OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4730/01**

MATHEMATICS

Mechanics 3

MONDAY 21 MAY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 A particle P is moving with simple harmonic motion in a straight line. The period is 6.1 s and the amplitude is 3 m. Calculate, in either order,

(i) the maximum speed of P, [3]

(ii) the distance of P from the centre of motion when P has speed 2.5 m s^{-1}. [3]

2 A tennis ball of mass 0.057 kg has speed 10 m s^{-1}. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to 7 m s^{-1}. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

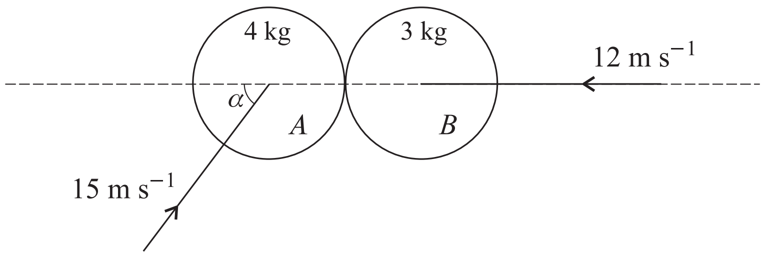
3 A particle P of mass 0.2 kg is projected horizontally with speed $u\text{ m s}^{-1}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\text{ m s}^{-1}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} = -2$. [2]

(ii) Hence show that $v = u\mathrm{e}^{-2x}$. [4]

(iii) Find u, given that $x = 2$ when $t = 4$. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, A is moving with speed 15 m s^{-1} at an angle α to the line of centres, where $\sin\alpha = 0.8$, and B is moving along the line of centres with speed 12 m s^{-1} (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

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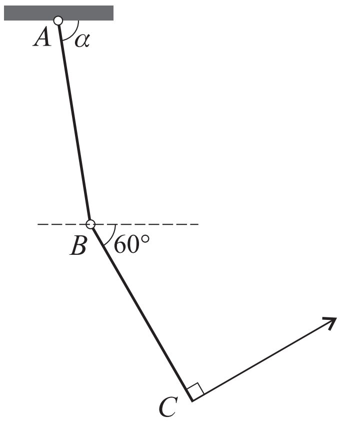

Two uniform rods AB and BC, each of length 1.4 m and weight 80 N, are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. They are held in equilibrium with AB at an angle α to the horizontal, and BC at an angle of $60°$ to the horizontal, by a light string, perpendicular to BC, attached to C (see diagram).

(i) By taking moments about B for BC, calculate the tension in the string. Hence find the horizontal and vertical components of the force acting on BC at B. [7]

(ii) Find α. [4]

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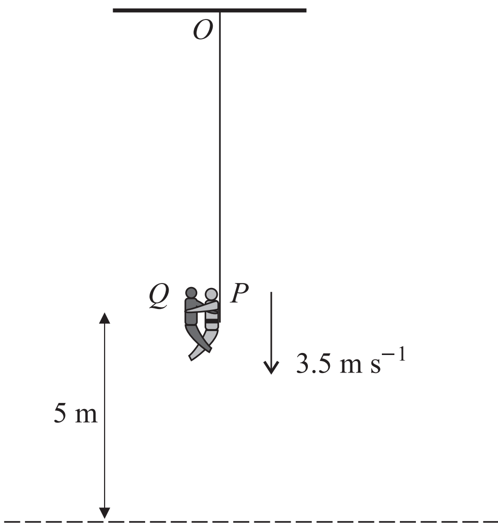

A circus performer P of mass 80 kg is suspended from a fixed point O by an elastic rope of natural length 5.25 m and modulus of elasticity 2058 N. P is in equilibrium at a point 5 m above a safety net. A second performer Q, also of mass 80 kg, falls freely under gravity from a point above P. P catches Q and together they begin to descend vertically with initial speed 3.5 m s^{-1} (see diagram). The performers are modelled as particles.

(i) Show that, when P is in equilibrium, $OP = 7.25$ m. [3]

(ii) Verify that P and Q together just reach the safety net. [5]

(iii) At the lowest point of their motion P releases Q. Prove that P subsequently just reaches O. [3]

(iv) State two additional modelling assumptions made when answering this question. [2]

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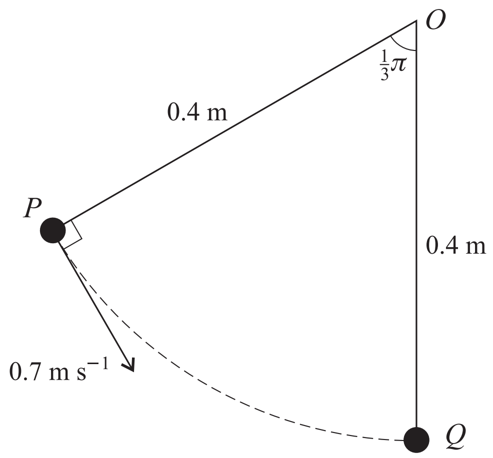

A particle P of mass 0.8 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. A particle Q is suspended from O by an identical string. With the string OP taut and inclined at $\frac{1}{3}\pi$ radians to the vertical, P is projected with speed 0.7 m s^{-1} in a direction perpendicular to the string so as to strike Q directly (see diagram). The coefficient of restitution between P and Q is $\frac{1}{7}$.

(i) Calculate the tension in the string immediately after P is set in motion. [4]

(ii) Immediately after P and Q collide they have equal speeds and are moving in opposite directions. Show that Q starts to move with speed 0.15 m s^{-1}. [4]

(iii) Prove that before the second collision between P and Q, Q is moving with approximate simple harmonic motion. [5]

(iv) Hence find the time interval between the first and second collisions of P and Q. [2]