OCR

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT   **4737/01**

MATHEMATICS

Decision Mathematics 2

WEDNESDAY 20 JUNE 2007   Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
                        List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **4** and **5**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **6** printed pages, **2** blank pages and an insert.

                OCR is an exempt Charity   **[Turn over**

1 Daniel needs to clean four houses but only has one day in which to do it. He estimates that each house will take one day and so he has asked three professional cleaning companies to give him a quotation for cleaning each house. He intends to hire the three companies to clean one house each and he will clean the fourth house himself. The table below shows the quotations that Daniel was given by the three companies.

	House 1	House 2	House 3	House 4
Allclean	£500	£400	£700	£600
Brightenupp	£300	£200	£400	£350
Clean4U	£500	£300	£750	£680

(i) Copy the table and add a dummy row to represent Daniel. [2]

(ii) Apply the Hungarian algorithm, **reducing rows first**, to find a minimum cost matching. You must show your working and say how each matrix was formed. [9]

(iii) Which house should Daniel ask each company to clean? Find the total cost of hiring the three companies. [2]

2 The table gives the pay-off matrix for a zero-sum game between two players, Amy and Bea. The values in the table show the pay-offs for Amy.

		Bea		
		Strategy X	Strategy Y	Strategy Z
Amy	Strategy P	4	−2	0
	Strategy Q	−1	5	4

Amy makes a random choice between strategies P and Q, choosing strategy P with probability p and strategy Q with probability $1 - p$.

(i) Write down and simplify an expression for the expected pay-off for Amy when Bea chooses strategy X. Write down similar expressions for the cases when Bea chooses strategy Y and when she chooses strategy Z. [4]

(ii) Using graph paper, draw a graph to show Amy's expected pay-off against p for each of Bea's choices of strategy. Using your graph, find the optimal value of p for Amy. [5]

Amy and Bea play the game many times. Amy chooses randomly between her strategies using the optimal value for p.

(iii) Showing your working, calculate Amy's minimum expected pay-off per game. Why might Amy gain more points than this, on average? [3]

(iv) What is Bea's minimum expected loss per game? How should Bea play to minimise her expected loss? [3]

3   The table shows the activities involved in a project, their durations and precedences, and the number of workers needed for each activity.

Activity	Duration (days)	Immediate predecessors	Number of workers
A	3	–	3
B	4	A	2
C	5	A	2
D	2	B, C	1
E	3	C	3
F	4	D	2
G	2	D, E	2

   (i) Draw an activity network to represent the project, using activity on arc. You are advised to make the diagram quite large. The activity network requires two dummy activities; explain why each of these is needed. [4]

   (ii) Carry out a forward pass to find the early times for the events. Record these at the vertices on your network. Also calculate and record the late times for the events. Find the minimum completion time for the project and list the critical activities. [6]

The number of workers required for each activity is shown in the table. Assume that each worker is able to do any of the activities. Once an activity has been started, it must run for its duration.

   (iii) Using graph paper, draw a resource histogram with each activity starting at its earliest possible start time. [3]

   (iv) Explain why if only four workers are available, the project cannot be completed in the minimum project completion time. Show how the project can be completed in one day more than the minimum project completion time when there are only four workers. [2]

       [Turn over

4 **Answer this question on the insert provided.**

The table shows a partially completed dynamic programming tabulation for solving a minimax problem.

Stage	State	Action	Working	Minimax
1	0	0	4	4
	1	0	3	3
	2	0	2	2
2	0	0	max(6, 4) = 6	3
		1	max(2, 3) = 3	
		2	max(3, 2) = 3	
	1	0	max(2, 4) =	
		1	max(4, 3) =	
		2	max(5, 2) =	
	2	0	max(2,	
		1	max(3,	
		2	max(4,	
3	0	0	max(5,	
		1	max(5,	
		2	max(2,	

(i) On the insert, complete the last two columns of the table. [9]

(ii) State the minimax value and write down the minimax route. [4]

(iii) Complete the diagram on the insert to show the network that is represented by the table. [3]

5 **Answer this question on the insert provided.**

The network represents a system of pipes through which fluid can flow from a source, *S*, to a sink, *T*.
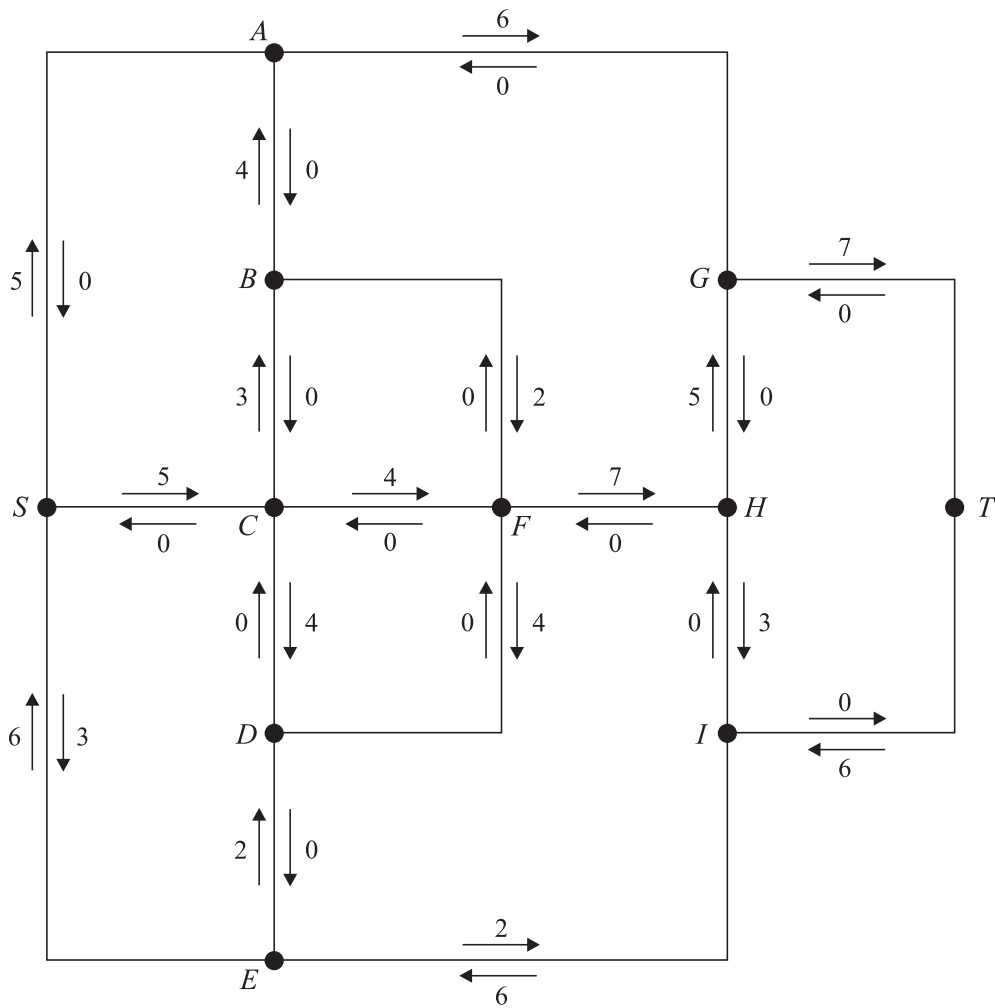
The arrows are labelled to show excess capacities and potential backflows (how much more and how much less could flow in each pipe). The excess capacities and potential backflows are measured in litres per second. Currently the flow is 6 litres per second, all flowing along a single route through the system.

(i) Write down the route of the 6 litres per second that is flowing from *S* to *T*. [1]

(ii) What is the capacity of the pipe *AG* and in which direction can fluid flow along this pipe? [2]

(iii) Calculate the capacity of the cut X = {*S, A, B, C, D, E*}, Y = {*F, G, H, I, T*}. [3]

(iv) Describe how a further 7 litres per second can flow from *S* to *T* and update the labels on the arrows to show your flow. Explain how you know that this is the maximum flow. [7]

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