost marks often did this through lack of care or not answering the precise question. As before, examiners advise candidates to be spatially aware, to use clearly labelled diagrams and to take care with basic geometry and trigonometry. Qs 4 and 8 (ii) caused the greatest difficulty. The majority of candidates appeared to have sufficient time to answer all the questions.

Comments on Individual Questions

1) The majority of candidates scored full marks. Where errors did occur it was with dimensions, the diagram (a few treated it as a cone) and finding an incorrect angle.

2) The majority of candidates scored full marks. Errors occurred when candidates were not consistent with the directions of motion after the collision. Clear diagrams with arrows are helpful to the candidate and examiner. Candidates who used $e = 1$ for perfect elasticity were more successful than those who used conservation of kinetic energy.

3) The majority of candidates realised that the speeds immediately before and after impact with the ground were required. Some overlooked this and used the initial speed of 21 m s^{-1}. Part (ii) was less well answered and for many it was the first point in the paper where marks were lost. As in previous papers, the change of direction was ignored when calculating the impulse.

4) The majority of candidates approached the problem through work done and change of energy. However, many candidates omitted either potential energy or the work done against the frictional force. A few candidates incorrectly used $\frac{1}{2}m(v-u)^2$ for the change in kinetic energy. Likewise, candidates who used "$F = ma$" often omitted the weight component or the friction. Disappointingly, a significant number of candidates were confused and treated the situation as if the skier was in equilibrium.

5) The given answer in part (i) helped candidates establish their solutions to this problem. Only a few candidates ignored resistance in part (ii). Part (iii) was very well answered with a large number of solutions correctly solving the quadratic in v^2. A small number of candidates glossed over details in their working to avoid the quartic equation.

6) This question was found to be straightforward for candidates who knew how to locate the centre of mass of the triangle at 4 cm to the right of *C* and 6 cm above *E*. Mistakes were made when medians were drawn and angles calculated. The majority showed understanding in the way they answered part (ii). Part (iii) was less well answered with some incorrectly resolving vertically and ignoring the force at *B*. Mistakes were also made in calculating moments about *B*. The distance *BD* was often given as 12 cm.

7) Problems encountered with this question frequently involved poor resolution of the tension in the string, not realising whether the contact force was zero or not, and confusion between v and ω. Many gained some marks in parts (i) and (ii) but often did not realise the change in situation in part (iii) and continued with the same tension.

8) Part (i) was well answered but part (ii) was correctly answered by a minority. A common error was to give the final answer to the distance to the missile as just the horizontal distance. A small number of candidates quoted the formula for the trajectory of a missile but did not know what to do with it. A few successfully differentiated it and scored full marks. For many, dealing with locating the moment at which the missile was moving downwards at an angle of 10° proved a stumbling block.

4730: Mechanics 3

General Comments

Candidates were very well prepared for examination and most demonstrated a good working knowledge of the topics examined. Qs 3, 5 and 6 were particularly well attempted.

The modal mark was 71. The reason for failing to obtain the maximum mark (72), for almost all of the candidates who obtained 71 marks, was the omission of the requirement to 'state ….' In either Q3(ii) or Q7(ii).

Comments on Individual Questions

1) This question was well attempted. Most of the errors that arose were made because of a misunderstanding of what was required. In particular some candidates found the tension when *P* is vertically below *O*, and thus had no occasion to apply the principle of conservation of energy.

2) Most candidates appreciated that impulse is a vector quantity. This was demonstrated by the use of components or by the use of a vector diagram. The main errors made were the omission of the mass in obtaining the components of the change in momentum, a sign error in obtaining the vertical component of the change in momentum, and choosing the acute angle instead of the obtuse angle in evaluating $(150 - \theta)^{\circ}$ from the equation $\sqrt{604}\sin(150 - \theta)^{\circ} = 28\sin 60^{\circ}$.

3) This question was very well attempted, with candidates taking moments of forces and resolving forces vertically, as appropriate, in parts (i) and (ii). The requirement to state the direction of the frictional force in part (ii) was frequently not met.

4) A majority of candidates used x as the symbol for the extension, rather than as defined by the question. In part (i) this was of no consequence and almost all candidates obtained the distance *OP* correctly.

   Part (ii) was poorly attempted by many candidates. In most of the poor attempts the 'proof' amounted to no more than '$\omega^2 = \lambda/\mathrm{mL} = 25$ and hence P moves with SHM'. In other cases problems arose because candidates obtained $\ddot{x} = g - 25x$. In none of these cases did candidates state that their x is the extension (and not the x of the question). Indeed in most such cases candidates got rid of the unwanted g by discarding the weight term from their originally correct Newton's second law equation.

   The relationship $v^2 = \omega^2(A^2 - x^2)$, and its particular case $v_{\max}^2 = \omega^2A^2$, were well known. They were used appropriately to find the amplitude in part (ii) and the required value of v in part (iii). However in almost all cases the answer for v was given as the positive square root of v^2 instead of the negative root.

   Many candidates used $x = A\cos\omega t$ instead of $x = A\sin\omega t$ in part (iii), without attempting to reconcile their expression for x with $x(0) = 0$. Similarly $v = -A\omega\sin\omega t$ was often used instead of $v = A\omega\cos\omega t$, without an attempt to reconcile the expression for v with $v(0) = 1.6$. Some candidates obtained incorrect answers for x and v by using the relevant value of ωt as 2 degrees instead of 2 radians.

5) Almost all of the candidates scored both marks in part (i) of this question.

Part (ii) was well attempted, although a significant minority of candidates omitted the constant of integration. Candidates should be aware of the critical importance of the constant, or its equivalent when using definite integrals, in questions of this type. In almost all cases for which the work leading to the expression for v^2 had gone awry, including that for which the constant was absent, the limit of v^2 is nevertheless g/k. Thus candidates whose expression for v^2 was incorrect obtained the printed answer for the limiting value and were able to score both of the available marks (following through the incorrect v^2 in the case of the accuracy mark).

Part (iii) was well attempted, with most of the candidates who obtained a correct expression for v^2 in part (ii) scoring all three marks in part (iii). In most other cases the expression found for v^2 was such as to allow both method marks in part (iii) to be scored. Almost all of the relevant candidates did so.

6) The topic of the question was clearly well understood by candidates and gaining full marks for the question was common.

Some candidates who made errors in part (ii), usually of sign, were nevertheless able to score all of the marks in part (iii). However some candidates failed to appreciate that the approximate value of 88.1°, or calculated values based on the 88.1°, should not be used in part (iii).

7) Part (a) of part (i) was often omitted or abandoned following attempts in which the candidates made no progress. Candidates who realised the need to resolve forces on P tangentially were usually successful in obtaining the printed equation for α.

Most candidates demonstrated knowledge of the method required in part (b), but very many worked in degrees instead of radians.

Part (ii) was omitted by some candidates, but those who did attempt this part used the principle of conservation of energy. However mistakes were made aplenty, in respect of both gravitational potential energy and elastic potential energy.

The requirement to state whether the speed is increasing or decreasing was often omitted. Some candidates included extensive analysis in their answer. However the expectation of the examiners was that candidates would simply observe, but not necessarily state, that P is beyond the equilibrium position when $AOP = 0.8$ radians, in concluding that P is slowing down.

4732: Probability and Statistics 1

General Comments

Most candidates showed a good understanding of much of the mathematics in this paper. There was a wide range of total marks, including some very high ones. Most candidates scored some marks on each question. A few candidates appeared to run out of time, although the majority finished comfortably. Many candidates failed to fill in the question numbers on the front page of their answer booklet.

A few candidates ignored the 'Instructions to Candidates' on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. Some missed the specific instruction in Q8(ii) to give answers to an appropriate degree of accuracy. In a few cases, marks were lost through premature rounding of intermediate answers. Particularly in probability questions, an exact answer, expressed as a fraction, is the best form of answer. Many candidates unnecessarily converted their fractional answers to decimals.

This paper did not require much algebraic understanding, but in one question which did so, Q9(iii), responses frequently showed poor algebraic technique. Arithmetic was often poor in Qs 2, 5 and 8.

In questions requiring written answers, candidates usually made some reference to the context. However, many candidates wrote long essays for parts worth only a single mark. Candidates should learn to use the mark tariff as some indication of how long to spend on each part.

The only part on which very few candidates scored was Q8(iv), where the wording of the question required a very specific reference to the context.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

Happily, few candidates appeared to be unaware of the existence of MF1. However, some candidates tried to use the less convenient version from the formula booklet involving, e.g., $\Sigma (x - \bar{x})^2$ (Q2) or $\Sigma (x - \bar{x})^2 f$ (Q5). The volume of arithmetic involved in this version led to errors in most cases. In other cases, the formula was simply misunderstood, e.g. $\Sigma (x - \bar{X})$ was interpreted as $\Sigma x - \bar{X}$. Some candidates used their own versions of the formulae for b and/or r, with varying success. When finding E(X), some candidates found Σxp correctly but went on to divide by the number of values of x or by Σx.

Candidates would benefit from direct teaching on the proper use of the formulae booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

The formulae for the mean and standard deviation of a frequency distribution are not given in MF1. Many candidates quoted them incorrectly, sometimes omitting the "f" or, more seriously, attempting $\Sigma x^2 f / \Sigma xf$ or $\Sigma x/n$ (where n is the number of classes). Others quoted them correctly but misunderstood them, calculating, for example, $\Sigma (xf)^2$ or $\Sigma x \times \Sigma f$ and $\Sigma x^2 \times \Sigma f$. A few tried $\Sigma (x - \bar{x})f / \Sigma f$, and got lost in the arithmetic.

Comments on Individual Questions

1) (i) This question was very well answered.

(ii) Most candidates scored full marks in this part. A few found $(\Sigma x)/4$ or $(\Sigma xp)/4$ or $(\Sigma xp)/6$.

2) (i) Most candidates chose the more convenient forms of the formulae for S_{xx} etc from MF1. Those who chose the other forms often made arithmetical errors. A few candidates thought that $r = (\Sigma xy)/(\Sigma x^2)$.

(ii) Most candidates saw the point that the sample was small but many expressed this inadequately, using words such as "extrapolation". Others gave inadequate reasons such as that the houses chosen might be untypical in some way, or that the sample is random and therefore possibly untypical. Some gave irrelevant comments such as the fact that some houses may contain children.

3) (i) This part was well answered.

(ii) Few candidates recognised that the answer could be written down without working. Many saw the need to use 4!, although many omitted to multiply by 3. Some multiplied by 3!, presumably because there were three possible odd numbers to use as the last digit. Some tried to use combinations. In this part and the next, some candidates found only the number of arrangements, rather than the probability.

(iii) Many candidates attempted to list all the possibilities but omitted some, using only 1 .. .. or only 12 .. .. .. and 21 .. .. Others included too many, using 1 .. .. and 2 .. .. A few used a complement method but included far too few possibilities. Some tried to use combinations.

4) (i)(a) Few candidates appeared to understand that symmetrical distributions have the property that mean = median.

(i)(b) Similarly few appreciated that a "stretching out" to the right will result in mean being greater than median.

(ii) Many candidates stated that the diagrams show more than two outcomes (0 to 8), whereas the geometric distribution depends on there being only two outcomes. Some suggested that the geometric distribution has probabilities that increase with x or that the probabilities should be constant. Some gave unclear answers such as "because the number of events is unknown". Many candidates quoted conditions for a geometric distribution, such as that trials need to be independent or that probability is constant.

(iii) Many candidates omitted this part. Some wrote "because the binomial only has two outcomes".

5) (i) This part was well answered although some candidates used, for example, $b = \Sigma xy / \Sigma x^2$. Those who used $\Sigma (x - \bar{x}) f^2$ etc. got lost in the arithmetic. A few found r instead of b.

(ii) A few candidates confused a and b.

(iii) Some candidates found the estimates, but did not answer the question.

6) (i) This part was well answered.

(ii) Many candidates did calculations such as $(^1/_3)^4 \times {}^2/_3$ or $1 - (^1/_3)^4 \times {}^2/_3$. Others used the long method but found only P(X = 1 or 2). Some added an extra term, either $(^1/_3)^3 \times {}^2/_3$ or $(^1/_3)^{-1} \times {}^2/_3$. Many candidates who attempted the quicker method found $1 - (^1/_3)^4$ instead of $1 - (^1/_3)^3$. Some found $P(X \geq 4)$ or $P(X > 4)$. A few found $^1/_{1/3}$.

(iii) This part was well answered, although a few candidates tried to find Σxp, usually stopping at $x = 4$.

7) (i) Most candidates found the correct probabilities to put on the tree, but many included extra branches representing coins removed after the total already removed was at least 3p. A few gave a correct tree except for one missing branch – the last branch in the series 1p, 1p, 2p. A small number of candidates answered the whole question **with** replacement.

(ii) Those who had drawn a correct tree usually answered this part correctly. Some candidates who had included the extra branches nevertheless answered this part correctly, although many included extra probabilities. Not many spotted the quick method: $^3/_{10} \times {}^7/_9 + {}^7/_{10}$.

(iii) Again, those who had drawn a correct tree usually answered this part correctly. Many, however, added extra probabilities.

8) (i) Many candidates ignored the percentages and just used the digits 1, 2, 3, 4, 5, 6, and 7. Others interchanged x and f, treating the percentages as the variable. Many others treated Household size as a continuous variable, with the first class either $0 \leq x < 1$ or $1 \leq x < 2$, and tried to interpolate. Others drew a cumulative frequency curve, but plotted points at (1, 34.1) etc. instead of (1.5, 34.1) etc. Both these latter methods led to non-integer answers which were not appropriate in this context.

(ii) Many candidates failed to state their assumption for the last class. Some made errors such as $\Sigma x/8$ or $\Sigma x^2 f / \Sigma xf$. Others quoted formulae correctly but misunderstood them, calculating, for example $\Sigma(xf)^2$, or $\Sigma x \times \Sigma f$ or $\Sigma f^{2\,2}$ or $\Sigma x^2 \times \Sigma f$. A few tried $\Sigma(x - \bar{x})^2 f / \Sigma f$, and got lost in the arithmetic. Candidates who treated the variable as continuous either used bogus "midpoints" or even "class widths" as the values of x. Some candidates ignored the instruction to give answers to "an appropriate degree of accuracy". Either two or three significant figures were accepted as appropriate. It is worth noting that this part included three elements: two instructions as well as a requirement for a calculation. Many candidates failed to read the question with care and ignored one or other or both of the instructions.

(iii) This part was worded to allow for a general answer, with only a small reference to context. It was well answered on the whole.

(iv) This part, on the other hand, was worded in such a way as to require detailed attention to the context. Most candidates gave only general answers.

(v) Many candidates understood the point. A few looked at the data with insufficient care and chose Withington because the percentage generally decreases with household size. A few chose Withington because its standard deviation was greater. Some chose Withington because they thought the "exact agreement" in Old Moat would give $r_s = 1$.

9) (i) This part was well answered.

(ii) Many candidates wrote (correctly) ${}^{11}C_0 q^{11} p^0 = 0.05$, but could not simplify this. Some just wrote $q^{11}p^0 = 0.05$, but kept both letters in their subsequent working, and so could make no relevant progress. This is an example of a question in which the rote use of the formula for a binomial distribution is unhelpful. A "common sense" approach leads to a first step of $q^{11} = 0.05$, thereby making the next step potentially simple. Having arrived at this statement, some candidates proceeded to use logarithms. In most cases they misunderstood the relevant rules of logarithms. Others gave as their next step $0.76^{11} = 0.05$, which is approximately correct. Probably this was arrived at by trial and improvement, but it is not sufficiently accurate to gain full marks. Use of the *n*th root key on the calculator was surprisingly rare. A significant number of candidates found q but failed to go on to find p. Many candidates wrote $(1 - p)^{11} = 0.05$ but then went on to write $1 - p^{11} = 0.05$.

(iii) A few candidates tried to use Σxp or q/p^2, but most started correctly with $11pq = 1.76$. Some kept both letters and made no progress. Many went on to give $11p(1 - p) = 1.76$, but had no idea how to solve this equation. Some thought that either $p = 0.16$ or $(1 - p) = 0.16$. Others multiplied out the bracket, but failed to rearrange the equation into the form $f(x) = 0$. Some of those who did rearrange correctly made errors when substituting into the formula for a quadratic equation. Others actually quoted the formula incorrectly, perhaps with a sign error or with b and b^2 interchanged.

4733: Probability and Statistics 2

General Comments

This paper was found relatively straightforward, although this comment should not detract from the fact that a large quantity of excellent work was seen. Many candidates answered questions both confidently and with commendable care, and a large number of scripts were seen in which the only marks deducted were for small slips or omissions; for instance, it was common for candidates to score a total of more than 45 out of 50 on the last four questions.

Many candidates have taken appropriate notice of the warning given in previous Reports to Centres that questions would in future tend not to contain instructions such as "State your hypotheses clearly" or "show all relevant probabilities".

The biggest problems with this paper seemed to be in understanding the questions. It is plain that a substantial number of candidates for this unit do not have English as their first language, and they will inevitably be handicapped in a subject where comprehension and interpretation are essential elements. In any case it is easy to recognise those who take care to read questions carefully.

It was disappointing that many candidates still refuse to give a final conclusion to hypothesis tests in terms of the scenario of the problem. This point has been made in all previous Report to Centres. Answers such as "Insufficient evidence that $p < 0.35$" or "insufficient evidence that the mean has changed" do not score the final mark.

Rather more candidates than usual found themselves short of time. Part of the reason for this was certainly the use of an over-long method to answer Q6(iii).

Comments on Individual Questions

1) For many this was a straightforward start, although weaker candidates made sign errors and as usual the weakest did not find a z-value.

2) This question clearly identified those who could understand the scenario (and it is once again emphasised that "understanding the problem" is an essential aspect of statistics in the real world). There were several common wrong answers to part (i), the commonest being 74. Part (ii) identified those candidates who could tell the difference between "equally likely" and "independent"; to get the first mark it was necessary to make some reference to the way in which the first number was chosen. In part (iii) those who understood what was happening generally scored both marks.

3) Again many found this question easy, though some tried to use a Poisson approximation, and as this is not valid, they scored few marks. A smaller number than usual made errors with the continuity correction.

4) Those who knew the appropriate formula had few difficulties in scoring full marks on part (i). Many also got part (ii) correct, although a variety of mistakes were seen, of which finding the variance by dividing the previous answer by 14 (instead of 70) was the most common. Clearly such candidates were confused as to what n means in this context.

5) Most got parts (i) and (iii) correct, apart from the usual continuity correction problems. In part (ii), which is a common type of examination question, those who could understand the scenario generally used the correct binomial distribution; others tended to attempt a Poisson distribution with mean 20×0.663, or even to give that number as their answer.

6) As usual, the question on continuous random variables proved a rich source of marks for many, and full marks were frequently obtained. In part (iii), most candidates attempted to find the value of the median, and quite a few did not know how to do so (integrating with limits 0 and 1 was common), while some thought that the equation $m^2 + m - 3 = 0$ would factorise. In fact it was unnecessary and time-wasting to do any of this. It is much quicker and easier to show that the probability that X is less than 11/9 is 0.453, and as this is less than 0.5 the median is clearly bigger than the mean.

7) As usual the question on significance tests using a discrete distribution was less well answered. Examiners have the impression that this topic is often under-emphasised by centres. Every year many candidates attempt to use a normal distribution, and as a normal approximation is not valid they score few marks. This year a further common error was in trying to use $P(< 2)$ (where 2 was the observed value), rather than $P(\leq 2)$. As usual those candidates who find merely $P(= 2)$ scored few marks; this is a serious error.

Part (ii) has not been set before. It was pleasing to see not only that many candidates answered it confidently and correctly, but that most who used tables showed both the straddling probability values (as required by the mark scheme). Some candidates used logarithms, almost always correctly.

8) Part (i) was generally well done, with marks being lost for the usual range of omissions. Perhaps because of the wording of the question, relatively few made the mistake of using $\mu = 100.7$. However, candidates should always ensure that they give a negative z-value if, as in this case, the sample mean is less than μ.

With the help of the given answer, many could obtain the result in part (ii)(a), and most could get something like the right form for (ii)(b), although the frequent wrong signs were probably caused by a failure to understand the concept of a Type II error in this context.

There is an important issue regarding the accuracy to which the final answers should be given. Irrespective of the number of significant figures involved, n should plainly be given to the nearest integer. Less obviously, a critical value given as 101 is not appropriate in a context where the values are going to be around 100 and 102. The rubric on the front of the question paper says that non-exact numerical answers should be given "correct to 3 significant figures unless a different degree of accuracy is … clearly appropriate", and there is no doubt that more than 3 significant figures are clearly appropriate here. Thus neither "$n = 123.6$" nor "$c = 101$" would score an accuracy mark.

4734: Probability and Statistics 3

General Comments

The general standard of the candidates was a little higher than in January 2006 and there were few with very low marks. Most candidates were familiar with the relevant statistical concepts, and usually could apply them accurately.

Candidates familiar with procedures sometimes lost marks by
- not using a correctly standardised test statistic,
- not giving an explicit comparison of a test statistic with the relevant critical value or critical region,
- not giving validity conditions in context.

Comments on Individual Questions

1) This proved to be a welcome start to the paper and most candidates scored well. Some did not earn the mark for the last part. It was hoped that candidates would realise that the marks for two papers taken by the same person would probably have a significant correlation (or similar). Some thought that being taught by the same teacher would affect independence.

2) This was also well done, although some candidates lost a mark in part (ii) by not expressing the equations in terms of y and x. The formal expression for the probability density function often omitted $x = 1$ in the range, but part (iv) was mostly well done.

3) As has been stated in previous reports, validity assumptions should be expressed contextually but this did not always happen. Some were expressed in an unacceptably terse manner such as *Normal* rather than *Breaking strength should be normally distributed*. The test was usually applied correctly although some used a normal rather than t-distribution.

4) This question also involved a small sample and an unknown variance so a t-distribution was required for the critical value, and this was used by a majority of candidates. Most obtained the correct negative value for the test statistic, but some changed it to a positive number. This is a practice to be discouraged. Being a straightforward test of technique, this question had generally very high scores.

5) Some did not understand what was required for the estimate in part (i) even though they could then calculate the confidence interval in part (ii). Part (iii) was easy for those who had read the relevant chapter of the Course Book but few interpreted the interval in the hoped-for way. This is, that if a large number of random samples of size 1200 were selected from the population and 90% confidence intervals calculated, then about 90% of the intervals would include p.

6) This was found to be difficult by many candidates but it was disappointing to find that so many were unable to obtain the cumulative distribution function in part (i). The procedure for obtaining the probability density function of Y, where $Y = T^3$, is described fully in the Course Book but many used some unjustified "short cuts". The answer was given in the question so, for full credit, all details were required. Part (iii) could be obtained either from g(y) or from f(t), and both methods were seen. However, the integration was found to be difficult and many gave the value of $\left[-2y^{-\frac{1}{2}}\right]_1^\infty$ or $\left[-2t^{-\frac{3}{2}}\right]_1^\infty$ as -2 .

7) Many excellent solutions were seen in part (i), and only a few gave incorrect hypotheses. Part (b) was almost always correct. Although the procedure for calculating χ^2 in part (c) was known, since the answer of 4.53 was given, the three values should have contained at least three decimal places to gain full credit. For part (d) the number of degrees of freedom was often found correctly but an unlikely significance level of 97½% was given instead of 2½%. Part (ii) required a goodness of fit test which was often seen and correctly carried out. The null hypothesis should have involved the relevant probabilities and needed to be carefully stated. "Colours in the ratio 2:2:1"was often seen.

4736: Decision Mathematics 1

General Comments

The full range of marks was achieved on this paper but some candidates found parts of it to be rather challenging at the top end, in particular those questions that required a deeper understanding of the material rather than just routinely applying standard algorithms. Most candidates were able to complete the examination in the time allowed.

It was disappointing to see basic arithmetic errors and candidates who could not solve simple simultaneous equations. Some candidates did not know the terminology, for example freely interchanging the words ‘node’ and ‘arc’ and it was fortunate for many candidates that spelling is not penalised on this paper; in particular the words ‘vertex’, ‘vertices’ and ‘column’ were frequently spelt incorrectly.

In general, the quality of the presentation of the candidates’ answers was a little poorer than in the past. Some candidates did not number their questions and several candidates presented very untidy work or squashed their answers into tiny little spaces. As in some previous papers, the omission of units was a common problem. Written explanations need not be lengthy but do need to be focussed.

Comments on Individual Questions

1) This was a straightforward start for many candidates and was generally answered well. Some candidates lost marks through poor presentation, such as not writing the weights of the bags when a diagram was used. A few candidates did not apply the first-fit method correctly, bags should have been put into the first bundle that had space rather than just arbitrarily back-filling, and some candidates went into a fifth bundle despite their being only four people who could carry a bundle each. In part (ii), some candidates sorted the list into increasing order instead of decreasing order and some sorted each person’s list instead of the entire list. The occasional omission of a value from the sorted list was treated as a misread.

2) A difficult question for many candidates. Most candidates realised that the variables a, b and c referred to apple, banana and cherry, respectively but fewer specified that the variable was the number of cakes of each type. A good number of the candidates who made a serious attempt at the question were able to find the coefficients 4, 3, 4 in part (ii) and most were able to give the constraint based on the total order size, although some used $\leq$ instead of $\geq$. The upper constraints for a, b and c were usually stated correctly but several candidates omitted the non-negativity constraints. Almost all candidates who attempted the question were able to give an appropriate expression for the objective function.

3) Most candidates found that the sum of the orders was 18, although some had done it by drawing a specific case and then counting. Some of the explanations for parts (i)(b) and (i)(c) were good, but often candidates talked too vaguely or referred to specific attempts to draw the diagram. The candidates who carefully considered the issues of the graph needing to be simple and needing to be connected usually gave the most convincing arguments. Several candidates drew correct graphs for parts (ii) and (iii), those who did not had usually either miscounted the number of arcs or had drawn graphs that were not simple.

4) Many good answers were seen. Some candidates did not show any working on the table in part (i), but most were able to find the minimum spanning tree. A few applied nearest neighbour instead and ended up with a total weight of 23 instead of 16. Answers to part (ii) suggested that several candidates were just guessing. The majority of candidates were able to find the lower bound and upper bound, although a few ignored vertex H completely. However several candidates did not interpret their answers as lengths, so the final answer was often given as 51 rather than 5100m, for example.

5) (i) This was done well by most candidates, but very badly by the weakest candidates. Candidates need to be careful when applying Dijkstra's algorithm, some numerical errors slipped in. Some candidates wrote down unnecessary extra temporary labels.

(ii) Most candidates realised what was required here, a few did not show the pairings and their totals, but most achieved the correct value for the number of speed cameras on the route. Several candidates did not attempt to write down a suitable route and some of those who did made a direct link from A to I rather than using AB and BI.

(iii) Sometimes done well, but many candidates either left this part out or did not see the connection with part (ii). Some candidates wrote down the shortest route from K to A and several claimed that the answer was either the same as in part (ii) or that it was 8 units longer.

6) (i) Most candidates were able to set up an initial Simplex tableau. Some candidates omitted the column corresponding to P and a few had sign errors in the objective row.
When slack variables have been added, the problem becomes:

maximise P

subject to $P - 3x + 5y - 4z = 0$

$x + 2y - 3z + s = 12$

$2x + 5y - 8z + t = 40$

and $x, y, z, s, t \geq 0$

This is represented in matrix form using the tableau:

1	-3	5	-4	0	0	0
0	1	2	-3	1	0	12
0	2	5	-8	0	1	40

(ii) Most candidates understood that the problem with the z-column was that the values in the constraint rows were not positive. Apart from candidates with the sign errors in the objective row, most candidates were able to choose the pivot correctly but the explanations of the pivot choice often omitted at least one part of the process. The pivot is chosen from column x because this has a negative value in the objective row, the ratios $12 \div 1 = 12$ and $40 \div 2 = 20$ are calculated and the smaller ratio chosen. This corresponds to the second row of the tableau so the pivot is the 1 in the x column.

(iii) Some good answers, candidates who showed the operations they were using were able to gain method marks even when they had made arithmetic slips. Most candidates were able to carry out pivot operations of the form

current row $\pm$ *multiple of pivot row*

to get a basis column with a 1 replacing the pivot value and a zero in the other rows. A worrying number of candidates were not able to read off the values of x, y, z and P from their tableau, many claiming that $y = 16$, or reading off the coefficients from the objective row.

(iv) This was a difficult concept and had not been tested before. Most candidates just gave the answer that 'the objective row still has negative values' and went on to the next part of the question. There were some candidates, not always those with the highest overall marks, who were able to interpret the cause of the problem on the tableau in terms of the original constraints and objective and hence could explain why P is unbounded.

(v) Some candidates wrote out the entire new tableau and the result of pivoting this tableau; it was sufficient to describe carefully which elements were changed, and to what values, and which were unchanged. Some candidates did not state the maximum value of P, as requested in the question.

(vi) The question instructed candidates how to show that Q has a finite maximum; some candidates could not carry out the arithmetic accurately and some did not appreciate that Q was not the same as P.

(vii) Again, the question told candidates how to start, but some could not even write down the constraints in this particular case. A number of candidates thought that it was perfectly acceptable to subtract inequalities. Even those who realised that they could work with the limiting case, when the constraints became equations, were often unable to solve the simultaneous equations correctly.

4737: Decision Mathematics 2

General Comments

A good spread of marks was achieved and most candidates were able to complete the examination in the time allowed. The quality of candidates' work has shown a noticeable decline with many spelling mistakes, especially of technical terms.

Comments on Individual Questions

1) A good starter question for most candidates. Some candidates carefully explained why the ratings needed to be subtracted from 6 but then did not do this when they applied the algorithm.

2) Most candidates were able to answer this question successfully apart from drawing the resource histogram. Several candidates produced graphs with no scales or labels and often they left a big 'hole' in the middle of the graph in an unnecessary attempt to keep each activity contained as a rectangular block. This continued confusion between resource histograms and cascade charts has been commented on before.

3) The first three parts of this question should have been straightforward for most candidates. Several candidates gave far too vague an explanation of dominance; the question specifically asked candidates to show that row *X* does not dominate row *Y*, this only required the use of the values in column *Y*. Similarly, to show that column *Y* does not dominate column *Z* only the relevant values in row *Y* were needed.

   Many of the answers to part (iv) were irrelevant to the question that had been asked. Candidates needed to consider the effect on the play-safe strategies of gradually reducing the value 5 in the cell where both players chose strategy *X*.

4) Many good attempts. In part (ii), there is no need to extend the p axis beyond the range 0 to 1, although it is desirable that the lines should extend as far as these limits. Some graphs had an extremely long p axis or an extremely short E axis, making it almost impossible to tell where the plotted lines were intended to be.

   Although most candidates were able to identify the appropriate pair of equations to give the optimal value for p, some just calculated all the intersections and then, incorrectly in this case, chose the pair that gave the biggest value of E. The candidates with the correct value for p were usually able to show that with this value Rowan would expect to lose on average 0.25 per game and hence Colin would expect to win 0.25 per game. The point that was missed by most candidates is that this assumes that Colin also plays optimally, and we do not yet know what Colin's optimal strategy is. If Colin plays less than optimally then he should expect to win less than 0.25 per game, on average.

5) In part (i), several candidates were not able to calculate the capacity of the given cut with the information presented in this form. The use of the labelling procedure was usually correct, and the use of the insert certainly helped with this. In part (iii), some candidates gave a flow augmenting route but then gave the total flow rather than the flow through this route. In part (v), some candidates marked the excess capacities and potential backflows rather than showing the flow using directed weighted arcs.

6) Many very good solutions, although some candidates spent time drawing out the network when they should have worked directly from the table. Some candidates slipped up with stage (3; 0) action 2, and said that min(2, 3) = 3, but this did not affect the final answer. The vertex (0; 0) was often omitted from the maximin route.

7) The majority of candidates were able to draw the bipartite graph and construct an appropriate alternating path to find the complete matching in part (i). The use of the activity network was generally correct, including the correct use of the precedence dummy, but several candidates left out the unique labelling dummy. Some candidates used large numbers of superfluous dummy activities, this was not penalised on this paper but did suggest that candidates had not properly appreciated the purpose of dummy activities. Very few candidates used activity on node, which is no longer part of the specification.

Advanced GCE Mathematics (3892 – 2, 7890 - 2)

January 2007 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	a	b	c	d	e	u
4721	Raw	72	63	55	48	41	34	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	58	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	54	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	54	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	59	52	45	38	32	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	61	54	47	40	33	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	25	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890/3892	300	240	210	180	150	120	0
7897892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	19.1	36.8	59.5	80.6	94.3	100	299
3892	66.7	77.8	88.9	88.9	100	100	9
7890	40.2	62.5	87.5	95.5	100	100	112
7892	50.0	83.3	83.3	83.3	91.7	100	12

For a description of how UMS marks are calculated see;
http://www.ocr.org.uk/exam_system/understand_ums.html

Statistics are correct at the time of publication

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