OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4724/01**

MATHEMATICS

Core Mathematics 4

THURSDAY 14 JUNE 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [2]

(ii) Hence find $\mathrm{f}'(x)$ and deduce that the gradient of the curve is negative at all points on the curve. [3]

2 Find the exact value of $\displaystyle\int_0^1 x^2 \mathrm{e}^x \, \mathrm{d}x$. [6]

3 Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

4 **(i)** Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3, and state the set of values of x for which the expansion is valid. [5]

(ii) Hence find the coefficient of x^3 in the expansion of $\dfrac{1+x^2}{(2+x)^2}$. [2]

5 A curve C has parametric equations

$$x = \cos t, \quad y = 3 + 2\cos 2t, \quad \text{where } 0 \leqslant t \leqslant \pi.$$

(i) Express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t and hence show that the gradient at any point on C cannot exceed 8. [4]

(ii) Show that all points on C satisfy the cartesian equation $y = 4x^2 + 1$. [3]

(iii) Sketch the curve $y = 4x^2 + 1$ and indicate on your sketch the part which represents C. [2]

6 The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point (2, 3) on the curve, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [8]

7 **(i)** Find the quotient and the remainder when $2x^3 + 3x^2 + 9x + 12$ is divided by $x^2 + 4$. [4]

(ii) Hence express $\dfrac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4}$ in the form $Ax + B + \dfrac{Cx + D}{x^2 + 4}$, where the values of the constants A, B, C and D are to be stated. [1]

(iii) Use the result of part **(ii)** to find the exact value of $\displaystyle\int_1^3 \frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4} \, \mathrm{d}x$. [5]

8 The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

9 Lines L_1, L_2 and L_3 have vector equations

$$L_1:\ \mathbf{r} = (5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) + s(-6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}),$$
$$L_2:\ \mathbf{r} = (3\mathbf{i} - 8\mathbf{j}) + t(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}),$$
$$L_3:\ \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + u(3\mathbf{i} + c\mathbf{j} + \mathbf{k}).$$

(i) Calculate the acute angle between L_1 and L_2. [4]

(ii) Given that L_1 and L_3 are parallel, find the value of c. [2]

(iii) Given instead that L_2 and L_3 intersect, find the value of c. [5]