OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

June 2006

3890-2/7890-2/MS/R/06

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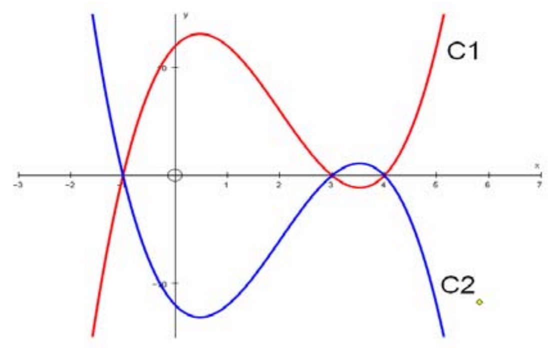
Mark Scheme 4721
June 2006

1	(i)	$\frac{21-3}{4-1}=\frac{18}{3}=6$	M1		Uses $\frac{y_2-y_1}{x_2-x_1}$
			A1	2	6 (not left as $\frac{18}{3}$)
	(ii)	$\frac{dy}{dx}=2x+1$	B1		
		$2\times3+1=7$	B1	2	
2	(i)	$27^{-\frac{2}{3}}=\frac{1}{27^{\frac{2}{3}}}=\frac{1}{9}$	M1		$\frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}}=9$ or 3^{-2} soi
			A1	2	$\frac{1}{9}$
	(ii)	$5\sqrt{5}=5^{\frac{3}{2}}$	B1	1	
	(iii)	$\frac{1-\sqrt{5}}{3+\sqrt{5}}=\frac{(1-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$	M1		Multiply numerator and denominator by conjugate
		$=\frac{8-4\sqrt{5}}{4}$	B1		$(\sqrt{5})^2=5$ soi
		$=2-\sqrt{5}$	A1	3	$2-\sqrt{5}$
3	(i)	$2x^2+12x+13=2(x^2+6x)+13$	B1		a = 2
			B1		b = 3
		$=2[(x+3)^2-9]+13$	M1		$13-2b^2$ or 13 – b^2 or $\frac{13}{2}$ - b^2 (their b)
		$=2(x+3)^2-5$	A1	4	c= –5
	(ii)	$2(x+3)^2-5=0$	M1		Uses correct quadratic formula or completing square method
		$(x+3)^2=\frac{5}{2}$	A1		x = $\frac{-12\pm\sqrt{40}}{4}$ or $(x+3)^2=\frac{5}{2}$
		$x=-3\pm\sqrt{\frac{5}{2}}$	A1	3	$x=-3\pm\sqrt{\frac{5}{2}}$ or -3 $\pm\frac{1}{2}\sqrt{10}$

4	(i)	$(x-4)(x-3)(x+1)$	B1		$x^2-7x+12$ or x^2-2x-3 or x^2-3x-4 seen
		$\equiv(x^2-7x+12)(x+1)$	M1		Attempt to multiply a quadratic by a linear factor or attempt to list an 8 term expansion of all 3 brackets
		$\equiv x^3+x^2-7x^2-7x+12x+12$ $\equiv x^3-6x^2+5x+12$	A1	3	$x^3-6x^2+5x+12$ **(AG)** obtained (no wrong working seen)
	(ii) (iii)		B1		+ve cubic with 3 roots (not 3 line segments)
			B1		(0, 12) labelled or indicated on *y*-axis
			B1	3	(– 1, 0), (3,0), (4, 0) labelled or indicated on *x*-axis
			M1		Reflect *their* (ii) in either *x*- or *y*-axis
			A1√	2	Reflect *their* (ii) in *x*-axis
5	(i)	$1<4x-9<5$ $10<4x<14$ $2.5<x<3.5$	M1		2 equations or inequalities both dealing with all 3 terms
			A1		2.5 and 3.5 seen oe
			A1	3	2.5 < *x* < 3.5 (or '*x* > 2.5 and *x* < 3.5')
	(ii)	$y^2\geq 4y+5$	B1		$y^2-4y-5=0$ soi
		$y^2-4y-5\geq 0$	M1		Correct method to solve quadratic
		$(y-5)(y+1)\geq 0$	A1		-1, 5 (**SR** If **both** values obtained from trial and improvement, award **B3**)
		$y\leq -1,\ y\geq 5$	M1		Correct method to solve inequality
			A1	5	$y\leq -1,\ y\geq 5$

6	(i)	$x^4-10x^2+25=0$ Let $y=x^2$ $y^2-10y+25=0$ $(y-5)^2=0$ $y=5$ $x^2=5$ $x=\pm\sqrt{5}$	*M1 dep*M1 A1 A1	 4	Use a substitution to obtain a quadratic or $(x^2-5)(x^2-5)=0$ Correct method to solve a quadratic 5 (not x = 5 with no subsequent working) $x=\pm\sqrt{5}$
	(ii)	$y=\dfrac{2x^5}{5}-\dfrac{20x^3}{3}+50x+3$ $\dfrac{dy}{dx}=2x^4-20x^2+50$	B1 B1	 2	$2x^4$ or $-20x^2$ oe seen $2x^4-20x^2+50$ (integers required)
	(iii)	$2x^4-20x^2+50=0$ $x^4-10x^2+25=0$ which has 2 roots	M1 A1	 2	*their* $\dfrac{dy}{dx}=0$ seen (or implied by correct answer) 2 stationary points **www in any part**
7	(i)	$y=x^2-5x+4$ $y=x-1$ $x^2-5x+4=x-1$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1 \quad x=5$ $y=0 \quad y=4$	 M1 M1 A1 A1	 4	Substitute to find an equation in *x* (or *y*) Correct method to solve quadratic *x* = 1, 5 *y* = 0, 4 (**N.B.** This final A1 may be awarded in part (ii) if y coordinates only seen in part (ii)) **SR** one correct (*x*,y) pair **www** **B1**
	(ii)	2 points of intersection	B1	1	
	(iii)	EITHER $x^2-5x+4=x+c$ has 1 solution $x^2-6x+(4-c)=0$ $b^2-4ac=0$ $36-4(4-c)=0$ $c=-5$ OR $\dfrac{dy}{dx}=1=2x-5$ $x=3 \quad y=-2$ $-2=3+c$ $c=-5$	 M1 M1 A1 A1 M1 A1 A1 A1	 4 4	$x^2-5x+4=x+c$ has 1 soln seen or implied Discriminant = 0 or $(x-a)^2=0$ soi $36-4(4-c)=0$ or $9=4-c$ $c=-5$ Algebraic expression for gradient of curve = non-zero gradient of line used $2x-5=1$ $x=3$ $c=-5$ **SR** c = -5 without any working **B1**

8	(i)	Height of box = $\frac{8}{x^2}$	*B1		Area of 1 vertical face $= \frac{8}{x^2} \times x$ $= \frac{8}{x}$
		4 vertical faces $= 4 \times \frac{8}{x}$ $= \frac{32}{x}$	*B1		
		Total surface area = $x^2 + x^2 + \frac{32}{x}$	B1 dep on both **		Correct final expression
		$A = 2x^2 + \frac{32}{x}$		3	
	(ii)	$\frac{dA}{dx} = 4x - \frac{32}{x^2}$	B1 B1 B1	3	$4x$ kx^{-2} $-32x^{-2}$
	(iii)	$4x - \frac{32}{x^2} = 0$	M1		$\frac{dA}{dx} = 0$ soi
		$4x^3 = 32$ $x = 2$	A1		$x = 2$
			M1 A1	4	Check for minimum Correctly justified
					SR If $x = 2$ stated **www** but with no evidence of differentiated expression(s) having been used in part (iii) **B1**

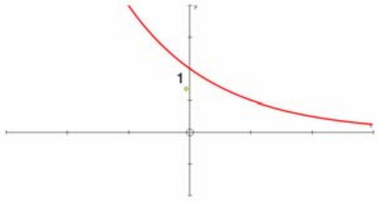
9	(i)	$\left(\frac{4+10}{2}, \frac{-2+6}{2}\right)$ (7, 2)	M1 A1	 2	Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ (7, 2) (integers required)
	(ii)	$\sqrt{(7-4)^2+(2--2)^2}$ $=\sqrt{3^2+4^2}$ $=5$	M1 A1	 2	Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ 5
	(iii)	$(x-7)^2+(y-2)^2=25$	B1√ B1√ B1	 3	$(x-7)^2$ and $(y-2)^2$ used (*their* centre) $r^2 = 25$ used (*their* r^2) $(x-7)^2+(y-2)^2=25$ cao <u>Expanded form:</u> $-14x$ and $-4y$ used B1√ $r = \sqrt{g^2+f^2-c}$ used B1√ $x^2+y^2-14x-4y+28=0$ B1 cao <u>By using ends of diameter:</u> $(x - 4)(x - 10) + (y + 2)(y - 6) = 0$ Both x brackets correct B1 Both y brackets correct B1 Final equation fully correct B1
	(iv)	Gradient of AB = $\frac{6--2}{10-4}=\frac{4}{3}$ Gradient of tangent = $-\frac{3}{4}$ $y--2=-\frac{3}{4}(x-4)$ $3x+4y=4$	B1 B1√ M1 A1 A1	 5	oe Correct equation of straight line through A, any non-zero gradient a, b, c need not be integers

Mark Scheme 4722
June 2006

1		$(3x-2)^4 = 81x^4 - 216x^3 + 216x^2 - 96x + 16$	M1 A1 A1 A1	4 **4**	Attempt binomial expansion, including attempt at coeffs. Obtain one correct, simplified, term Obtain a further two, simplified, terms Obtain a completely correct expansion
2	(i)	$u_2 = -1, u_3 = 2, u_4 = -1$	B1 B1	2	For correct value –1 for u_2 For correct values for both u_3 and u_4
	(ii)	Sum is $(2+(-1)) + (2+(-1)) + ... + (2+(-1))$ i.e. $50 \times (2+(-1)) = 50$	M1 M1 A1	3 **5**	For correct interpretation of Σ notation For pairing, or $50 \times 2 - 50 \times 1$ For correct answer 50
3		$y = 4x^{\frac{1}{2}} + c$ Hence $5 = 4 \times 4^{\frac{1}{2}} + c \Rightarrow c = -3$ So equation of the curve is $y = 4x^{\frac{1}{2}} - 3$	M1 A1 A1 M1 A1√ A1	6 **6**	For attempt to integrate For integral of the form $kx^{\frac{1}{2}}$ For $4x^{\frac{1}{2}}$, with or without +*c* For relevant use of (4, 5) to evaluate *c* For correct value –3 (or follow through on integral of form $kx^{\frac{1}{2}}$) For correct statement of the equation in full (aef)
4	(i)	Intersect where $x^2 + x - 2 = 0 \Rightarrow x = -2, 1$	M1 A1	2	For finding *x* at both intersections For both values correct
	(ii)	Area under curve is $\left[4x - \frac{1}{3}x^3\right]_{-2}^{1}$ i.e. $(4 - \frac{1}{3}) - (-8 + \frac{8}{3}) = 9$ Area of triangle is 4½ Hence shaded area is 9 – 4½ = 4½	M1 M1 A1 M1 A1 A1	6	For integration attempt with any one term correct For use of limits – subtraction and correct order For correct area of 9 Attempt area of triangle (½*bh* or integration) Obtain area of triangle as 4 ½ Obtain correct final area of 4 ½
		OR Area under curve is $\int_{-2}^{1} (2 - x - x^2)\,dx$ $= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1}$ $= (-\frac{1}{3} - \frac{1}{2} + 2) - (\frac{8}{3} - 2 - 4)$ $= 4\frac{1}{2}$	M1 M1 A1 M1 A1 A1	**8**	Attempt subtraction – either order For integration attempt with any one term correct Obtain $\pm\left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]$ For use of limits – subtraction and correct order Obtain ± 4 ½ - consistent with their order of subtraction Obtain 4 ½ only, following correct method only

5	(i)	$\sin^2 x = 1-\cos^2 x \Rightarrow 2\cos^2 x+\cos x-1=0$ Hence $(2\cos x-1)(\cos x+1)=0$ $\cos x=\frac{1}{2} \Rightarrow x=60^\circ$ $\cos x=-1 \Rightarrow x=180^\circ$	M1 M1 A1 A1	4	For transforming to a quadratic in cos x For solution of a quadratic in cos x For correct answer 60° For correct answer 180° [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
	(ii)	$\tan 2x=-1 \Rightarrow 2x=135$ or 315 Hence $x=67.5^\circ$ or 157.5°	M1 M1 A1 A1	4	For transforming to an equation of form tan2x = k For correct solution method, i.e. inverse tan followed by division by 2 For correct value 67.5 For correct value 157.5
		OR $\sin^2 2x=\cos^2 2x$ $2\sin^2 2x=1$ $\quad 2\cos^2 2x=1$ $\sin 2x=\pm\frac{1}{2}\sqrt{2}$ $\quad \cos 2x=\pm\frac{1}{2}\sqrt{2}$ Hence $x=67.5^\circ$ or 157.5°	M1 M1 A1 A1		Obtain linear equation in cos 2x or sin 2x Use correct solution method For correct value 67.5 For correct value 157.5 [Max 3 out of 4 if any extra answers present in range, or in radians] **SR** answer only is B1, B1 justification – ie graph or substitution is B2, B2
				8	
6	(i)	(a) $100+239\times 5=£1295$	M1 A1	2	For relevant use of $a + (n-1)d$ For correct value 1295
		(b) $\frac{1}{2}\times 240\times(100+1295)=£167400$	M1 A1	2	For relevant use of $\frac{1}{2}n(a+l)$ or equivalent For correct value 167400
	(ii)	$100r^{239}=1500 \Rightarrow r=1.01139...$ Hence total is $\dfrac{100(1.01139^{240}-1)}{1.01139-1}=£124359$	B1 M1 A1 M1 A1	5	For correct statement of $100r^{239}=1500$ Attempt to find r For correct value 1.01 For relevant use of GP sum formula For correct value 124359 (3 s.f. or better)
				9	

7	(i)	$AC^2 = 11^2 + 8^2 - 2\times 11\times 8\times\cos 0.8$ $= 62.3796...$ Hence AC=7.90 cm	M1 A1 A1	3	Attempt to use the cosine formula Correct unsimplified expression Show the given answer correctly
	(ii)	Area of sector = $\frac{1}{2}\times 7.90^2 \times 1.7 = 53.0$ Area of triangle = $\frac{1}{2}\times 7.90^2 \times \sin 1.7 = 30.9$ Hence shaded area = 22.1 cm^2	M1 M1 A1	3	Attempt area of sector using $(\frac{1}{2})r^2\theta$ Attempt area of ΔACD, using $(\frac{1}{2})r^2\sin\theta$, or equiv Obtain 22.1
	(iii)	(arc) $DC = 7.90\times 1.7 = 13.4$ (line) $DC^2 = 7.90^2 + 7.90^2 - 2\times 7.90\times 7.90\times\cos 1.7$ $DC = 11.9$ Hence perimeter = 25.3cm	M1 A1 M1 A1	4 **10**	Use $r\theta$ to attempt arc length Obtain 13.4 Attempt length of line DC using cosine rule or equiv. Obtain 25.3
8	(i)	$f(2) = 12 \Rightarrow 4a + 2b = 6$ $f(-1) = 0 \Rightarrow a - b = 12$ Hence a = 5, b = –7	M1 A1 M1 A1 M1 A1	6	For equating f(2) to 12 For correct equation 4a + 2b = 6 For equating f(–1) to 0 For correct equation a – b = 12 For attempt to find a and b For both values correct
	(ii)	Quotient is $2x^2 + x - 9$ Remainder is 8	B1 M1 A1 M1 A1	5 **11**	For correct lead term of $2x^2$ For complete division attempt or equiv For completely correct quotient For attempt at remainder – either division or f(–2) For correct remainder

9	(i)		M1 A1 B1	3	Attempt sketch of any exponential graph, in at least first quadrant Correct graph – must be in both quadrants For identification of (0, 1)
	(ii)	$A \approx \frac{1}{2} \times 0.5 \times \left\{1 + 2\left(0.5^{\frac{1}{2}} + 0.5 + 0.5^{\frac{3}{2}}\right) + 0.5^2\right\}$ ≈ 1.09	B1 M1 A1 A1	4	State, or imply, at least three correct *y*-values For correct use of trapezium rule, inc correct *h* For correct unsimplified expression For the correct value 1.09, or better
	(iii)	$\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow x\log_{10}\frac{1}{2} = \log_{10}\frac{1}{6}$ $x = \frac{\log_{10}\frac{1}{6}}{\log_{10}\frac{1}{2}} = \frac{-\log_{10}6}{-\log_{10}2}$ Hence $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1	4	For equation $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ and attempt at logs Obtain $x\log\left(\frac{1}{2}\right) = \log\left(\frac{1}{6}\right)$, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
		OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $\Rightarrow x\log_{10}2 = \log_{10}6$ $x = \frac{\log_{10}6}{\log_{10}2}$ $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		For equation $2^x = 6$ and attempt at logs Obtain *x* log 2 = log 6, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
		OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $2^{x-1} = 3$ $(x-1)\log_{10}2 = \log_{10}3$ Hence $x = 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		Attempt to rearrange equation to $2^n = 3$ Obtain $2^{x-1} = 3$ For attempt at logs For showing the given answer correctly
		OR $x = \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= \frac{\log_{10}6}{\log_{10}2}$ $x\log_{10}2 = \log_{10}6$ $\log_{10}2^x = \log_{10}6$ $2^x = 6$ $\left(\frac{1}{2}\right)^x = \frac{1}{6}$	M1 A1 M1 A1	**11**	Use log 2 + log 3 = log 6 Obtain *x*log 2 = log 6 Attempt to remove logarithms Show $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ correctly

Mark Scheme 4723
June 2006

1	Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$	**M1**		any non-zero constant k
	Obtain $2(4x+1)^{-\frac{1}{2}}$	**A1**		or equiv, perhaps unsimplified
	Obtain $\frac{2}{3}$ for value of first derivative	**A1**		or unsimplified equiv
	Attempt equation of tangent through (2, 3)	**M1**		using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
	Obtain $y = \frac{2}{3}x + \frac{5}{3}$ or $2x - 3y + 5 = 0$	**A1**	**5**	or equiv involving 3 terms
2	Either: Attempt to square both sides	**M1**		producing 3 terms on each side
	Obtain $3x^2 - 14x + 8 = 0$	**A1**		or inequality involving < or >
	Obtain correct values $\frac{2}{3}$ and 4	**A1**		
	Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
	Obtain $\frac{2}{3} < x < 4$	**A1**	**5**	or correctly expressed equiv; allow ≤ signs
	Or: Attempt solution of two linear equations or inequalities	**M1**		one eqn with signs of $2x$ and x the same, second eqn with signs different
	Obtain value $\frac{2}{3}$	**A1**		
	Obtain value 4	**B1**		
	Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
	Obtain $\frac{2}{3} < x < 4$	**A1**	**(5)**	or correctly expressed equiv; allow ≤ signs
3 (i)	Attempt evaluation of cubic expression at 2 and 3	**M1**		
	Obtain –11 and 31	**A1**		
	Conclude by noting change of sign	**A1√**	**3**	or equiv; following any calculated values provided negative then positive
(ii)	Obtain correct first iterate	**B1**		using x_1 value such that $2 \le x_1 \le 3$
	Attempt correct process to obtain at least 3 iterates	**M1**		using any starting value now
	Obtain 2.34	**A1**	**3**	answer required to 2 d.p. exactly; 2→2.3811→2.3354→2.3410; 2.5→2.3208→2.3428→2.3401; 3→2.2572→2.3505→2.3392

4	**(i)** State $\ln y = (x-1)\ln 5$	**B1**		whether following $\ln y = \ln 5^{x-1}$ or not; brackets needed
	Obtain $x = 1 + \dfrac{\ln y}{\ln 5}$	**B1**	**2**	**AG**; correct working needed; missing brackets maybe now implied
	(ii) Differentiate to obtain single term of form $\dfrac{k}{y}$	**M1**		any constant k
	Obtain $\dfrac{1}{y\ln 5}$	**A1**	**2**	or equiv involving y
	(iii) Substitute for y and attempt reciprocal	**M1**		or equiv method for finding derivative without using part **(ii)**
	Obtain $25\ln 5$	**A1**	**2**	or exact equiv
5	**(i)** State $\sin 2\theta = 2\sin\theta\cos\theta$	**B1**	**1**	or equiv; any letter acceptable here (and in parts **(ii)** and **(iii)**)
	(ii) Attempt to find exact value of $\cos\alpha$	**M1**		using identity attempt or right-angled triangle
	Obtain $\frac{1}{4}\sqrt{15}$	**A1**		or exact equiv
	Substitute to confirm $\frac{1}{8}\sqrt{15}$	**A1**	**3**	**AG**
	(iii) State or imply $\sec\beta = \dfrac{1}{\cos\beta}$	**B1**		
	Use identity to produce equation involving $\sin\beta$	**M1**		
	Obtain $\sin\beta = 0.3$ and hence 17.5	**A1**	**3**	and no other values between 0 and 90; allow 17.4 or value rounding to 17.4 or 17.5
6	**(i)** Either: Obtain f(–3) = –7	**B1**		maybe implied
	Show correct process for compn of functions	**M1**		
	Obtain –47	**A1**	**3**	
	Or: Show correct process for compn of functions	**M1**		using algebraic approach
	Obtain $2-(2-x^2)^2$	**A1**		or equiv
	Obtain –47	**A1**	**(3)**	
	(ii) Attempt correct process for finding inverse	**M1**		as far as $x = \ldots$ or equiv
	Obtain either one of $x = \pm\sqrt{2-y}$ or both	**A1**		or equiv perhaps involving x
	Obtain correct $-\sqrt{2-x}$	**A1**	**3**	or equiv; in terms of x now
	(iii) Draw graph showing attempt at reflection in $y = x$	**M1**		
	Draw (more or less) correct graph	**A1**		with end-point on x-axis and no minimum point in third quadrant
	Indicate coordinates 2 and $-\sqrt{2}$	**A1**	**3**	accept –1.4 in place of $-\sqrt{2}$
7	**(a)** Obtain integral of form $k(4x-1)^{-1}$	**M1**		any non-zero constant k

		Obtain $-\frac{1}{2}(4x-1)^{-1}$	**A1**		or equiv; allow + *c*
		Substitute limits and attempt evaluation	**M1**		for any expression of form $k'(4x-1)^n$
		Obtain $\frac{2}{21}$	**A1**	**4**	or exact equiv
	(b)	Integrate to obtain ln *x*	**B1**		
		Substitute limits to obtain ln 2*a* – ln *a*	**B1**		
		Subtract integral attempt from attempt at area of appropriate rectangle	**M1**		or equiv
		Obtain 1 – (ln 2*a* – ln *a*)	**A1**		or equiv
		Show at least one relevant logarithm property	**M1**		at any stage of solution
		Obtain 1 – ln 2 and hence $\ln(\frac{1}{2}\mathrm{e})$	**A1**	**6**	**AG**; full detail required
8	**(i)**	State *R* = 13	**B1**		or equiv
		State at least one equation of form *R* cos α = *k*, *R* sin α = *k*′, tan *α* = k''	**M1**		or equiv; allow sin / cos muddles; implied by correct α
		Obtain 67.4	**A1**	**3**	allow 67 or greater accuracy
	(ii)	Refer to translation and stretch	**M1**		in either order; allow here equiv terms such as ‘move’, ‘shift’; with both transformations involving constants
		State translation in positive *x* direction by 67.4	**A1**√		or equiv; following their α; using correct terminology now
		State stretch in *y* direction by factor 13	**A1**√	**3**	or equiv; following their *R*; using correct terminology now
	(iii)	Attempt value of $\cos^{-1}(2 \div R)$	**M1**		
		Obtain 81.15	**A1**√		following their *R*; accept 81
		Obtain 148.5 as one solution	**A1**		accept 148.5 or 148.6 or value rounding to either of these
		Add their α value to second value correctly attempted	**M1**		
		Obtain 346.2	**A1**	**5**	accept 346.2 or 346.3 or value rounding to either of these; and no other solutions
9	**(i)**	Attempt to express *x* in terms of *y*	***M1**		obtaining two terms

	Obtain $x = e^{\frac{1}{2}y} + 1$	A1		or equiv
	State or imply volume involves $\int \pi x^2$	B1		
	Attempt to express x^2 in terms of y	*M1		dep ***M**; expanding to produce at least 3 terms
	Obtain $k\int (e^y + 2e^{\frac{1}{2}y} + 1)\,dy$	A1		any constant k including 1; allow if dy absent
	Integrate to obtain $k(e^y + 4e^{\frac{1}{2}y} + y)$	A1		
	Use limits 0 and p	M1		dep ***M *M**; evidence of use of 0 needed
	Obtain $\pi(e^p + 4e^{\frac{1}{2}p} + p - 5)$	A1	8	**AG**; necessary detail required
(ii)	State or imply $\frac{dp}{dt}$ = 0.2	B1		maybe implied by use of 0.2 in product
	Obtain $\pi(e^p + 2e^{\frac{1}{2}p} + 1)$ as derivative of V	B1		
	Attempt multiplication of values or expressions for $\frac{dp}{dt}$ and $\frac{dV}{dp}$	M1		
	Obtain $0.2\pi(e^4 + 2e^2 + 1)$	A1√		following their $\frac{dV}{dp}$ expression
	Obtain 44	A1	5	or greater accuracy

Mark Scheme 4724
June 2006

Q		Answer	Marks		Guidance
1		$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$	B1		s.o.i. e.g. $2x\frac{dy}{dx} + y$
		$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1		
		Substitute (1,2) into their differentiated equation and attempt to solve for $\frac{dy}{dx}$. [Allow subst of (2,1)]	M1 dep at least 1 x **B1**		Or attempt to solve their diff equation for $\frac{dy}{dx}$ and then substitute (1,2)
		$\frac{dy}{dx} = -2$	A1	4	
2	(i)	$1 + (-2)(-3x) + \frac{(-2)(-3)}{1.2}(-3x)^2$ (+ … ignore)	M1		State or imply; accept $-3x^2$ & $-9x^2$
		$= 1 + 6x$	B1		Correct first 2 terms
		… $+27x^2$	A1	3	Correct third term
	(ii)	$(1+2x)^2(1-3x)^{-2}$	M1		For changing into suitable form, seen/implied
		Attempt to expand $(1+2x)^2$ & select (at least) 2 relevant products and add	M1		Selection may be after multiplying out
		55 (Accept $55x^2$)	A2√	4	If (i) is $a + bx + cx^2$, f.t. $4(a+b)+c$
		<u>SR 1</u> For expansion of $(1+2x)^2$ with 1 error, A1√			
		<u>SR 2</u> For expansion of $(1+2x)^2$ & > 1 error, A0			
		<u>Alternative Method</u>			
		For correct method idea of long division	M1		
		1 ….. $+10x$ ….. $+55x^2$	A1,A1,A1(4)		
3	(i)	$\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$	M1		Correct format + suitable method
		$\frac{1}{x}$	A1		seen in (i) or (ii)
		$-\frac{1}{3-x}$	A1	3	ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately
	(ii)	$\int \frac{1}{x}(dx) = \ln x$ or $\ln\lvert x\rvert$	B1		
		$\int \frac{1}{3-x}(dx) = -\ln(3-x)$ or $-\ln\lvert 3-x\rvert$	B1		Check sign carefully; do not allow $\ln(x-3)$
		Correct method idea of substitution of limits	M1		Dep on an attempt at integrating
		ln 2 (+ ln 1 − ln 1) − ln 2 = 0	A1	4	Clearly seen; WWW **AG**
		<u>Alternative Method</u>			
		If ignoring PFs, ln x(3 − x) immediately	B2		$\ln x(x-3) \to 0$
		As before	M1,A1	(4)	
	(iii)	Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)	B1	1	

4	(i) Working out $\mathbf{b}-\mathbf{a}$ or $\mathbf{a}-\mathbf{b}$ or $\mathbf{c}-\mathbf{a}$ or $\mathbf{a}-\mathbf{c}$	M1)		Irrespective of label
	$= \pm(-3\mathbf{i}-\mathbf{j}-\mathbf{k})$ or $\pm(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	A1)		If not scored ,these 1st 3 marks can be
	Method for finding magnitude of <u>any</u> vector	M1)		awarded in part (ii)
	Method for finding scalar product of <u>any</u> 2 vectors	M1		
	Using $\cos\theta = \frac{a.b}{\lvert a\rvert\lvert b\rvert}$ AEF for <u>any</u> 2 vectors	M1		
	<u>[Alternative cosine rule method</u> $\lvert\overrightarrow{BC}\rvert = \sqrt{6}$	B1		
	Cosine rule used	M1		'Recognisable' form
	$45.3°, 0.79(0), \frac{\pi}{3.97}$ (45.289378, 0.7904487)	A1	6	Do not accept supplement (134.7 etc)
	(ii) Use of $\frac{1}{2}\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert\sin\theta$	M1		Accept $\lvert\frac{1}{2}\overrightarrow{AB} \times \overrightarrow{AC}\rvert$
	3.54 (3.5355) or $\frac{5\sqrt{2}}{2}$	A1	2	Accept from correct supp (134.7 etc)
5	(i) $\frac{dA}{dt}$ or kA^2 seen	M1		
	$\frac{dA}{dt} = kA^2$	A1	2	
	(ii) Separate variables + attempt to integrate	*M1		Accept if based on $\frac{dA}{dt} = kA^2$ or A^2
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	A1		
	Subst one of (0,0),(1,1000) or (2,2000) into eqn.	dep*M1		Equation must contain k and/or c
	Subst another of (0,),(1,1000) or (2,2000) into eqn	dep*M1		This equation must contain k <u>and</u> c
	Substitute $A = 3000$ into eqn with k and c subst	dep*M1		
	$t = \frac{7}{3}$ ISW	A1	6	Accept 2.33, 2h 20 m
6	(i) Attempt to connect du and dx e.g. $\frac{du}{dx} = e^x$	M1		But not $du = dx$
	Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i.	A1		
	Simplification to $\int \frac{u-1}{u}(du)$ WWW	A1	3	AG
	(ii) Change $\frac{u-1}{u}$ to $1-\frac{1}{u}$ or use parts	M1		If parts, may be twice if $\int \ln x\,dx$ is involved
	$\int \frac{1}{u}\,du = \ln u$	A1		Seen anywhere in this part
	<u>Either</u> attempt to change limits <u>or</u> resubstitute	M1 (indep)		Expect new limits e+1 & 2
	Show as $e+1-\ln(e+1)-\{2 \text{ or } (1+1)\}+\ln 2$	A1		
	WWW show final result as $e-1-\ln\left(\frac{e+1}{2}\right)$	A1	5	AG

7	(i)	Produce at least 2 of the 3 relevant eqns in λ and μ	M1		e.g. $1 + 3\lambda = -8 + \mu$, $-2 + \lambda = 2 - 2\mu$
		Solve the 2 eqns in λ & μ as far as λ = … or μ = …	M1		
		1st solution: $\lambda = -2$ or $\mu = 3$	A1		
		2nd solution: $\mu = 3$ or $\lambda = -2$ f.t.	A1√		
		Substitute their λ and μ into 3rd eqn and find 'a'	M1		
		Obtain $a = 2$ & clearly state that a cannot be 2	A1	6	
	(ii)	Subst their λ or μ (& poss a) into either line eqn	M1		
		Point of intersection is $-5\mathbf{i} - 4\mathbf{j}$	A1	2	Accept any format No f.t. here
		N.B. In this question, award marks irrespective of labelling of parts			
8	(i)	Integration method			
		Attempt to change $\cos^2 6x$ into $f(\cos 12x)$	M1		
		$\cos^2 6x = \frac{1}{2}(1 + \cos 12x)$	A1		with $\cos^2 6x$ as the subject of the formula
		$\int = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$	A1		AG Accept $\frac{1}{2}\left(x + \frac{1}{12}\sin 12x\right)$
		Differentiation method			
		Differentiate RHS producing $\frac{1}{2} + \frac{1}{2}\cos 12x$ ---(E)	B1		
		Attempt to change $\cos 12x$ into $f(\cos 6x)$	M1		Accept $+/-2\cos^2 6x +/- 1$
		Simplify (E) WWW to $\cos^2 6x$ + satis finish	A1	3	
	(ii)	Parts with $u = x, \mathrm{d}v = \cos^2 6x$	*M1		
		$x\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right) - \int\left(\frac{1}{2}x + \frac{1}{24}\sin 12x\right)\mathrm{d}x$	A1		Correct expression only
		$\int \sin 12x \, \mathrm{d}x = -\frac{1}{12}\cos 12x$	B1		Clear indication somewhere in this part
		Correct use of limits to whole integral	dep*M1		Accept ( ) (−0)
		$\frac{\pi^2}{288} - \frac{\pi^2}{576} - \frac{1}{288} - \frac{1}{288}$	A1		AE unsimp exp. Accept 12x24,sin π here
		$\frac{\pi^2}{576} - \frac{1}{144}$	+A1	6	Tolerate e.g. $\frac{2}{288}$ here
		S.R. If final marks are A0 + A0, allow SR A1 for			0.01/0.010/0.0101/0.0102/0.0101902

9	(i)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		Used, not just quoted
		$\frac{dx}{dt} = -4\sin t$ or $\frac{dy}{dt} = 3\cos t$	*B1		
		$\frac{dy}{dx} = -\frac{3\cos t}{4\sin t}$ or $\frac{3\cos t}{-4\sin t}$ ISW	dep*A1	3	Also $\frac{-3\cos t}{4\sin t}$ provided B0 not awarded
		SR: M1 for Cartesian eqn attempt + B1 for $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ + **A1** as before(must be in terms of t)			
	(ii)	$y - 3\sin p = \left(\text{their } \frac{dy}{dx}\right)(x - 4\cos p)$	M1		Accept p or t here
		<u>or</u> $y = \left(\text{their } \frac{dy}{dx}\right)x + c$ & subst cords to find c			Ditto
		$4y\sin p - 12\sin^2 p = -3x\cos p + 12\cos^2 p$	A1		Correct equation cleared of fractions
		<u>or</u> c = $\frac{12\sin^2 p + 12\cos^2 p}{4\sin p}$			
		$3x\cos p + 4y\sin p = 12$ WWW	A1	3	**AG** Only p here. Mixture earlier $\to$ A0
	(iii)	Subst $x = 0$ and $y = 0$ separately in tangent eqn	M1		to find R & S
		Produce $\frac{3}{\sin p}$ and $\frac{4}{\cos p}$	A1		Accept $\frac{12}{4\sin p}$ and/or $\frac{12}{3\cos p}$
		Use $\Delta = \frac{1}{2}\left(\frac{3}{\sin p}.\frac{4}{\cos p}\right) = \frac{12}{\sin 2p}$ WWW	A1	3	**AG**
	(iv)	Least area = 12	B1		
		$p = \frac{1}{4}\pi$ as final or only answer	B2	3	These B marks are independent.
		S.R. $45° \to$ B1 ;			S.R. [-12 and e.g. $-\pi/4 \to$ B1]

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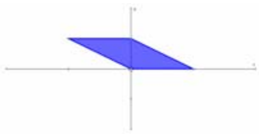
1.	i) $\begin{pmatrix} 7 & 4 \\ 0 & -1 \end{pmatrix}$	B1 B1	 2	Two elements correct All four elements correct
	(ii) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1		**A** – **B** correctly found
	$k = 3$	B1	2 $\boxed{4}$	Find k
2	(i) 	M1 A1	 2	For 2 other correct vertices For completely correct diagram
	(ii) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	B1 B1	2 $\boxed{4}$	Each column correct
3.	(i) $2 + 3\text{i}$	B1	1	Conjugate seen
	(ii)	M1		Attempt to sum roots or consider x terms in expansion or substitute $2 - 3\text{i}$ into equation and equate imaginary parts
	$p = -4$	A1		Correct answer
		M1		Attempt at product of roots or consider last term in expansion or consider real parts
	$q = 13$	A1	4 $\boxed{5}$	Correct answer

4.	$\Sigma r^3 + \Sigma r^2$	M1		Consider the sum as two separate parts
	$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1		Correct formula stated
	$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	A1		Correct formula stated
	$\frac{1}{12}n(n+1)(n+2)(3n+1)$	M1		Attempt to factorise and simplify or expand both expressions
		A1	5	Obtain given answer correctly or complete verification
			$\boxed{5}$	
5.	(i) -7i	B1		Real part correct
		B1	2	Imaginary part correct
	(ii) 2 + 3i	B1		iz stated or implied or $i^2 = -1$ seen
		B1		Real part correct
	-5 + 12i	B1	3	Imaginary part correct
	(iii) $\frac{1}{5}(4 - 7i)$ or equivalent	M1		Multiply by conjugate
		A1		Real part correct
		A1	3	Imaginary part correct
			$\boxed{8}$	**N.B. Working must be shown**
6..	(i) Circle , Centre *O* radius 2	B1 B1		Sketch showing correct features
	One straight line	B1		
	Through *O* with +ve slope	B1		
	In 1st quadrant only	B1	5	
	(ii) $1 + i\sqrt{3}$	M1		Attempt to find intersections by trig, solving equations or from graph
		A1	2	Correct answer stated as complex number
			$\boxed{7}$	

7.	(i)	M1		Attempt at matrix multiplication
	$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	 3	Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
	(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
	(iii)	B1 M1 A1 A1	 4 $\boxed{8}$	State that conjecture is true for $n = 1$ or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion
8.	(i)	M1		Correct expansion process shown
	$a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	A1		Obtain correct unsimplified expression
	$a^2 - 2a$	A1	3	Obtain correct answer
	(ii)	M1		Solve their det $\mathbf{M} = 0$
	$a = 0$ or $a = 2$	A1A1ft	3	Obtain correct answers
	(iii) (a)	B1 B1		Solution, as inverse matrix exists or $\mathbf{M}$ non-singular or det$\mathbf{M} \neq 0$
	(b)	B1 B1	4 $\boxed{10}$	Solutions, eqn. 1 is multiple of eqn 3

9.	(i)	M1 A1	2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	2	Attempt to expand and simplify Obtain given answer correctly
	(iii) $(n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$ $\frac{1}{2}n(n+1)(2n+1)$	B1 B1 M1 M1 A1 A1	6 $\boxed{\textbf{10}}$	Correct Σr stated $\Sigma 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression Obtain given answer correctly

10.	(i) $\alpha + \beta + \gamma = 2$ $\alpha\beta\gamma = -4$	B1 B1		Write down correct values
	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1	3	
	(ii)	M1		Sum new roots
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		Obtain numeric value using their (i)
	$p = -5$	A1ft	3	p is negative of their answer
	(iii)	M1*		Expand three brackets
		A1		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		DM1		Use their (i) results
		A1ft		Obtain 2
	$q = -2$	A1ft	5	q is negative of their answer
			$\boxed{11}$	**Alternative for (ii) & (iii)**
		M2		Substitute $x = u - 1$ in given equation
		A1		Obtain correct unsimplified equation for u
		M1		Expand
		A2		Obtain $u^3 - 5u^2 + 10u - 2 = 0$
		A1 A1		State correct values of p and q.

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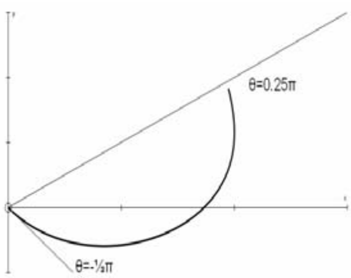
June 2006

1	Correct expansion of sin x	B1 Quote or derive $x - {}^1/_6 x^3$
	Multiply their expansion by $(1 + x)$	M1 Ignore extra terms
	Obtain $x + x^2 - x^3/6$	A1√ On their sin x; ignore extra terms; allow 3!
		SC Attempt product rule M1 Attempt f(0), f ′ (0), f″(0) … (at least 3) M1 Use Maclaurin accurately cao A1
2 (i)	Get $\sec^2 y \dfrac{dy}{dx} = 1$ or equivalent	M1
	Clearly use $1 + \tan^2 y = \sec^2 y$	M1 May be implied
	Clearly arrive at A.G.	A1
(ii)	Reasonable attempt to diff. to $\dfrac{-2x}{(1+x^2)^2}$	M1 Use of chain/quotient rule
	Substitute their expressions into D.E.	M1 Or attempt to derive diff. equ^n.
	Clearly arrive at A.G.	A1
		SC Attempt diff. of $(1+x^2)\dfrac{dy}{dx} = 1$ M1,A1 Clearly arrive at A.G. B1
3 (i)	State $y = 0$ (or seen if working given)	B1 Must be = ; accept x-axis; ignore any others
(ii)	Write as quad. in x^2	M1 $(x^2 y - x + (3y-1) = 0)$
	Use for real x, $b^2 - 4ac \geq 0$	M1 Allow > ; or < for no real x
	Produce quad. inequality in y	M1 $1 \geq 12y^2 - 4y$; $12y^2 - 4y - 1 \leq 0$
	Attempt to solve inequality	M1 Factorise/ quadratic formula
	Justify A.G.	A1 e.g. diagram / table of values of y
		SC Attempt diff. by product/quotient M1 Solve $dy/dx = 0$ for two real x M1 Get both $(-3, -{}^1/_6)$ and $(1, ½)$ A1 Clearly prove min./max. A1 Justify fully the inequality e.g. detailed graph B1
4 (i)	Correct definition of cosh x or cosh $2x$	B1
	Attempt to sub. in RHS and simplify	M1 or LHS if used
	Clearly produce A.G.	A1
(ii)	Write as quadratic in cosh x	M1 $(2\cosh^2 x - 7\cosh x - 4 = 0)$
	Solve their quadratic accurately	A1√ Factorise/quadratic formula
	Justify one answer only	B1 State cosh $x \geq 1$/graph; allow ≥ 0
	Give $\ln(4 + \sqrt{15})$	A1 cao; any one of $\pm \ln(4 \pm \sqrt{15})$ or decimal equivalent of ln ( )
5 (i)	Get $(t + ½)^2 + ¾$	B1 cao
(ii)	Derive or quote $dx = \dfrac{2}{1+t^2} dt$	B1
	Derive or quote $\sin x = 2t/(1 + t^2)$	B1
	Attempt to replace all x and dx	M1
	Get integral of form $A/(Bt^2+Ct+D)$	A1√ From their expressions, C≠ 0
	Use complete square form as $\tan^{-1}(f(t))$	M1 From formulae book or substitution
	Get A.G.	A1

6 (i) Attempt to sum areas of rectangles	M1 ($h.3^h + h.3^{2h} + \ldots + h.3^{(n-1)h}$)
Use G.P. on $h(1+3^h+3^{2h}+\ldots+3^{(n-1)h})$	M1 All terms not required, but last term needed (or 3^{1-h}); or specify a, r and n for a G.P.
Simplify to A.G.	A1 Clearly use $nh = 1$
(ii) Attempt to find sum areas of different rect.	M1 Different from (i)
Use G.P. on $h(3^h+3^{2h}+\ldots+3^{nh})$	M1 All terms not required, but last term needed; G.P. specified as in (i), or deduced from (i)
Simplify to A.G.	A1
(iii) Get 1.8194(8), 1.8214(8) correct	B1,B1 Allow $1.81 \le A \le 1.83$
7 (i) Attempt to solve r=0, $\tan\theta = -\sqrt{3}$	M1 Allow $\pm\sqrt{3}$
Get $\theta = -\frac{1}{3}\pi$ only	A1 Allow -60°
(ii) $r = \sqrt{3} + 1$ when $\theta = \frac{1}{4}\pi$	B1,B1 AEF for r, 45° for θ
(iii)	B1 Correct r at correct end-values of θ; Ignore extra θ used
	B1 Correct shape with r not decreasing
(iv) Formula with correct r used	M1 r^2 may be implied
Replace $\tan^2\theta = \sec^2\theta - 1$	B1
Attempt to integrate <u>their</u> expression	M1 Must be 3 different terms leading to any 2 of $a\theta + b\ln(\sec\theta/\cos\theta) + c\tan\theta$
Get $\theta + \sqrt{3}\ln\sec\theta + \frac{1}{2}\tan\theta$	A1 Condone answer x2 if ½ seen elsewhere
Correct limits to $\frac{1}{4}\pi + \sqrt{3}\ln\sqrt{2} + \frac{1}{2}$	A1 cao; AEF
8 (i) Attempt to diff. using product/quotient	M1
Attempt to solve $dy/dx = 0$	M1
Rewrite as A.G.	A1 Clearly gain A.G.
(ii) Diff. to $f'(x) = 1 \pm 2\,\mathrm{sech}^2 x$	B1 Or $\pm 2\,\mathrm{sech}^2 x - 1$
Use correct form of N-R with their expressions from correct f(x)	M1
Attempt N-R with x_1= 2 from previous M1	M1 To get an x_2
Get x_2 = 1.9162(2) (3 s.f. min.)	A1
Get x_3 = 1.9150(1) (3 s.f. min.)	A1 cao
(iii) Work out e_1 and e_2 (may be implied)	B1√ -0.083(8), -0.0012 (allow ± if both of same sign); e_1 from 0.083 to 0.085

Use $e_2 \approx ke_1^2$ and $e_3 \approx ke_2^2$	M1
Get $e_3 \approx e_2^3/e_1^2$ = - 0.0000002(or 3)	A1√ ± if same sign as B1√
	SC B1 only for x_4 - x_3
9 (i) Rewrite as quad. in e^y	M1 Any form
Solve to $e^y = (x \pm \sqrt{(x^2 + 1)})$	A1 Allow $y = \ln($ $)$
Justify one solution only	B1 $x - \sqrt{(x^2 + 1)} < 0$ for all real x
	SC Use $C^2 - S^2 = 1$ for $C = \pm\sqrt{(1+x^2)}$ M1
	Use/state $\cosh y + \sinh y = e^y$ A1
	Justify one solution only B1
(ii) Attempt parts on $\sinh x. \sinh^{n-1}x$	M1
Get correct answer	A1 $(\cosh x.\sinh^{n-1}x - \int \cosh^2 x.(n-1)\sinh^{n-2}x \, dx)$
Justify $\sqrt{2}$ by $\sqrt{(1+\sinh^2 x)}$ for $\cosh x$ when limits inserted	B1 Must be clear
Replace $\cosh^2 = 1 + \sinh^2$; tidy at this stage	M1
Produce I_{n-2}	A1
Gain A.G. clearly	A1
(iii) Attempt $4I_4 = \sqrt{2} - 3I_2$,$2I_2 = \sqrt{2} - I_0$	M1 Clear attempt at iteration (one at least seen)
Work out $I_0 = \sinh^{-1}1 = \ln(1 + \sqrt{2}) = \alpha$	B1 Allow I_2
Sub. back completely for I_4	M1
Get $^1/_8(3 \ln(1+\sqrt{2}) - \sqrt{2})$	A1 AEEF

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1 (a) Identity = $1+0\text{i}$	B1	For correct identity. Allow 1
Inverse = $\frac{1}{1+2\text{i}}$	B1	For $\frac{1}{1+2\text{i}}$ seen or implied
$= \frac{1}{1+2\text{i}} \times \frac{1-2\text{i}}{1-2\text{i}} = \frac{1}{5} - \frac{2}{5}\text{i}$	B1 **3**	For correct inverse **AEFcartesian**
(b) Identity = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	B1	For correct identity
Inverse = $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$	B1 **2**	For correct inverse
	$\boxed{5}$	
2 (a) $(z_1 z_2 =)\ 6\text{e}^{\frac{5}{12}\pi\text{i}}$	B1	For modulus = 6
	B1	For argument = $\frac{5}{12}\pi$
$\left(\frac{z_1}{z_2} = \frac{2}{3}\text{e}^{-\frac{1}{12}\pi\text{i}} =\right) \frac{2}{3}\text{e}^{\frac{23}{12}\pi\text{i}}$	M1	For subtracting arguments
	A1 **4**	For correct answer
(b) $\left(w^{-5} =\right) 2^{-5}\,\text{cis}\left(-\frac{5}{8}\pi\right)$	M1	For use of de Moivre
	A1	For $-\frac{5}{8}\pi$ seen or implied
$= \frac{1}{32}\left(\cos\frac{11}{8}\pi + \text{i}\sin\frac{11}{8}\pi\right)$	A1 **3**	For correct answer (allow 2^{-5} and $\text{cis}\frac{11}{8}\pi$)
	$\boxed{7}$	

3 *EITHER* $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a})\times[8, 3, -6]$	M1*	For attempt at vector product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\mathbf{n} = \pm[-12, 50, 9]$	A1 √	For obtaining **n**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \dfrac{\lvert\mathbf{n}\rvert}{\lvert[8, 3, -6]\rvert}$	M1 (dep*)	For dividing $\lvert\mathbf{n}\rvert$ by magnitude of $[8, 3, -6]$
$= \dfrac{\sqrt{2725}}{\sqrt{109}}$	A1	For either magnitude correct
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a})\,.\,[8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$\cos\theta = \pm\dfrac{109}{\sqrt{134}\sqrt{109}} = \pm\sqrt{\dfrac{109}{134}}$	A1 √	For correct $\cos\theta$ **AEF**. f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134}\sin\theta$	M1 (dep*) A1	For using trigonometry for perpendicular distance For correct expression for d in terms of θ
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{c}-\mathbf{a} = \pm[11, 3, -2]$	B1	For vector joining lines
$(\mathbf{c}-\mathbf{a})\,.\,[8, 3, -6]$	M1*	For attempt at scalar product of $\mathbf{c}-\mathbf{a}$ and $[8, 3, -6]$
$x = \dfrac{109}{\sqrt{109}} = \sqrt{109}$	A1 √	For finding projection of $\mathbf{c}-\mathbf{a}$ onto $[8, 3, -6]$ f.t. from incorrect $\mathbf{c}-\mathbf{a}$
$d = \sqrt{134-109}$	M1 (dep*) A1	For using Pythagoras for perpendicular distance For correct expression for d
(d =) 5	A1	For correct distance **CAO**
OR $\mathbf{CP} = \pm[-11+8t, -3+3t, 2-6t]$	B1	For finding a vector from $C(12, 5, 3)$ to a point on the line
$\mathbf{CP}\,.\,[8, 3, -6] = 0$	M1*	For using scalar product for perpendicularity
$t = \pm 1$ *OR* $P = (9, 5, -1)$	A1 √	For correct point. f.t. from incorrect **CP**
$d = \sqrt{3^2+0^2+4^2}$	M1 (dep*) A1	For finding magnitude of **CP** For correct expression for d
(d =) 5	A1 **6**	For correct distance **CAO**
		SR Obtain $\mathbf{CP} = [11, 3, -2]-[8, 3, -6] = \pm[3, 0, 4]$ B1 Verify $[3, 0, 4]\,.\,[8, 3, -6] = 0$ M1* $d = \sqrt{3^2+0^2+4^2}$ = 5 M1(dep*) A1 A1 (maximum 5 / 6)
	$\boxed{6}$	

4 Integrating factor $e^{\int -\frac{x^2}{1+x^3}dx}$	M1	For correct process for finding integrating factor
$= e^{-\frac{1}{3}\ln(1+x^3)} = \left(1+x^3\right)^{-\frac{1}{3}}$	A1	For correct IF, simplified (here or later)
$\Rightarrow \frac{d}{dx}\left(y\left(1+x^3\right)^{-\frac{1}{3}}\right) = \frac{x^2}{\left(1+x^3\right)^{\frac{1}{3}}}$	M1	For multiplying through by their IF
$\Rightarrow y\left(1+x^3\right)^{-\frac{1}{3}} = \frac{1}{2}\left(1+x^3\right)^{\frac{2}{3}}\ (+c)$	M1 A1	For integrating RHS to obtain $A\left(1+x^3\right)^k$ *OR* $\ln A\left(1+x^3\right)^k$ For correct integration (+*c* not required here)
$\Rightarrow 1 = \frac{1}{2}+c \quad \Rightarrow \quad c = \frac{1}{2}$	M1 A1 √	For substituting (0, 1) into GS (including + *c*) For correct *c*. f.t. from their GS
$\Rightarrow y = \frac{1}{2}\left(1+x^3\right)+\frac{1}{2}\left(1+x^3\right)^{\frac{1}{3}}$	A1	For correct solution. **AEF** in form $y = f(x)$
	$\boxed{8}$	
5 (i) *EITHER* $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
$\mathbf{n} = \mathbf{a}\times\mathbf{b} = \pm k[-10, 10, -2]$	M1 A1 √	For finding perpendicular to plane For correct **n**. f.t. from incorrect **b**
Use (2, 1, 5) *OR* (0, –1, 5)	M1	For substituting a point into equation $ax+by+cz=d$ where $[a, b, c]$ = their **n**
$\Rightarrow 5x-5y+z=10$	A1	For correct cartesian equation **AEF**
OR $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$	B1	For stating 2 vectors in the plane
e.g. $\mathbf{r} = [2, 1, 5]+\lambda[2, 2, 0]+\mu[2, 3, 5]$	M1	For stating parametric equation of plane
$[x, y, z] = [2+2\lambda+2\mu, 1+2\lambda+3\mu, 5+5\mu]$	A1 √	For writing 3 equations in *x*, *y*, *z* f.t. from incorrect **b**
	M1	For eliminating λ and μ
$\Rightarrow 5x-5y+z=10$	A1 **5**	For correct cartesian equation **AEF**
(ii) $[2t, 3t-4, 5t-9]$	B1 **1**	For stating a point *A* on l_1 with parameter *t* **AEF**
(iii) $\pm[2t+5, 3t-7, 5t-13]$	M1	For finding direction of l_2 from *A* and (–5,3, 4)
$\pm[2t+5, 3t-7, 5t-13].[2, 3, 5] = 0$	M1	For using scalar product for perpendicularity with any vector involving *t*
$\Rightarrow t = 2$	A1	For correct value of *t*
$\frac{x+5}{9} = \frac{y-3}{-1} = \frac{z-4}{-3}$ *OR* $\frac{x-4}{9} = \frac{y-2}{-1} = \frac{z-1}{-3}$	A1 **4**	For a correct equation **AEFcartesian**
		SR For $2p+3q+5r=0$ and no further progress award B1
	$\boxed{10}$	

6 (i) $\left(m^2+4=0\Rightarrow\right) m=\pm 2\mathrm{i}$	B1	For correct solutions of auxiliary equation (may be implied by correct CF)
CF = $A\cos 2x + B\sin 2x$	B1	For correct CF (**AEtrig** but not $Ae^{2\mathrm{i}x}+Be^{-2\mathrm{i}x}$ only)
PI = $p\sin x\ (+\ q\cos x)$	B1	State a trial PI with at least $p\sin x$
$-p\sin x\ (-q\cos x)+4p\sin x\ (+4q\cos x)=\sin x$	M1	For substituting PI into DE
$\Rightarrow p=\frac{1}{3},\quad q=0$	A1	For correct p and q (which may be implied)
$\Rightarrow y=A\cos 2x+B\sin 2x+\frac{1}{3}\sin x$	B1 √ **6**	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI
(ii) $(0,0)\Rightarrow A=0$	B1 √	For correct equation in A and/or B f.t. from their GS
$\frac{\mathrm{d}y}{\mathrm{d}x}=2B\cos 2x+\frac{1}{3}\cos x\Rightarrow\frac{4}{3}=2B+\frac{1}{3}$	M1	For differentiating their GS and substituting values for x and $\frac{\mathrm{d}y}{\mathrm{d}x}$
$A=0,\ B=\frac{1}{2}$	A1	For correct A and B Allow $A=-\frac{1}{4}\mathrm{i},\ B=\frac{1}{4}\mathrm{i}$ from CF $Ae^{2\mathrm{i}x}+Be^{-2\mathrm{i}x}$
$\Rightarrow y=\frac{1}{2}\sin 2x+\frac{1}{3}\sin x$	A1 **4** $\boxed{10}$	For stating correct solution **CAO**
7 (i) $C+\mathrm{i}S=1+e^{\mathrm{i}\theta}+e^{2\mathrm{i}\theta}+e^{3\mathrm{i}\theta}+e^{4\mathrm{i}\theta}+e^{5\mathrm{i}\theta}$	M1	For using de Moivre, showing at least 3 terms
$=\dfrac{e^{6\mathrm{i}\theta}-1}{e^{\mathrm{i}\theta}-1}$	M1 A1	For recognising GP For correct GP sum
$=\dfrac{e^{3\mathrm{i}\theta}-e^{-3\mathrm{i}\theta}}{e^{\frac{1}{2}\mathrm{i}\theta}-e^{-\frac{1}{2}\mathrm{i}\theta}}\cdot\dfrac{e^{3\mathrm{i}\theta}}{e^{\frac{1}{2}\mathrm{i}\theta}}=\dfrac{e^{3\mathrm{i}\theta}-e^{-3\mathrm{i}\theta}}{e^{\frac{1}{2}\mathrm{i}\theta}-e^{-\frac{1}{2}\mathrm{i}\theta}}e^{\frac{5}{2}\mathrm{i}\theta}$	A1 **4**	For obtaining correct expression **AG**
(ii) $C+\mathrm{i}S=\dfrac{2\mathrm{i}\sin 3\theta}{2\mathrm{i}\sin\frac{1}{2}\theta}\cdot e^{\frac{5}{2}\mathrm{i}\theta}$	M1 A1	For expressing numerator and denominator in terms of sines For $k\sin 3\theta$ and $k\sin\frac{1}{2}\theta$
$\mathrm{Re}\Rightarrow C=\sin 3\theta\cos\frac{5}{2}\theta\ \mathrm{cosec}\frac{1}{2}\theta$	A1	For correct expression **AG**
$\mathrm{Im}\Rightarrow S=\sin 3\theta\sin\frac{5}{2}\theta\ \mathrm{cosec}\frac{1}{2}\theta$	B1 **4**	For correct expression
(iii) $C=S\ \Rightarrow\ \sin 3\theta=0,\ \tan\frac{5}{2}\theta=1$	M1	For either equation deduced **AEF** Ignore values outside $0<\theta<\pi$
$\theta=\frac{1}{3}\pi,\frac{2}{3}\pi$	A1	For both values correct and no extras
$\theta=\frac{1}{10}\pi,\frac{1}{2}\pi,\frac{9}{10}\pi$	A2 **4** $\boxed{12}$	For all values correct and no extras. Allow A1 for any 1 value *OR* all correct with extras

8 (i) $r^4 . a \neq a . r^4$	B1 **1**	For stating the non-commutative product in the given table, or justifying another correct one
(ii) Possible subgroups order 2, 5	B1 B1 **2**	For either order stated For both orders stated, and no more (Ignore 1)
(iii) (a) $\{e, a\}$ **(b)** $\{e, r, r^2, r^3, r^4\}$	B1 B1 **2**	For correct subgroup For correct subgroup
(iv) order of $r^3 = 5$ $(ar)^2 = ar.ar = r^4a.ar = e$ $\Rightarrow$ order of $ar = 2$ $(ar^2)^2 = ar^2ar.r = ar^2r^4a.r = ara.r = e$ $\Rightarrow$ order of $ar^2 = 2$	B1 M1 A1 A1 **4**	For correct order For attempt to find $(ar)^m = e$ *OR* $(ar^2)^m = e$ For correct order For correct order
(v) $\begin{array}{c\|cccc} & ar & ar^2 & ar^3 & ar^4 \\ \hline ar & e & r & r^2 & r^3 \\ ar^2 & r^4 & e & r & r^2 \\ ar^3 & r^3 & r^4 & e & r \\ ar^4 & r^2 & r^3 & r^4 & e \end{array}$	 B1 B1 B1 B1 B1 **5** **14**	If the border elements $ar\ ar^2\ ar^3\ ar^4$ are not written, it will be assumed that the products arise from that order For all 16 elements of the form *e* or r^m For all 4 elements in leading diagonal = *e* For no repeated elements in any completed row or column For any two rows or columns correct For all elements correct

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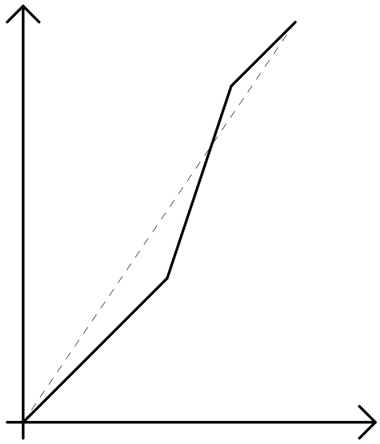
1		Momentum before = $3M$ - 1200×3 Momentum after = 1200×5	B1 B1	Ignore g if included; accept inconsistent directions (or loss of momentum of loaded wagon = $3M$ B1 gain of momentum of unloaded wagon = 1200(5 + 3) B1)
		$3M - 3600 = 6000$	M1	Equation with all terms; accept with g
		$3(1200 + m) - 3600 = 6000$	A1	For any correct equation in m, M
		$m = 2000$	A1 5	
2	**(i)**		M1	For resolving forces in the **i** direction or for relevant use of trigonometry
		$2.5 = 6.5\sin\theta$	A1	
		$\theta = 22.6^{o}$	A1 3	AG Accept verification
	(ii)		M1	For resolving forces in the **j** direction or for using Pythagoras or relevant trigonometry.
		$R = 6.5\cos 22.6^{o}$	A1	
		$R = 6$	A1 3	

3	(i)		B1	Line segment *AB* (say) of +ve slope from origin
			B1	Line segment *BC* (say) of steeper +ve slope and shorter time interval than those for *AB*. ***SR***: If the straight line segments are joined by curves, this B1 mark is not awarded
			B1	Line segment *CD* (say) of less steep slope compared with *BC*. (An (x, t) graph is accepted and the references to more/less steep are reversed.)
		Time intervals 80, 40, 40	B1	May be implied; any 2 correct
		t = 80, 120, 160	B1	
	(ii)	Line joining (0, 0) and (160, 360)	B1 ft 6	
	(iii)	v = 360/160	M1	Woman's velocity (= 2.25)
			M1	For equation of man's displacement in relevant interval
		$s = 120 + 4.5(t - 80)$	A1	Accept omission of -80
		$2.25t$	M1	Woman's displacement, awarded even if t is interpreted differently in man's expression
		$t = 106\frac{2}{3}$ (107)	A1 5	Accept also 106.6, 106.7 but not 106
		SR Construction method		
		Plotting points on graph paper	M1	Candidates reading the <u>displacement</u> intersection from graph, then dividing this distance by the woman's speed to find t, also get v = 360/160 M1 as above for the woman's velocity.
		t between 104 and 109 inclusive	A1	
4	(i)	Displacement is 20 m	B1 1	20+c (from integration) B0
	(ii)		M1	For using $s(t) = \int v(t)dt$
		$s(t) = 0.01t^3 - 0.15t^2 + 2t$ (+A)	A1	Can be awarded prior to cancelling
		$10 - 15 + 20 + A = 20$	M1	For using $s(10)$ = cv (20)
		Displacement is $0.01t^3 - 0.15t^2 + 2t + 5$	A1 4	AG
	(iii)		M1	For using $a(t) = dv/dt$
		$a = 0.06t - 0.3$	A1	
		$0.06t - 0.3 = 0.6$	DM1	For starting solving $a(t) = 0.6$ depends on previous M1
		$t = 15$	A1	
		Displacement is 35 m	B1 5	

5	(i)	 $R = mg$ $m = 2.55$	M1 M1 A1 3	For using $F = 5$ and $F = \mu R$ Accept 2.5 or 2.6
	(ii)a	$P\cos\alpha = 6$ $R = P\sin\alpha + 25$ $0.2R = 6$ $0.2(P\sin\alpha + 25) = 6$ $\alpha = 39.8^\circ$	B1 M1 A1ft B1 M1 A1	 For resolving vertically with 3 distinct forces Or $P\sin\alpha$ + (cv m)g For using $F = 6$ and $F = \mu R$. Can be implied by $0.2(P\sin\alpha + 25) = 6$ For an equation in $P\sin\alpha$ (=5)after elimination of R Accept a r t 40°
	(ii)b	$P^2 = 6^2 + 5^2$ or $P\cos39.8^\circ = 6$ or $P\sin39.8^\circ = 5$ $P = 7.81$	M1 A1 8	For eliminating or substituting for α with cv(6). Evidence is needed that 5 is the value of $P\sin\alpha$ (rather than the original frictional force) Accept a r t 7.8
6	(i)	10500 + 3000 + 1500 Driving force below 15000 gives retardation	M1 A1 2	For summing 3 resistances Accept generalised case or specific instance
	(ii)	$35000 - 15000 = 80000a$ Acceleration is 0.25 ms^{-2}	M1 A1 2	Newton's second law for whole train AG Accept verification
	(iii)	 $35000 - 10500 - 8500 = 0.25m$ Mass is 64000 kg	M1 A1 A1 3	For applying Newton's second law to E only, at least 2 forces out of the relevant 3.
	(iv)	 $-15000 - 15000 = 80000a$ OR $-3000-10500-15000=(80000 - m)a$ $-1500 = ma$ Mass is 4000 kg	M1 A1 M1 A1 A1 5	For applying Newton's second law with all appropriate forces $a = -0.375$ For applying Newton's second law to B only, only 1 force Or cv(a)
	(v)	$-15000 - 10500 \pm T = 64000(-0.375)$ $T = \pm1500$ ➔ forward force on E of 1500 N OR (working with A and B) $-1500 - 3000 \pm T = (80000 - 64000)(-0.375)$ $T = \pm1500$ ➔ forward force on E of 1500	B1ft B1 2 B1ft B1	Follow through cv (m_E, a), or accept use of m_E, a Follow through cv (m_E, a), or accept use of m_E, a

7	(i)	$0 = 6 + (\pm)1.5a$	M1	For using $v = u + at$ with $v = 0$

		$a = (\mp)4\text{ms}^{-2}$	A1		
		$-mg\sin 15^\circ - F = ma$	M1		For applying Newton's second law with 2 forces
		$-0.1\times 9.8\sin 15^\circ - F = 0.1\times(-4)$	A1		
		$R = 0.1g\cos 15^\circ$	B1		
		$0.146357\ldots = \mu\, 0.946607\ldots$	M1		For using $F = \mu R$
		Coefficient is 0.155	A1	7	Anything between 0.15 and 0.16 inclusive
	(ii)	$mg\sin 15^\circ > \mu\, mg\cos 15^\circ$ (or $\tan 15^\circ > \mu$)	M1		For comparing weight component with frictional force (or tan 'angle of friction' with μ)
		➔ particle moves down	A1	2	Awarded if conclusion is correct even though values are wrong
	(iii)	$(6 + 0)\div 2 = s\div 1.5$	M1		For using $(u + v) \div 2 = s\div t$
		$s = 4.5$	A1		
		$mg\sin 15^\circ - F = ma$	M1		For using Newton's second law with 2 forces
		$0.25364\ldots - 0.146357\ldots = 0.1a$	A1		Values must be correct even if not explicitly stated. Note that the correct value of friction may legitimately arise from a wrong value of μ and a wrong value of R
		$v^2 = 2(1.07285\ldots)4.5$	M1		For using $v^2 = 2as$ with any value of a
		Speed is 3.11 ms^{-1}	A1	6	Accept anything rounding to 3.1 from correct working

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1		$mgh = 35 \times 9.8 \times 4$ $mgh/t = 1372/10$ 137 W	M1 A1 M1 A1	4	watch out for extras or 0.137 kW	**4**
2		$v^2 = 2gh$ $u = \sqrt{4g}$ or $\sqrt{39.2}$ or 6.26 $v = \sqrt{2.8g}$ or $\sqrt{27.44}$ (5.24) $I = \pm\ 0.3(6.26 + 5.24)$ 3.45 Ns	M1 A1 A1 M1 A1✓	5	kinematics or energy speed of impact (±) speed of rebound (±) must be sum of mags. of vels. ✓ must be positive	**2** **5**
3	(i)	d = 2.25 h = 1.125 or 1.12 or 1.13 or 9/8	B1 B1	2	3/8x6 *OG* (be generous) horizontal distance	
	(ii)	$T_1 + T_2 = 12$ resolving vertically $T_1 \times 6\cos30° = 12xh$ (their h) mom(*O*) (their h ok for A1) $T_1 = 2.60$ N or $3\sqrt{3}/2$ $T_2 = 9.40$ N ✓ $(12 - T_1)$	M1 M1 A1 A1 A1✓	5	if not then next M1 ok or mom(*A*)$T_2 \times 6\cos30° = 12(6\cos30° - h)$ or $T_2 = 9.40$ or $T_1 = 2.60$ or ✓$(12 - T_2)$	**7**
		above ✓ depends on at least one of the M marks ($T_s > 0$)				
4	(i)	P = 13500 W	B1	1	or 13.5 kW	
	(ii)	500 = 13500/v v = 27 ms^{-1}	M1 A1	2		
	(iii)	15000/25 – 500 = 950a a = 0.105 or 2/19	M1 A1 A1	3	2 parts to F A0 for 900a or 100/950	
	(iv)	15000/26–500 – 950.9.8sin5° = 950a a = (–) .773 ms^{-2}	M1 A1 A1	3	3 parts to F A0 for 900a s.c. accept 0.77	**9**
5	(i)	$\bar{x} = 9$ c of m of Δ 4 cm above BD $(324 + 108)(m)\bar{y} = 324(m) \times 9 + 108(m) \times (18+4)$ $432\bar{y}$ 324 x 9 (18^2 x 9) 108 x (18 +4) $\bar{y} = 12.25$	B1 B1 M1 A1 A1 A1 A1	7	ignore any working 8 cm below C/see their diagram $432\bar{y} = 108 \times 8 + 18^2(12 + 9)$ from C left hand side 1st term on right hand side 2916 2nd term on right hand side 2376 5292÷432 or 49/4	
	(ii)	tanθ = 5.75/9 θ = 32.6° or 147.4°	M1 A1✓	2	must be …/9 ✓$\tan^{-1}$ ((18-their $\bar{y}$)/9) or 180°..	**9**

6	(i)	T = 4.9 N T = 0.3 x 0.2 x ω^2 ω = 9.04 rads^{-1}	B1 M1 A1 A1	 4	B0 for 0.5g or $0.3v^2/0.2$ and $\omega = v/0.2$	**6**
	(ii)	cosθ = √0.6/0.8 (0.968) Tcosθ = 0.5 x 9.8 T = 5.06 N	B1 M1 A1 A1	 4	(θ=14.5°) angle to vert. or equiv. angle consistent with diagram can be their angle	
	(iii)	Tsinθ = 0.5 x v^2/0.2 v = 0.711 ms^{-1}	M1 A1 A1	 3	must be a component of T (sinθ = ¼) can be their angle	**11**
7	(i)	vsin50° 0=$v^2\sin^2 50°$-2x9.8x13 (must be 13) v = 20.8 ms^{-1}	B1 M1 A1	 3	initial vertical component or mx9.8x13 = ½m(vsin50°)^2 sin/cos mix ok for above M1	
	(ii)	45 = vcos50°.t t = 3.36 √their v (3.13 for v=22.4) s = vsin50° x t - ½ x 9.8 x t^2 s = – 1.6 to – 2.0 inclusive (– 1.68) ht above ground = 0.320 m	M1 A1√ M1 A1 A1 A1	 6	see alternative below other methods include other t_s ignore ht adjustments can be their v and their t can be implied from next A1	
	(iii)	v_v = vsin50° - 9.8xt v_v = -17.0 √their v,t(-13.5 for 22.4) speed=√(v_v^2+(vcos50°)^2) speed = 21.6 ms^{-1} √ their v and v_v (19.7 for v = 22.4)	M1 A1√ M1 A1√	 4	or v_v^2=2g(15-their ans to ii) √ above for v_v or ½mv^2 – mgx1.68 = ½mx20.8^2 (4 marks) M1/A1√ s,v /M1 solve/ A1 √	**13**
	(ii)	y = xtanθ –gx^2/2$v^2\cos^2$θ y=45tan50°-9.8.45^2/2.$v^2\cos^2$50° calculate y y = – 1.6 to – 2.0 inclusive	B1 M1 A1 M1 A1		**Alternative 1st 5 marks** substitute v and 50° and x=45 can be their v should be – 1.68	

8	(i)	10 = 4 + m.x e = … or rationale for x = 2 m = 3	M1 M1 A1	 3	conservation of momentum	
	(ii)	v = 6 e = 4/5 or 0.8	B1 M1 A1	 3	 allow sign errors for M mark watch out for lost minuses	
	(iii)	10 – 5 = 2x + y (5 = -2a + b) (-5 = 2c + d) e = 0.8 = (y-x)/10 y = x + 8 (a + b = 8) (c – d = 8) x = -1 (a=1) (c=1) y = 7 (b=7) (d=-7) $½.2.5^2 + ½.1.5^2 - ½.2.1^2 - ½.1.7^2$ 12 J	M1 A1 M1 A1 A1 A1 M1 A1	 8	 look for consistency or 1 in opp. direction to 1st K.E. lost. Must be 4 parts (37.5 – 25.5)	**14**

±1 in 3rd sig. fig. except where stated

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1	(i)		M1		For using I = Δ(mv) in the direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\cos\theta = 0.4\text{x}25$	A1		
		Direction at angle 120^o to original motion	A1	3	Accept $\theta = 60^o$ with θ correctly identified.
	(ii)		M1		For using I = Δ(mv) perp. to direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\sin60^o = 0.4v$	A1ft		
		Speed is 43.3 ms^{-1}	A1	3	
2			M1		For applying Newton's 2nd Law.
			M1		For using a = v(dv/dx).
		$2v(dv/dx) = -(2v + 3v^2)$	A1		
			M1		For separating variables and attempting to integrate.
		2/3ln(2 + 3v) = -x (+C)	A1ft		ft absence of minus sign,
		[2/3ln14 = C]	M1		For using v(0) = 4.
		[2/3ln2 = -x + 2/3ln14]	M1		For attempting to solve v(x) = 0 for x.
		Comes to rest after travelling 1.30m	A1	8	AG

3	**(i)**		M1		For taking moments about C for the whole structure.
		1.4R = 0.35x360 + 1.05x200	A1		
		Magnitude is 240N	A1		AG
			M1		For taking moments about A for the rod AB.
		0.7x240 = 0.35x200 + 1.05T	A1		
		Tension is 93.3N	A1	6	
	OR **(i)**		M1		For taking moments about A for AB and AC.
		$0.7R_B = 70 + 1.05T$ and $0.7R_C = 126 + 1.05T$	A1		
			M1		For eliminating T or for adding the equations, and then using $R_B + R_C = 560$.
		$0.7(560 – R_B) – 0.7R_B = 126 – 70$ or $0.7\text{x}560 = 70 + 126 + 2.1T$	A1		For a correct equation in R_B only or T only
		Magnitude is 240N	A1		AG
		Tension is 93.3N	A1	6	
	(ii)	Horizontal component is 93.3 N to the left	B1ft		
		Y = 240 - 200	M1		For resolving forces vertically.
		Vertical component is 40 N downwards	A1	3	

4	(i)		M1		For using Newton's 2nd Law perp. to string with a = L$\ddot{\theta}$.
		L(m)$\ddot{\theta}$ = -(m)gsinθ or (m)$\ddot{s}$ = -(m)gsin(s/L)	A1		
		$\ddot{\theta} \approx$ -kθ or $\ddot{s}$ = -ks [and motion is therefore approx. simple harmonic]	B1		
			M1		For using T = 2π/n and k = w^2 or T = $2\pi\sqrt{L/g}$ for simple pendulum.
		Period is 3.14s.	A1	5	AG
	(ii)		M1		For using $\dot{\theta}^2 = n^2(\theta_0{}^2 - \theta^2)$ or the principle of conservation of energy
		$\dot{\theta}^2$ =4(0.1^2 – 0.06^2) or ½ m(2.45$\dot{\theta}$)^2 = 2.45mg(cos0.06 – cos0.1)	A1		
		Angular speed is 0.16 rad s^{-1}.	A1	3	(0.1599… from energy method)
	OR	(in the case for which (iii) is attempted before (ii))			
	(ii)	[$\dot{\theta}$= -0.2sin2t]	M1		For using $\dot{\theta}$= d(Acos nt)/dt
		$\dot{\theta}$= -0.2sin(2x0.464)	A1ft		
		Angular speed is 0.16 rad s^{-1}.	A1	3	
	(iii)		M1		For using θ= Acos nt or Asin(π/2 – nt) or for using θ= Asin nt and T =$t_{0.1}$ – $t_{0.06}$
		0.06 = 0.1cos2t or 0.1sin(π/2 – 2t) or 2T = π/2 – sin^{-1}0.6	A1ft		ft angular displacement of 0.04 instead of 0.06
		Time taken is 0.464s	A1	3	

5			M1		Σ mv conserved in **i** direction.
		2x12cos60° – 3x8 = 2a + 3b	A1		
			M1		For using NEL
		For LHS of equation below	A1		
		0.5(12cos60° + 8) = b - a	A1		Complete equation with signs of a and b consistent with previous equation.
			M1		For eliminating a or b.
		Speed of B is 0.4ms^{-1} in **i** direction	A1		
		a = -6.6	A1		
		Component of A's velocity in **j** direction is 12sin60°	B1		May be shown on diagram or implied in subsequent work.
		Speed of A is 12.3ms^{-1}	B1ft		
			M1		For using θ = tan^{-1}(**j**comp/$\pm$ **i** comp)
		Direction is at 122.4° to the **i** direction	A1ft	12	Accept θ = 57.6° with θ correctly identified.
6	**(i)**	T = 1470x/30	B1		
		[49x = 70x9.8]	M1		For using T = mg
		x = 14	A1		
		Distance fallen is 44m	A1ft	4	
	(ii)	PE loss = 70g(30 + 14)	B1ft		
		EE gain = 1470x14^2/(2x30)	B1ft		
		[½ 70v^2 = 30184 – 4802]	M1		For a linear equation with terms representing KE, PE and EE changes.
		Speed is 26.9ms^{-1}	A1	4	AG
	OR (ii)	[0.5 v^2 = 14g – 68.6 + 30g]	M1		For using Newton's 2nd law (vdv/dx = g – 0.7x), integrating (0.5 v^2 = gx – 0.35x^2 + k), using v(0)^2 = 60g➔ k = 30g, and substituting x = 14.
		For 14g + 30g	B1ft		
		For $\mp$68.6	B1ft		Accept in unsimplified form.
		Speed is 26.9ms^{-1}	A1	4	AG
	(iii)	PE loss = 70g(30 + x)	B1ft		
		EE gain = 1470x^2/(2x30)	B1ft		
		[x^2 – 28x – 840 = 0]	M1		For using PE loss = KE gain to obtain a 3 term quadratic equation.
		Extension is 46.2m	A1	4	
	OR (iii)		M1		For identifying SHM with n^2 = 1470/(70x30)
			M1		For using v$_{max}$ = An
		$A = 26.9/\sqrt{0.7}$	A1		
		Extension is 46.2m	A1	4	

7	**(i)**	½ $0.3v^2$ + ½ $0.4v^2$	B1		
		$\pm 0.3g(0.6\sin\theta)$	B1		
		$\pm 0.4g(0.6\theta)$	B1		
		$[0.35v^2 = 2.352\theta - 1.764\sin\theta]$	M1		For using the principle of conservation of energy.
		$v^2 = 6.72\theta - 5.04\sin\theta$	A1	5	AG
	(ii)		M1		For applying Newton's 2nd Law radially to P and using $a = v^2/r$
		$0.3(v^2/0.6) = 0.3g\sin\theta - R$	A1		
		[½ $(6.72\theta - 5.04\sin\theta)$ = $0.3g\sin\theta - R$]	M1		For substituting for v^2.
		Magnitude is $(5.46\sin\theta - 3.36\theta)$N	A1		AG
		$[5.46\cos\theta - 3.36 = 0]$	M1		For using $dR/d\theta = 0$
		Value of θ is 0.908	A1	6	
	(iii)	$[T - 0.3g\cos\theta = 0.3a]$	M1		For applying Newton's 2nd Law tangentially to P
		$[0.4g - T = 0.4a]$	M1		For applying Newton's 2nd Law to Q [If $0.4g - 0.3g\cos\theta = 0.3a$ is seen, assume this derives from $T - 0.3g\cos\theta = 0.3a$ ……. M1 and $T = 0.4g$ ……. M0]
		Component is $5.6 - 4.2\cos\theta$	A1	3	
	OR (iii)	$0.4g - 0.3g\cos\theta = (0.3 + 0.4)a$	B2		
		Component is $5.6 - 4.2\cos\theta$	B1	3	
	OR (iii)	$[2v(dv/d\theta) = 6.72 - 5.04\cos\theta]$	M1		For differentiating v^2 (from (i)) w.r.t. θ
		$2(0.6a) = 6.72 - 5.04\cos\theta$	M1		For using $v(dv/d\theta) = ar$
		Component is $5.6 - 4.2\cos\theta$	A1	3	

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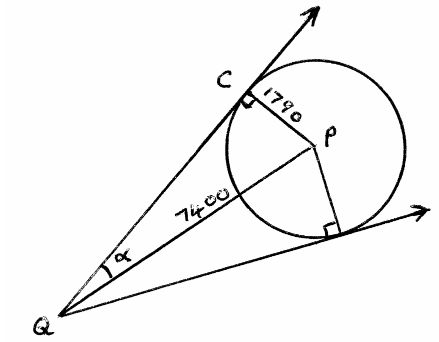
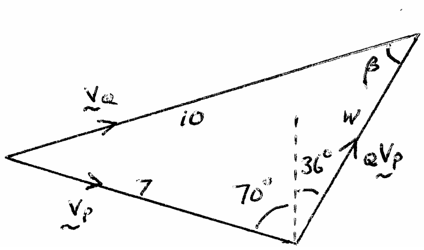
1	$\int x\rho\,dx = \int_0^a k(a+2x)\,x\,dx$	M1	for $\int ...(a+2x)x\,dx$
	$= k\left[\frac{1}{2}a\,x^2 + \frac{2}{3}x^3\right]_0^a \quad (= \frac{7}{6}ka^3)$	A1	
	$\int \rho\,dx = k\int_0^a (a+2x)\,dx \quad = k\left[a\,x + x^2\right]_0^a$ $= 2ka^2$	B1	for $...\left[a\,x + x^2\right]_0^a$
	$\bar{x} = \dfrac{\frac{7}{6}ka^3}{2ka^2}$	M1	*Dependent on first M1*
	$= \frac{7}{12}a$	A1 **5**	Accept 0.583*a*
2 (i)	$I = \frac{1}{2}\times 8\times 0.15^2 \quad (= 0.09\text{ kg m}^2)$	B1	
	Using $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$ $25^2 = 10^2 + 2\alpha\times 75$ $\alpha = 3.5\text{ rad s}^{-2}$	M1A1	
	Couple is $I\alpha = 0.09\times 3.5$ $= 0.315\text{ N m}$	M1 A1 ft **5**	ft from wrong *I* and / or α , but ft requires M1M1
	OR Increase in KE is $\frac{1}{2}\times 0.09\times(25^2 - 10^2)$ M1A1 ft $= 23.625\text{ J}$ M1 Couple is $\dfrac{23.625}{75} = 0.315\text{ N m}$ A1 ft		WD by couple is $L\times 75$ ft requires M1M1
(ii)	By conservation of angular momentum $(0.09 + I_2)\times 9 = 0.09\times 25$ $I_2 = 0.16\text{ kg m}^2$	M1 A1 ft A1 **3**	Using angular momentum

3	$\int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2$	M1	
	$= \frac{1}{2}$	A1	
	Mass per unit area $\rho = 48\ \text{kg m}^{-2}$	B1	
	$I = \int \frac{4}{3}(\rho y \delta x)(\frac{1}{2}y)^2$	M1	For integral of y^3
	$= \int \frac{1}{3}\rho y^3\,dx$	A1	
	$= \frac{1}{3}\rho \int_1^2 \frac{1}{x^6}\,dx$	A1 ft	
	$= \frac{1}{3}\rho\left[-\frac{1}{5x^5}\right]_1^2$	A1	For correct integration of $\frac{1}{x^6}$
	$= \frac{31}{480}\rho = \frac{31}{480}\times 48$		
	$= 3.1\ \text{kg m}^2$	A1 **8**	
4 (i)	$RC = 2a\cos\theta$	B1	or $RC^2 = 2a^2 + 2a^2\cos 2\theta$
	EPE $= \frac{5mg}{2a}(2a\cos\theta)^2$	M1	
	GPE $= mga\sin 2\theta + 2mg(2a\sin 2\theta)$	M1	One term sufficient for M1
	$V = 10mga\cos^2\theta + 5mga\sin 2\theta$	A1	
	$\frac{dV}{d\theta} = -20mga\cos\theta\sin\theta + 10mga\cos 2\theta$ $= -10mga\sin 2\theta + 10mga\cos 2\theta$	B1	Correct differentiation of $\cos^2\theta$ (or $\cos 2\theta$) and $\sin 2\theta$
	For equilibrium, $10mga(\cos 2\theta - \sin 2\theta) = 0$ $\tan 2\theta = 1$	M1	For using $\frac{dV}{d\theta} = 0$
	$\theta = \frac{1}{8}\pi$	A1 **7**	Accept $22\frac{1}{2}°$, 0.393
(ii)	$\frac{d^2V}{d\theta^2} = -20mga\cos 2\theta - 20mga\sin 2\theta$	B1 ft	
	When $\theta = \frac{1}{8}\pi$, $\frac{d^2V}{d\theta^2}$ $(= -20\sqrt{2}\,mga) < 0$	M1	Determining the sign of V''
	Hence the equilibrium is unstable	A1 **3**	Correctly shown
	OR Other method for determining whether V has a maximum or a minimum M1 Correct determination A1 ft Equilibrium is unstable A1		Correctly shown

5 (i)	$I = \frac{1}{3}(20)(0.3^2 + 0.9^2) + 20 \times 0.9^2$ $= 22.2 \text{ kg m}^2$	M1 M1 A1 (ag) **3**	MI of lamina about any axis Use of parallel (or perp) axes rule Correctly obtained
	OR $I = \frac{1}{3} \times 20 \times 0.3^2 + \frac{4}{3} \times 20 \times 0.9^2$ M1M1 $= 22.2 \text{ kg m}^2$ A1		As above
(ii)	Total moment is $20 \times 9.8 \times 0.9\cos\theta - 44.1$ Angular acceleration is zero when moment is zero $\cos\theta = \frac{44.1}{20 \times 9.8 \times 0.9} = 0.25$	M1 M1 A1 (ag) **3**	
(iii)	Maximum angular speed when $\cos\theta = 0.25$ $\theta = 1.318$ Work done against couple is 44.1×1.318 By work energy principle, $\frac{1}{2}I\omega^2 = 20 \times 9.8 \times 0.9\sin\theta - 44.1\theta$ $\omega = 3.19 \text{ rad s}^{-1}$	M1 A1 M1 A1 ft A1 **5**	Equation involving work, KE and PE

6 (i)	As viewed from *P*  $\sin\alpha = \frac{1790}{7400}$ $\alpha = 14.0°$ Bearing of relative velocity is $50-\alpha = 036°$ or $50+\alpha = 064°$	M1 A1 (ag) B1 ft **3**	For 64 or ft $50+\alpha$
(ii)	Velocity diagram  $\frac{\sin\beta}{7} = \frac{\sin 106}{10}$ $\beta = 42.3°$ Bearing of $\mathbf{v}_Q$ is $36+\beta = 078.3°$	B1 M1 A1 A1 **4**	Correct diagram *(may be implied)* *Correct triangle must be intended* Accept $78°$
(iii)	$\frac{w}{\sin 31.7} = \frac{10}{\sin 106}$ $w = 5.47\ \mathrm{m\,s^{-1}}$	M1 A1 **2**	*If cosine rule is used, M1 also requires an attempt at solving the quadratic*
	Alternative for (ii) and (iii) $\begin{pmatrix} w\sin 36 \\ w\cos 36 \end{pmatrix} = \begin{pmatrix} 10\sin\theta \\ 10\cos\theta \end{pmatrix} - \begin{pmatrix} 7\sin 110 \\ 7\cos 110 \end{pmatrix}$ B1 Obtaining an equation in θ only, and solving it M1 $\theta = 78.3°$ A2 Obtaining an equation in *w* only, and solving it M1 $w = 5.47\ \mathrm{m\,s^{-1}}$ A1		e.g. $10\sin\theta - 7.2654\cos\theta = 8.3173$ or A1A1 if another angle found first

(iv)	$QC = \sqrt{7400^2 - 1790^2} = 7180 \text{ m}$ Time taken is $\frac{7180}{5.468}$ $= 1310 \text{ s}$	M1 M1 A1 ft **3**	(Or M2 for other complete method for finding the time) For attempt at relative distance $\div w$ (not awarded for $7400 \div w$) or 21.9 minutes ft is $7180 \div w$
(v)	Bearing of *CP* is $90 + 36 = 126°$	B1 **1**	

7 (i)	$I = \frac{1}{3}m(3a)^2 + m(2a)^2$ $= 7ma^2$ $mg(2a\sin\theta) = I\alpha$ $\alpha = \dfrac{2g\sin\theta}{7a}$	M1 A1 M1 A1 **4**	Using parallel axes rule
(ii)	By conservation of energy $\frac{1}{2}I\omega^2 = mg(2a\cos\frac{1}{3}\pi - 2a\cos\theta)$ $\frac{7}{2}ma^2\omega^2 = mga(1-2\cos\theta)$ $\omega = \sqrt{\dfrac{2g(1-2\cos\theta)}{7a}}$	M1 A1 A1 (ag) **3**	Equation involving KE and PE Need to see how $\frac{1}{3}\pi$ is used Correctly obtained
(iii)	$mg\cos\theta - R = m(2a\omega^2)$ $R = mg\cos\theta - \frac{4}{7}mg(1-2\cos\theta)$ $= \frac{1}{7}mg(15\cos\theta - 4)$	M1 A1 A1	For radial acceleration $r\omega^2$
	$mg\sin\theta - S = m(2a\alpha)$ $S = mg\sin\theta - \frac{4}{7}mg\sin\theta$ $= \frac{3}{7}mg\sin\theta$	M1 A1 A1 **6**	For transverse acceleration $r\alpha$
	OR $S(2a) = I_G\alpha = (3ma^2)\alpha$ M1A1 $S = \frac{3}{7}mg\sin\theta$ A1		Must use I_G
(iv)	When $\cos\theta = \frac{1}{3}$, $\sin\theta = \frac{\sqrt{8}}{3}$, $\tan\theta = \sqrt{8}$ $R = \frac{1}{7}mg$, $S = \frac{\sqrt{8}}{7}mg$ Angle with R is $\tan^{-1}\dfrac{S}{R} = \tan^{-1}\sqrt{8} = \theta$ so the resultant force is vertical Magnitude is $\sqrt{R^2+S^2}$ $= \frac{1}{7}mg\sqrt{1+8} = \frac{3}{7}mg$	M1 A1 M1 A1 **4**	
	OR When resultant force is F vertically upwards $S = F\sin\theta$, hence $F = \frac{3}{7}mg$ M1A1 $R = F\cos\theta$, so $\frac{1}{7}mg(15\cos\theta - 4) = \frac{3}{7}mg\cos\theta$ M1 $\cos\theta = \frac{1}{3}$ A1		
	OR Horizontal force is $R\sin\theta - S\cos\theta$ $= \frac{1}{7}mg(15\cos\theta-4)\sin\theta - \frac{3}{7}mg\sin\theta\cos\theta$ M1 $= \frac{1}{7}mg\sin\theta(12\cos\theta - 4)$ $= 0$ when $\cos\theta = \frac{1}{3}$ A1		

	Vertical force is $R\cos\theta + S\sin\theta$ $= \frac{1}{7}mg \times \frac{1}{3} + \frac{3}{7}mg \times \frac{8}{9} = \frac{3}{7}mg$ M1A1		

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geq$ 3sfs, ISW for later rounding
Penalise 2 sfs only once in paper.

1(i)	Negative, because (grad or coeff of x in 1st equn or x-value or reg coeff or B or –0.6) is negative	B1 **1**	Neg because x incr & y decr
(ii)	$x = -1.6 \times 7.0 + 21$ $x = 9.8$	M1 A1 **2**	Sub y=7.0 in 2nd eqn. Allow 1 sign error If sub in both must choose 2nd
(iii)	$y = -0.6(-1.6y + 21) + 13$ or similar $\bar{x} = 5,\ \bar{y} = 10$	M1 A1A1 **3**	Obtain correct eqn in 1 variable. Allow 1 num'l error Allow without bars
Total		**6**	
	In qus 2 & 3 "prod" means "product of two probabilities"		
2(i)	$^4/_7$ or 0.571 (3 sfs)	B1 **1**	
(ii)	$^5/_8 \times {^4/_7} + {^3/_8} \times {^5/_8}$ $= {^{265}/_{448}}$ or 0.592 (3 sfs)	M1M1 A1 **3**	M1: one correct prod or add any two prods M1: all correct
(iii)	$^3/_8 \times {^5/_8} + {^5/_8} \times {^3/_7}$ $= {^{225}/_{448}}$ or 0.502 (3 sfs)	M1M1 A1 **3**	M1: one correct prod or add any two prods M1: all correct
Total		**7**	
3(i)	$\frac{7!}{3! \times 2(!)}$ $= 420$	M1M1 A1 **3**	M1: 7!/(a factorial); or … ÷ (3! x 2(!)) M1: all correct
(ii)	$\frac{5!}{2(!)}$ $= 60$	M1 A1 **2**	M1: 5! seen (not part of a C) or 5 x 4! or 120 seen or … ÷ 2(!) alone
(iii)	$1 - {^4/_7} \times {^3/_6}$ or $1 - {^4C_2} / {^7C_2}$ or $1 - {^4P_2} / {^7P_2}$ or $^3/_7 \times {^2/_6} + {^3/_7} \times {^4/_6} + {^4/_7} \times {^3/_6}$ oe or $^3C_2 / {^7C_2} + {^3C_1} \times {^4C_1} / {^7C_2}$ $= {^5/_7}$ or 0.714 (3 sfs)	M1M1 A1 **3**	M1: 1– prod or 1-.../$^7C_2$ or 1–$^4C_2$ /.. (or Ps) or add 3 prods or add 2 correct prods or $^3C_2 / {^7C_2}$ or $^3C_1 \times {^4C_1} / {^7C_2}$ or add $\geq$ 5 out of 7 correct prods M1: all correct
Total		**8**	

4(i)	0.4207 or 0.421 (3 sfs) or $0.8^{25}+25\text{x}0.8^{24}\text{x}0.2 + .. {}^{25}C_4\text{x}0.4^{21}\text{x}0.2^4$ 0.579(3)	B1 B1 **2**	or 1 – 0.6167 or 0.3833 (3 sfs) or 1- (6 correct terms, 0 to 5)
(ii)	${}^{10}C_3$ x $(1\text{-}0.27)^7$ x 0.27^3 = 0.261 (3 sfs)	M1 A1 **2**	
(iii)	Allow "=" thro'out $1 – 0.73^n > 0.95$ $0.73^9 = 0.059$ or $0.73^n < 0.05$ $0.73^{10} = 0.043$ $n\log0.73 < \log0.05$ oe $n = 10$	 M1 M1 A1 **3**	or 1 - ${}^nC_0$ x 0.27^0 x $0.73^n > 0.95$ oe allow incorrect sign M1 must be correct ft (1 – 0.27) from (ii) for M1M1 10 with incorrect sign in wking: SCB2 10 with just $0.73^9 = 0.059$: M1M1A1
Total		**7**	
5(i)	$^1/_3 + {}^1/_4 + p + q = 1$ oe 0 x $^1/_3$ + 1 x $^1/_4 + 2p + 3q = 1^1/_4$ oe equalize coeffs, eg mult eqn (i) by 2 or 3 Or make p or q subject of (i) or (ii) $p = {}^1/_4$, $q = {}^1/_6$ oe	B1 B1 M1 A1A1 **5**	 allow one error. ft their equns subst or subtr not nec'y
(ii)	Σx^2p (not /4 or /3 etc) $(= 2^3/_4)$ – $(1^1/_4)^2$ = 1.1875 or $1^3/_{16}$ oe sd = √(their 1.1875) = 1.09 (3 sfs)	M1 M1 A1 B1f **4**	≥ 2 non-zero terms correct. dep +ve result indep if +ve result or $(x\text{-}1^1/_4)^2p$ (≥ 2 (non-0) terms correct): M2 ft (i) (0≤ p, q <1) or letters p, q both M1s cao dep 1st M1 &√(+ve no.) eg √2.75 = 1.66
Total		**9**	

6(i)(a)	Ranks: 2 4 7 5 3 1 6 6 4 1 3 5 7 2 7 1 6 3 2 5 4 1 7 2 5 6 3 4 Σd^2 (= 60) $r_s = 1 - \frac{6\times 60}{7\times 48}$ $= -^1/_{14}$ or -0.071 (3 dps)	M1 A1 M1 M1 A1 **5**	$\geq$ 5 ranks correct in each set all correct dep ranks attempted even if opp orders, allow arith errors Correct formula with n = 7, dep 2nd M1 calc r for ranks: S_{xx}=S_{yy}= 140 – 28²/7. S_{xy} = 110-28²/7 (= 28) (= -2) corr subst in one corr S (any version):M1 corr subst in $r = S_{xy} / \sqrt{(S_{xx}S_{yy})}$:M1 -0.07 without wking: M1A1M2A0 No mks unless $\|r_s\| \leq 1$
(b)	Little (or no) connection (agreement, rel'nship) between dist and commission Allow disagreement	B1ft **1**	ft their r_s Must refer to context. Not "little corr'n between dist and com" not "strong disagreement" Ignore other comment
(c)	Unchanged. No change in rank	B1B1 **2**	
(ii)(a)	= –1	B1 **1**	indep
(b)	Close to –1 or, eg ≈ – 0.9	B1	cao not referring to "corr'n" rather than r allow "neg", not neg corr'n or neg skew
Total		**10**	

7(i)	Midpoints attempted $\geq$ 2 classes $\sum xf$ / 100 or $\sum xf / \sum f$ attempted $\geq$ 2 terms x within class, not class width Mean = 27.2 (to 3 sfs) (not 27.25) art 27.2 from fully correct wking $\sum x^2 f$ or $\sum x - \bar{x})^2 f$ $\geq$ 2 terms $\sqrt{(\sum x^2 f/100 - \bar{x}^2)}$ or $\sqrt{((\sum x - \bar{x})^2 f/100)}$ or $/\sum f$ fully corr method, not $\sqrt{\text{neg}}$ = 40.5 to 41.1 (3 sfs)	M1 M1 A1 M1 M1 A1 6	Correct (149.5) / With 150 / Tot = 2000 2720.5/100 / 2725/100 / Allow Ms & poss As 27.2 / 27.25 240702.25 / 242050 40.82 / 40.96 allow class widths for 2nd M1 only
(ii)	Recog LQ in 1st class & UQ in 3rd class Graph: Interp: Attempt 25(.25)th value LQ = 3.0 to 4.3 Attempt 75(.75)th value UQ =27 to 29 Subtract IQR = 23 or 24 or 25	B1 M1 M1 A1 4	 both nec'y dep B1or M1 integer. dep M2
(iii)(a) (b) (c)	Increase Increase No change	B1 1 B1 1 B1 1	Ignore "probably" etc
Total		**13**	
8(i)	Geometric. Each attempt (or result or try) indep	B1 B1 2	 In context. Not "events,. trials, outcomes" . Ignore extra
(ii)(a)	$(^2/_3)^3 \times {}^1/_3$ $= {}^8/_{81}$ or 0.0988 (3 sfs)	M2 A1 3	$(^2/_3)^2 \times {}^1/_3$ or $(^2/_3)^4 \times {}^1/_3$: allow other numerical "p" ($0<p<1$):M1
(b)	$(^2/_3)^3$ $1 - (^2/_3)^3$ $= {}^{19}/_{27}$ or 0.704 (3sfs)	M1 M1 A1 3	not $(^2/_3)^3 \times \ldots$ or ${}^1/_3 + {}^2/_3 \times {}^1/_3 + (^2/_3)^2 \times {}^1/_3$ M2 $1 - (^2/_3)^4$ or $1 - (\text{"}q\text{"})^4$ M1 or 3 terms, with 2 correct M1 or 3 correct terms + 1 extra M1 or "p" + "qp" + "q^2p" M1 or 1 – sum of 3 correct terms M1 "p" means num value, not ${}^1/_3$
(iii)	3	B1f 1	or ${}^1/_{\text{"}p\text{"}}$
(iv)	$1 - {}^{19}/_{27}$ | (1 – 0.7037) or 0.2963 $(^8/_{27})^2 \times {}^{19}/_{27}$ | $0.2963^2 \times 0.7037$ $= {}^{1216}/_{19683}$ | = 0.0618 (3 sfs)	M1 M1 A1 3	ft (b) for M1M1 must see method if ft Allow figs rounded to 2 sfs for M1M1 cao. allow art 0.0618 or 0.0617
Total		**12**	

Total 72 marks

Mark Scheme 4733
June 2006

1		$\mu = \frac{3}{37}\int_3^4 x^3 dx = \frac{3}{37}\left[\frac{x^4}{4}\right]_3^4$ [= $3\frac{81}{148}$] $\frac{3}{37}\int_3^4 x^4 dx = \frac{3}{37}\left[\frac{x^5}{5}\right]_3^4$ = $12\frac{123}{185}$ or 12.665 $\sigma^2 = 12\frac{123}{185} - 3\frac{81}{148}^2$ = **0.0815**	M1 M1 A1 A1 M1 A1 **6**	Integrate xf(x), limits 3 & 4 *[can be implied]* [$\frac{525}{148}$ or 3.547] Attempt to integrate x^2f(x), limits 3 & 4 Correct indefinite integral, any form $\frac{2343}{185}$ or in range [12.6, 12.7] *[can be implied]* Subtract their μ^2 Answer, in range [0.0575, 0.084]
2	(i)	Find P($R \geq 6$) or P($R < 6$) = 0.0083 or 0.9917 Compare with 0.025 [can be from N] [0.05 if "empty LH tail stated] Reject H_0	M1 A1 B1 **4** A1√	Find P(= 6) from tables/calc, OR RH critical region P(≥ 6) in range [0.008, 0.0083] or P(< 6) = 0.9917 OR CR is 6 with probability 0.0083/0.9917 Explicitly compare with 0.025 [or 0.975 if consistent] OR state that result is in critical region Correct comparison and conclusion, √ on their p
	(ii)	n = 9, P(≤ 1) = 0.0385 [> 0.025] n = 10, P(≤ 1) = 0.0233 [< 0.025] Therefore n = **9**	M1 A1 B1 **3**	At least one, or n = 8, P(≤ 1) = 0.0632 Both of these probabilities seen, don't need 0.025 Answer n = 9 only, indep't of M1A1, *not* from P(= 1)
3	(i)	$(140 - \mu)/\sigma = -2.326$ $(300 - \mu)/\sigma = 0.842$ Solve to obtain: μ = **257.49** σ = **50.51**	M1 B1 A1√ M1 A1 A1 **6**	One standardisation equated to Φ^{-1}, allow "1–", σ^2 Both 2.33 and 0.84 at least, ignore signs Both equations completely correct, √ on their z Solve two simultaneous equations to find one variable μ value, in range [257, 258] σ in range [50.4, 50.55]
	(ii)	Higher as there is positive skew	B1 B1 **2**	"Higher" or equivalent stated Plausible reason, allow from normal calculations
4	(i)	Each element equally likely to be selected (and all selections independent) OR each possible sample equally likely	B1 **1**	One of these two. "Selections independent" alone is insufficient, but don't need this. An example is insufficient.
	(ii)	B(6, 5/8) ${}^6C_4\, p^4 (1 - p)^2$ = **0.32187**	M1 M1 A1√ **3**	B(6, 5/8) stated or implied, allow e.g. 499/799 Correct formula, any p Answer, a.r.t. 0.322, can allow from wrong p
	(iii)	N(37.5, 225/16) $\frac{39.5-37.5}{3.75}$ = 0.5333 $1 - \Phi(0.5333)$ = **0.297**	B1 B1 M1 dep A1 dep M1 A1 **6**	Normal, mean 37.5, or 37.47 from 499/799, 499/800 14.0625 or 3.75 seen, allow 14.07/14.1 or 3.75 Standardise, wrong or no cc, np, npq, no $\sqrt{n}$ Correct cc, $\sqrt{npq}$, signs can be reversed Tables used, answer < 0.5, p = 5/8 Answer, a.r.t. 0.297 SR: np < 5: Po(np) stated or implied, B1

5	(i)	B(303, 0.01) $\approx$ Po(3.03)	B1 B1 **2**	B(303, 0.01) stated, allow p = 0.99 or 0.1 Allow Bin implied clearly by parameters Po(3.03) stated or implied, can be recovered from (ii)
	(ii)	$e^{-3.03}(1+3.03+\frac{3.03^2}{2})$ = 0.4165 **AG**	M1 A1 **2**	Correct formula, $\pm$ 1 term or "1 – " or both Convincingly obtain 0.4165(02542) [Exact: 0.41535]
	(iii)	302 seats $\Rightarrow \mu$ = 3.02 $e^{-3.02}(1+3.02)$ = 0.1962 0.196 < 0.2 So **302** seats.	M1 M1 A1 A1 A1 **5**	Try smaller value of μ Formula, at least one correct term Correct number of terms for their μ 0.1962 [or 0.1947 from exact] Answer 302 only
	SR: SR: SR:	B(303, 0.99): B1B0; M0; M1 then N(298.98,2.9898) or equiv, standardise: M1A1 total 4/9 p = 0.1: B(303, 0.1), N(30.3, 27.27) B1B0; Standardise 2 with np & $\sqrt{npq}$, M1A0; N(0.1n, 0.09n); standardise with np & $\sqrt{npq}$; solve quadratic for $\sqrt{n}$; n = 339: M1M1M1A1, total 6/9 B(303, 0.01) ≈ N(3.03, 2.9997): B1B0; M0A0; M1A0		
6	(i)	Customers arrive independently	B1 **1**	Valid reason in context, allow "random"
	(ii)	1 – 0.9921 = **0.0079**	M1 A1 **2**	Poisson tables, "1 –", or correct formula $\pm$ 1 term Answer, a.r.t. 0.008 [1 – 0.9384 = 0.0606: M1A0]
	(iii)	N(48, 48) $z = \frac{55.5-48}{\sqrt{48}}$ = 1.0825 1 – Φ(1.0825) = **0.1394**	B1 B1√ M1 dep A1 dep M1 A1 **6**	Normal, mean 48 Variance or SD same as mean√ Standardise, wrong or no cc, $\mu = \lambda$ Correct cc, $\sqrt{\lambda}$ Use tables, answer < 0.5 Answer in range [0.139, 0.14]
	(iv)	$e^{-\lambda}$ < 0.02 λ > –ln 0.02 = 3.912 0.4t = 3.912: t = 9.78 minutes t = 9 minutes 47 seconds	M1 M1 A1 M1 A1 **5**	Correct formula for P(0), OR P(0 | λ = 4) at least ln used OR λ = 3.9 at least by T & I 3.91(2) seen OR λ = 3.91 at least by T & I Divide λ by 0.4 or multiply by 150, any distribution 587 seconds $\pm$ 1 sec *[inequalities not needed]*

7	(i)	$\frac{c-4000}{60/\sqrt{50}} = 1.645$ Solve $c = 4014$ [4013.958] Critical region is **> 4014**	M1 B1 A1√ M1 A1 A1√ **6**	Standardise unknown with √50 or 50 *[ignore RHS]* $z = 1.645$ or -1.645 seen Wholly correct eqn, √ on their z [1 – 1.645: M1B1A0] Solve to find c Value of c, a.r.t. 4014 Answer "> 4014", allow ≥, √ on their c, needs M1M1
	(ii)	Use "Type **II** is: accept when H_0 false" $\frac{4020-4014}{60/\sqrt{50}}$ $= 0.7071$ [0.712 from 4013.958] $1 - \Phi(0.7071)$ $=$ **0.240** [0.238 from 4013.958]	M1dep depM1 A1√ A1 M1 A1 **6**	Standardise 4020 and 4014√ , allow 60^2, cc With √50 or 50 Completely correct LHS, √ on their c z-value in range [0.707, 0.712] Normal tables, answer < 0.5 Answer in range [0.2375, 0.2405]
	(iii)	Smaller Smaller cv, better test etc	B1 B1 **2**	"Smaller" stated, no invalidating reason Plausible reason
	(iv)	Smaller Smaller cv, larger prob of Type **I** etc	B1 B1 **2**	"Smaller" stated, no invalidating reason Plausible reason
	(v)	No, parent distribution known to be normal	B2 **2**	"No" stated, convincing reason SR: If B0, "No", reason that is not invalidating: B1

Mark Scheme 4734
June 2006

1		Add two Poisson distributions	M1		
		With mean 17	A1		
		$P(27)=e^{-17}17^{27}/27!$ or $P(\le 27)-P(\le 26)$	M1		Use formula or table
		0.00634 or 0.0063, 0.0064 from tables	A1	**4**	M1A1 0.0052 from N(17,17)
2		$H_0: p_1=p_2=p_3=p_4$,			Indication of equality of proportions
		(H_1: They are not all equal**)**	B1		
		Expected values under H_0=150	B1		
		$X^2 = (12^2+23^2+15^2+20^2)/150$	M1		At least one correct term
		=8.653	A1		Accept art 8.65 or 8.66
		Critical value with 3 d.f. = 7.815	B1		
		(X^2 > 7.185 so) reject H_0 and accept that proportions are different.	B1√	**6**	ft critical value
3		Assume population of differences has a normal distribution.			
		or sample random	B1		Either assumption.
		H_0: $\mu_B - \mu_A=0$, H_1: $\mu_B - \mu_A > 0$	B1		AEF
		$t=(23.43-22.84)/\sqrt{(0.548/10)}$	M1		
		=2.520	A1		
		CV=1.833	B1		Seen
		2.52 > CV so reject H_o	M1		Allow from CV 2.262 (2-tail), 1.812,1.734
		Accept that there is evidence that mean time has reduced.	A1 √	**7**	ft wrong CV
4	(i)	EITHER: $\int_{q_3}^{4}\frac{1}{12}x\mathrm{d}x=\frac{1}{4}$ or $\int_{1}^{2}\frac{4}{3x^3}dx+\int_{2}^{q_3}\frac{1}{12}xdx=\frac{3}{4}$	M1*		
		$[x^2/24]$ OR $[-2/(3x^2)] + [x^2/24]$	A1		Either
		$(16-q_3^2)/24=1/4$ or $1/3 + q_3^2/24 = ¾$	dep *M1		Form equation and attempt to solve
		$q_3=\sqrt{10}$	A1	**4**	Accept to 3 SF
		If they find F(x): M1A1, M1A1			
	(ii)	$E(X^2)=\int_1^2\frac{4}{3x}\mathrm{d}x+\int_2^4\frac{x^3}{12}\mathrm{d}x$			
		$E(X) = \int_1^2\frac{4}{3x^2}\mathrm{d}x+\int_2^4\frac{x^2}{12}\mathrm{d}x$	M1		Either correct
		$\left[\frac{4}{3}\ln x\right]_1^2+\left[\frac{x^4}{48}\right]_2^4$	A1		
		$\left[\frac{-4}{3x}\right]_1^2+\left[\frac{x^3}{36}\right]_2^4$	A1		
		$a= E(X^2)/E(X)$	M1		
		a=2.6659, 2.67	A1	**5**	Or exact value, (3ln2)/5 + 9/4 or equiv.

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5	(i)	(48×72/150) or (48/150)(72/150)×150	M1		Multiply and divide relevant values
			A1	2	All correct
	(ii)	No, no expected value less than 5	B1	1	
	(iii)	H_0:Volume and day are independent			Attributes specified
		(H_1:Volume and day are not independent)	B1		
		Critical value for 4 df=13.28	B1		
		Test statistic > 13.28, reject H_0	M1		
		Accept that volume and day are not independent	A1	4	
	(iv)	Choose Friday	B1		
		Highest volume	B1	2	Not reference to E values
6	(i)	(a) No	B1		Must be with reason
		0.43 belongs to relevant interval	B1		
		(b)Yes			
		0.43 is outside relevant interval	B1	3	
	(ii)	$H_0:p_R = p_T$, $H_1:p_R \neq p_T$	B1		Proportions
		Estimate of p = 74/165	B1		
		Variance estimate of difference			
		$=(\frac{74}{165})(\frac{91}{165})(\frac{1}{80}+\frac{1}{85})$	B1		May be implied by later work
		$z=(28/80-46/85)/\sigma_{est}$	M1		Standardising
			A1		Completely correct expression
		= -2.468	A1		+ or - , 2.47
		Compare correctly with CV			
		-2.468<-2.326, or 2.468 > 2.326	M1		
		Reject H_0 and accept that the proportions differ on the island.	A1	8	Conclusion in context
7	(i)	$T_1 \sim N(2.2,0.75^2)$, $T_2 \sim N(1.8,0.70^2)$			
		Use T_2 - ½ T_1 normal	M1		Or ½$T_1 - T_2$
		μ=0.7	A1		
		$\sigma^2=0.7^2+ \frac{1}{4}\times0.75^2$ (0.630625)	A1		
		$(0-\mu)/\sigma$	M1		From reasonable σ^2 not just sum
		-0.881	A1		+ or -
		Probability 0.189	A1	6	
	(ii)	Use sum of 5 Ts	M1		
		μ=9.4	A1		
		σ^2=2.5225	A1		
		$z=(10-\mu)/\sigma$	M1		Standardising, must be σ
		Probability 0.6473,0.647	A1	5	
	(iii)	Calculation of variance	B1	1	

8	**(i)**	$s_B^2 = \frac{1}{49}(630.194 - \frac{176.35^2}{50})$	M1		Any equivalent formula
		$=0.1675$	A1		May be implied by later work
		H_0: $\mu_B - \mu_A = 0$, H_1: $\mu_B - \mu_A > 0$	B1		aef
		$z=0.115/\sqrt{(0.049/40 + 0.1675/50)}$	M1		Standardising but not from pooled variance estimate
		$=1.700$	A1		art 1.70
		$z > 1.645$, reject H_0	M1		Compare correctly with 1.645
		and accept that $\mu_B > \mu_A$	A1 √	7	ft their calculated z
	(ii)	$z = 0.09/\sqrt{(0.004575)}$	M1		Correct form
		$= 1.331$	A1		
		H_0 not rejected for $\alpha < 9.16$	M1 A1	4	Accept < 9.2, ≤ 9.2 . M1 for correct method for 9.2, A1 for inequality
	(iii)	(a) Not necessary	B1		Ignore any reason
		(b) Not necessary since samples large enough for CLT to be applied (normality of sample means giving normality of difference)	M1		Mention of CLT implied by "sample large"
			A1	3	Sample mean (approx) normal. (Do not award if population or sample said to be normal)

Mark Scheme 4735
June 2006

1	**(i)**	(a) True (b) False **(c)** True	B2	**2**	B0 for 0,1 correct, B1 for 2 correct, B2 for 3 correct.
	(ii)	Var(2X-Y)= 4Var(X)+Var(Y)-4Cov(X,Y) 6=11-4Cov(X,Y) Cov(X,Y)=5/4	M1 A1 M1 A1	**4**	Using formula Obtain cov cao
2		EITHER: sample is random OR twin pairs chosen independently	B1		
		H_0:m_F=m_S , H_1:m_F > m_S	B1		For both using medians
		Use of B(60,0.5)	M1		
		Normal approx with μ=30, σ^2=15	A1		Both
		EITHER: z=(36.5-30)/ $\sqrt{15}$	M1		Standardising
		=1.678	A2		A1 if correct apart from missing or wrong cc
		OR:CR is (X-30-0.5)/ $\sqrt{15}$ >1.645	M1		Setting up inequality
		X≥37	A2		A1 if correct apart from missing or wrong c.c.
		EITHER: 1.678> 1.645 OR: Sample value 37 in CR	M1		Correct comparison
		There is evidence that the first-born male twins are taller than the second -born twin in a majority of cases.	A1		Conclusion in context
		OR: p-value: 0.0467 > 1.645	M1		
		Completion	A1	**9**	
		NB: Exact Bin (60,0.5) p-value is 0.04623 from graphical calculator: full credit			
3	(i)	P(C)=P(C│F)P(F)+P(C│F′)P(F′) =0.98×0.05 + 0.04×0.95 0.087 **AG**	M1 A1 A1	**3**	Use of formula
	(ii)	P(F│C)= $\frac{0.05\times0.98}{0.05\times0.98+0.95\times0.04}$	M1A1		
		=0.5632	A1	**3**	art 0.563 or 49/87
	(iii)	P(F│C′)=P(C′│F)P(F) / P(C′)	M1		Conditional prob.
		0.02×0.05/0.913 [0.001095]	A1		
		5000×above = 5.476., 5.48.	M1A1√	**4**	ft a conditional prob.

4	**(i)**	$M_X(t)=\int_a^b \frac{1}{b-a}e^{xt}dt$	M1		Correct integral with limits
		= $\left[\frac{e^{xt}}{(b-a)t}\right]_a^b$	B1		Correct integral
		$=\frac{e^{bt}-e^{at}}{(b-a)t}$ AG	A1	**3**	
	(ii)	Product of mgfs	M1		
		$\left(\frac{1-e^{-t}}{t}\right)\left(\frac{e^{t}-1}{t}\right)$	A1	**2**	
	(iii)	$M_S(t)=\left(\frac{e^{\frac{1}{2}t}-e^{-\frac{1}{2}t}}{t}\right)^2$	M1		Square of $M_Y(t)$
		$= (e^t-2+e^{-t})/t^2$	A1dep		Correctly shown
		mgfs of S and T are same		depA1	Correctly shown
		S and T have identical distributions	B1	**4**	
5	(i)	$^{13}C_4$	M1		Use of formula
		715	A1	**2**	
	(ii)	1234,1235,1236,1237,1245,1246, 1345	B2		B1 for 5 or 6
		7/715	B1√ ft (i)	**3**	
	(iii)	Wilcoxon Rank Sum Test		B1	
		H_0:m_X=m_Y, H_1:m_X≠m_Y	B1		Both, involving medians
		Use P(R ≤13)	M1		
		2×7/715×100 =1.958% < 2%	M1		Comparing correctly
		Reject H_0, evidence of difference in medians at a significance level of (smaller than) 2%		A1	**5**
		SR: If tables used, B1B1 M1 for CV with correct comparison for rejection M1 for rejection at 2% (not <) Max 4/5			

6	(i)	$G'(t)=[0.8(1-0.2t)+0.16t]/(1-0.2t)^2$	M1 A1		Quotient or product rule
		$G'(t)=0.8/0.8^2=5/4$ **AG**	A1	**3**	
	(ii)	$G(t)=0.8t\,(1-0.2t)^{-1}$	M1		Use binomial expansion
		$=0.8t(1+0.2t+0.04t^2+\ \ )$	A1		At least 2 correct terms
		$P(Y=r)=0.8(0.2)^{r-1}$	A1		
		$r=1,2,3,\ldots.$	A1	**4**	OR from G(0.8)
	(iii)	EITHER: $Y\sim G(0.8)$	B1		Parameter not required
		$\mathrm{Var}(Y)=(1-0.8)/0.8^2$	M1		
		$=0.3125$	A1		
		OR: $G''(t)=0.32/(1-0.2t)^3$	B1		
		Use $G''(1)+G'(1)-(G'(1))^2$	M1		
		0.3125	A1	**3**	
	(iv)	$G_T(t)=0.8^6t^6(1-0.2t)^{-6}$	B1		$(G_Y(t))^6$
		$P(T\geq 8)=1-0.8^6(1+6\times 0.2)$	M1		Two terms in bracket
		$=0.42328$	A1	**3**	art 0.423
7	(i)	$E(X)=\frac{1}{2}(n+1)$	B1		
		$\mathrm{Var}(X)=\frac{1}{n}\sum r^2-\frac{1}{4}(n+1)^2$	M1		Use of variance formula
		$=\frac{1}{6}(n+1)(2n+1)-\frac{1}{4}(n+1)^2$	A1		
		$=\frac{1}{12}(n^2-1)$ AG	A1	**4**	Correctly obtained
	(ii)	$E(N_1)=E(X_1)+E(X_2)-1$	M1		
		$=\frac{1}{2}(n+1)+\frac{1}{2}(n+1)-1$			
		$=n$, **(** so N_1 is an unbiased estimator of n**)**	A1	**2**	
	(iii)	$P(M=r)=$			
		EITHER: $P(X_1<r,X_2=r)+P(X_1=r,X_2<r)$	M1		
		$=((r-1)/n)(1/(n-1))+(1/n)(r-1)/(n-1)$	A1		
		$=2(r-1)/[n(n-1)]$ AG, $r=2,3,4,\ldots$	A1		
		OR: Choose 1 from r-1 and 1 from 1	M1		
		${}^{r-1}C_1\times{}^1C_1/{}^nC_2$	A1		
		$=(r-1)/[\frac{1}{2}n(n-1)]$=AG	A1	**3**	
	(iv)	$E(M)=\dfrac{2}{n(n-1)}\sum_{r=2}^{n}r(r-1)$	M1		
		$=\frac{2}{3}(n+1)$	A1		
		$N_2=\frac{3}{2}M-1$	A1√	**3**	ft $E(M)$
	(v)	$\mathrm{Var}(N_1)<\mathrm{Var}(N_2)$ or equivalent	M1		Stated or implied
		$\frac{1}{6}(n^2-n-2)<\frac{9}{4}\mathrm{Var}(M)$	A1		
		$\mathrm{Var}(M)>\frac{2}{27}(n^2-n-2)$	A1√	**3**	ft N_2

Mark Scheme 4736
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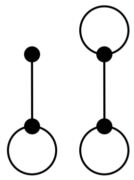
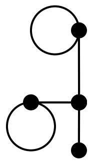
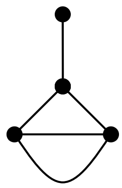
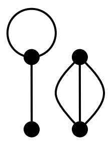
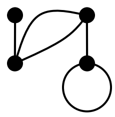
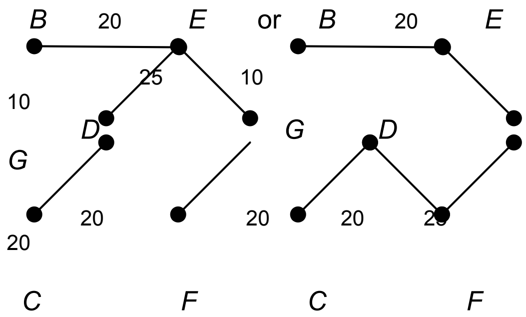
1	**(i)**	2 4 3 3 2 5 4 Box 1 **2 4 2** Box 2 **3 3** Box 3 **5** Box 4 **4**	 M1 A1 [2]	 For packing these seven weights into boxes with no more than 8 kg total in each box For this packing
	(ii)	5 4 4 3 3 2 2 Box 1 **5 3** Box 2 **4 4** Box 3 **3 2 2**	B1 M1 A1 [3]	For putting the weights into decreasing order (may be implied from packing) For packing the seven weights into three boxes with no more than 8 kg total in each box For this packing
	(iii)	15×2^2 = **60** seconds	M1 A1 [2]	For a correct calculation For 60 or 60 seconds or 1 minute $\boxed{7}$
2	**(i)**	   graph *A* graph *B* graph *C* other solutions:  or 	 M1 A1 [2] --------- M1 A1 [2] --------- M1 A1 [2]	Graphs may be in any order For a reasonable attempt For a graph that is topologically equivalent to one of these graphs For a different reasonable attempt For a graph that is topologically equivalent to one of these graphs For another different reasonable attempt For a graph that is topologically equivalent to one of these graphs
	(ii)	The graphs each have four **odd** nodes, but Eulerian graphs have no odd nodes.	B1 [1]	For any recognition that the nodes are not all even $\boxed{7}$

3	(i)	**Travelling salesperson**	B1 [1]	Identifying TSP by name
	(ii)	***A* – *B* – *E* – *G* – *F* – *D* – *C* – *A*** 130 (minutes) Shortest possible time **≤ 130 minutes**	M1 A1 B1 B1 [4]	For starting with *A* – *B* – *E* – *G* - … For this closed tour For 130 For less than or equal to their time, **with units**
	(iii)	Order of connecting: ***B***, ***E***, ***G***, ***F***, ***D***, ***C***  Lower bound = 10 + 15 + 95 = **120 minutes**	B1 M1 A1 M1 M1 A1 [6]	For a valid vertex order (or arc order) for their starting point For a diagram or listing showing a tree connecting the vertices *B*, *C*, *D*, *E*, *F* and *G* but not *A* For a diagram showing one of these trees (vertices must be labelled but arc weights are not needed) For stating or using the total weight of their tree For stating or using *AB* and *AD* or 10 + 15 For 120 or calculating 25 + their 95, **with units**
	(iv)	***A* – *B* – *E* – *G* – *F* – *C* – *D* – *A*** or this in reverse	M1 A1 [2]	For a reasonable attempt For a valid tour of weight 125 $\boxed{13}$

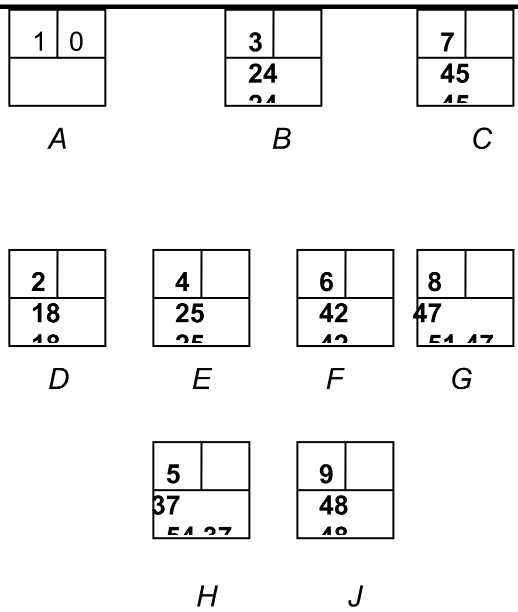
4	(i)	**$x \le 2$** **$y \ge 1$** **$y \le 2x$** **$x + y \le 4$**	B1 B1 B1 B1 [4]	Strict inequalities used, penalise first time only All inequalities reversed, penalise first time only
	(ii)	**(2, 1), (2, 2)** **(½, 1)** **$(1\frac{1}{3}, 2\frac{2}{3})$**	B1 B1 B1 [3]	Both of these This vertex in any exact form This vertex in any exact form or correct to 3 sf
	(iii)	x y $P = x + 2y$ 2 1 4 2 2 6 ½ 1 2½ $1\frac{1}{3}$ $2\frac{2}{3}$ $6\frac{2}{3}$ $x = 1\frac{1}{3}$, $y = 2\frac{2}{3}$ (may be given in coordinate form) $P = 6\frac{2}{3}$	M1 A1 A1 [3]	Evidence of checking value at any vertex or using a sliding profit line Their x and y values at maximum in any exact form or correct to 3 sf Their maximum P value in any exact form or correct to 3 sf
	(iv)	x y $Q = 2x - y$ 2 1 3 2 2 2 ½ 1 0 $1\frac{1}{3}$ $2\frac{2}{3}$ 0 $Q =$ **0** (x, y) can be any point on the **line segment joining (½, 1) and $(1\frac{1}{3}, 2\frac{2}{3})$**	M1 A1 A1 [3]	Evidence of checking value at any vertex or using a sliding profit line 0 (cao) The edge of the feasible region where $y = 2x$ No follow through
	(v)	$P = Q \Rightarrow 2x - y = x + 2y$ $\Rightarrow x = 3y$ $y = \frac{1}{3}x$ lies entirely in the shaded region	M1 A1 A1 [3]	For considering $P = Q$, or equivalent For this line, or any equivalent reasoning For explanation of why there are no solutions $\boxed{16}$

<table>
<tr><td>5</td><td>(i)</td><td>$2x - 5y + 2z + s = 10$
$2x + 3z + t = 30$</td><td>B1 [1]</td><td>Slack variables used correctly</td></tr>
<tr><td></td><td>(ii)</td><td><table><tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td></td></tr><tr><td>1</td><td>-1</td><td>2</td><td>3</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>2</td><td>-5</td><td>2</td><td>1</td><td>0</td><td>10</td></tr><tr><td>0</td><td>2</td><td>0</td><td>3</td><td>0</td><td>1</td><td>30</td></tr></table></td><td>M1

A1 [2]</td><td>For overall structure correct, including two slack variable columns and column for RHS (condone omission of P column or labels)
For a completely correct initial tableau, with no extra constraints added (condone variations in order of rows or columns)</td></tr>
<tr><td></td><td>(iii)</td><td>Pivot on x column since it is the only column with a negative value in the objective row
$10 \div 2 = 5$ $5 < 15$ so pivot on this row
$30 \div 2 = 15$</td><td>B1

B1 [2]</td><td>For negative in objective row, top row, pay-off row, or equivalent
For these two divisions shown</td></tr>
<tr><td></td><td>(iv)</td><td>New row 2 = row 2 $\div$ 2
New row 1 = row 1 + new row 2
New row 3 = row 3 – 2 $\times$ new row 2</td><td>B1
B1 [2]</td><td>For dealing with the pivot row correctly
For dealing with the other rows correctly
May be coded by rows of table</td></tr>
<tr><td></td><td></td><td><table><tr><td>1</td><td>0</td><td>-0.5</td><td>4</td><td>0.5</td><td>0</td><td>5</td></tr><tr><td>0</td><td>1</td><td>-2.5</td><td>1</td><td>0.5</td><td>0</td><td>5</td></tr><tr><td>0</td><td>0</td><td>5</td><td>1</td><td>-1</td><td>1</td><td>20</td></tr></table></td><td>M1
M1
A1 [3]</td><td>For updating their pivot row correctly
For a reasonable attempt at updating other rows
For correct values in tableau (condone consistent order of rows or columns). Do not follow through errors in initial tableau or pivot choice.</td></tr>
<tr><td></td><td></td><td>$x = 5$, $y = 0$, $z = 0$
$P = 5$
Not the maximum feasible value of P since there is still a negative value in the objective row</td><td>B1
B1
B1 [3]</td><td>For reading off x, y and z from their tableau
For reading off P from their tableau
'No' seen or implied and a correct reason

$\boxed{13}$</td></tr>
</table>

6	(a)		M1 M1 A1 B1 B1	ANSWERED ON INSERT Values correct at *B*, *D* and *E* (condone temporary labels implied from permanent labels) Both 54 and 37 seen at *H* and both 51 and 47 seen at *G* (method) All temporary labels correct <u>and no extras</u> All permanent labels correct Order of labelling correct (condone boxes consistently swapped over)
		A* – *E* – *H* - *J **48** metres	B1 B1 [7]	For this route, including end vertices (cao) For 48 (cao)
	(b) (i)	*A* and *J* are the only odd nodes 48 + 300 = **348** metres	B1 M1 A1 [3]	Identifying odd nodes (or by implication) For their 48 + 300 (or their 300) 348 (cao)
	(ii)	Odd nodes *A*, *B*, *H*, *J* *AB* = **24** *AH* = **37** *AJ* = **48** *HJ* = **11** *BJ* = **38** *BH* = **34** Repeat *AB* and *HJ* = 35 300 – 30 = 270 metres Shortest distance = 270 + 35 = **305** metres	B1 B1 B1 M1 M1 A1 [6]	Identifying odd nodes (or by implication) For distances from *A* – or from their Dijkstra For distances *HJ*, *BJ*, *BH* correct Choosing their least pairing or by implication Or by implication 305 (cao) **16**

Mark Scheme 4737
June 2006

1	(i)	4+4+8+7+6 = **29** litres per second	B1 [1]	For 29
	(ii)	**4-1-2+3+3+5** = 12 litres per second **0 – 5 – 4 + 3 + 0 + 5** = -1 So minimum flow across cut is **0**	M1 A1 M1 A1 [4]	For using upper and lower capacities correctly For showing how 12 (given) was worked out For a substantially correct calculation For 0, from an appropriate calculation
	(iii)	Flow in arc ***CE*** $\geq$ **2** and flow in arc ***CF*** $\geq$ **3**, so at least 5 litres per second must flow into *C* At most 4 litres per second flow into *A*, of which at least 1 flows out to *B* and 2 flow out to *E*, so at most 1 litre per second can flow along *AD*	M1 A1 M1 A1 [4]	For any reasonable attempt (eg *CE* = 2, *CF* = 3) For correct reasoning For identifying **$\leq$ 4 in** and **$\geq$ 3 out** or equivalent For a correct conclusion
	(iv)	Either a diagram or a description of a flow of 11 litres per second. Arcs *AD*, *AE*, *BE*, *CE, CF* must all be at their minimum capacities.	M1 A1 A1 [3]	For a flow of 11 litres per second from *S* to *T* Flow satisfies all lower capacities Flow satisfies all upper capacities
	(v)	**11 $\leq$** maximum **flow $\leq$ 12**	B1 B1 [2]	11 as lower bound 12 as upper bound (max flow = 12 $\Rightarrow$ B0, B1) $\boxed{14}$

Question	Answer	Marks	Guidance
2 (i)	The route for which the minimum weight on the route is greatest	B1 B1 [2]	For identifying **route minima** For identifying **what has been maximised** ('maximises the minimum' ⇒ B0 B1)

(ii)

Stage	State	Action	Working	Maximin
1	**0**	**0**		18
	1	**0**		15
	2	**0**		15
2	**0**	**0**	min(16,18) =**16**	**16**
		1	min(13,15) =**13**	
		2	min(14,15) =**14**	
	1	**0**	min(19,18) =**18**	**18**
		1	min(13,15) =**13**	
		2	min(18,15) =**15**	
3	**0**	**0**	min(20,16) =**16**	**16**
		1	min(16,18) =**16**	

Maximin routes: **(3; 0) – (2; 0) – (1; 0) – (0; 0)**
(3; 0) – (2; 1) – (1; 0) – (0; 0)
Maximum load = **16 tonnes**

Marks	Guidance
B1 B1 [2]	Stage and state columns completed correctly Action column completed correctly
M1 A1 M1 A1 M1 A1 [6]	For calculating minima for stage 2 state 0 For maximin values identified (may be implied from working seen for stage 3) For calculating minima for stage 2 state 1 For maximin values identified (may be implied from working seen for stage 3) For calculating minima for stage 3 For maximin value identified (Forwards working scores M0, M0, M0)
B1 B1 B1 [3]	For first correct route For second correct route For 16 tonnes (**with units**)

Question	Answer	Marks	Guidance
(iii)	**18** tonnes **(3; 0) – (2; 0) – (2; 1) – (1; 0) – (0; 0)**	B1 B1 [2]	For 18 For this route $\boxed{15}$

3	(i)	**3** ***Y***	M1 A1 [2]	For 3 (allow -3) For *Y* (cao)
	(ii)	**5 > 3, -2 > -4, 5 > -1 and 6 > 0** or using signs of differences +2, +2, +6, +6 **3 > -2, -5 > -6, 1 > 0, 4 > 2** or equivalent, or using differences Reduced matrix: Colin's strategy Rose's strategy $\begin{array}{c|ccc} & W & X & Y \\ \hline A & -1 & 4 & -3 \\ B & 5 & -2 & 5 \\ D & -5 & 6 & -4 \end{array}$	M1 A1 M1 A1 B1 [5]	For an appropriate comparison, or implied For all four comparisons seen For an appropriate comparison, or implied For all four comparisons seen For correct reduced matrix, with rows and columns labelled *A*, *B*, *D* and *W*, *X*, *Y* Cao
	(iii)	Row minima are -3, -2, -5 Play-safe for Rose is ***B*** Column maxima are 5, 6, 5 Play-safes for Colin are ***W* and *Y*** **Not stable**	 M1 M1 A1 [3]	Follow through their 3×3 reduced matrix For identifying row *B* For identifying columns *W* and *Y* For 'no' or 'not stable'
	(iv)	**5 is added** throughout the matrix to make the entries non-negative. In this augmented reduced matrix, $9p_1 + 3p_2 + 11p_3$ is the expected number of points won by Rose when Colin plays **strategy *X***	M1 A1 [2]	For 'add 5' or equivalent For identifying that this is when Colin plays strategy *X*
	(v)	$p_1 = \frac{7}{48}$, $p_2 = \frac{27}{48}$, $p_3 = \frac{14}{48}$ $\Rightarrow m \le \frac{298}{48}$ (or $6\frac{5}{24}$, 6.2083, 6.21) in all three cases $\Rightarrow M = \frac{58}{48}$ (or $\frac{29}{24}$, $1\frac{5}{24}$, 1.2083, 1.21)	 M1 A1 [2]	 For attempting to evaluate *m* cao (in any appropriate form) **14**

4 (i)

ANSWERED ON INSERT

Activity	Duration	Immediate predecessors
A	6	**-**
B	4	**-**
C	5	***A***
D	1	***A, B***
E	5	***A, D***
F	4	***D***
G	2	***C, E, F***

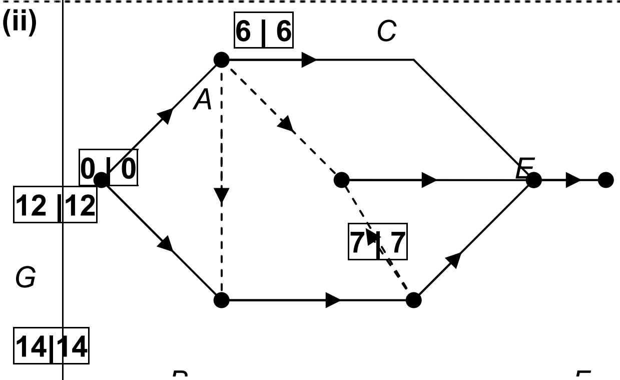
B1 For predecessors for activities *A*, *B* and *C* correct

B1 For predecessors for activities *D, F* and, *G* correct

B1 For predecessors for activity *E* correct

[3]

(ii)



Minimum completion time = **14** hours
Critical activities: ***A, D, E, G***

M1 For carrying out forward pass (no more than one independent error)

A1 For all early event times correct

M1 For carrying out backwards pass (no more than one independent error)

A1 For all late event times correct

B1 For 14 cao

B1 For *A*, *D*, *E*, *G* only cao

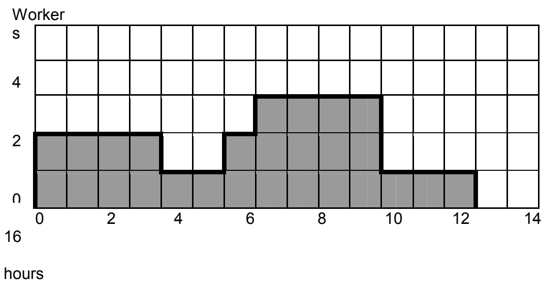
[6]

(iii) **Increased by 2** (hours)
Becomes 16 (hours)

B1 For stating that time increases by 2, or equivalent

[1]

(iv)



Number of workers required = **3**

B1 For a resource histogram with no overhanging cells

M1 For a reasonable attempt, ft their start times if possible

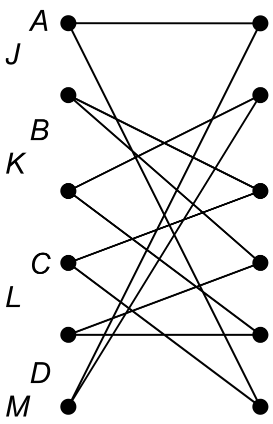
A1 For a completely correct histogram (cao)

B1 For 3 or follow through their histogram if possible

[4]

$\boxed{\textbf{14}}$

5 (i)



M1 — For a substantially correct attempt

A1 — For a completely correct bipartite graph

[2]

ANSWERED ON INSERT

(ii)

C – N *E – M* *F - K*
A – J *B – L* *D - O*

M1 — For pairing *F – K*, *C – N*, *E – M*

A1 — For all correct (Diagram only ϕ M1, A0)

[2]

(iii)

	J	*K*	*L*	*M*	*N*	*O*
A	2	5	2	2	5	2
B	2	5	2	0	5	5
C	5	0	5	5	2	2
D	**2**	**5**	**0**	**5**	**5**	**2**
E	**5**	**2**	**5**	**2**	**0**	**5**
F	**2**	**2**	**5**	**5**	**2**	**2**

B1 — For '5' in all the entries that should be 5

B1 — For '2' in all the entries that should be 2

B1 — For '0' in all the entries that should be 0

[3]

(iv)

Reduce rows

0	3	0	0	3	0
2	5	2	0	5	5
5	0	5	5	2	2
2	5	0	5	5	2
5	2	5	2	0	5
0	0	3	3	0	0

Columns are already reduced

Or, reduce columns

0	5	2	2	5	0
0	5	2	0	5	3
3	0	5	5	2	0
0	5	0	5	5	0
3	2	5	2	0	3
0	2	5	5	2	0

Rows are already reduced

M1 — For a substantially correct attempt from their matrix

A1 — For a correct reduction of rows and columns (or columns and rows) for their matrix

Cannot cross out 0's using fewer than 6 lines so matching is complete

A – J *B - M* *C – K* *D - L* *E – N* *F – O*

A – O *B – M* *C – K* *D - L* *E – N* *F – J*

First matching: Fred and Jenny
Second matching: Jenny and Olivia

M1 — For achieving a reduced cost matrix with a complete matching of zero cost (without unnecessary augmenting)

A1 — 0's in correct cells (not ft)

B1 — For this matching or ft their reduced cost matrix

B1 — For this matching or ft their reduced cost matrix

B1 — For the names for their first matching

B1 — For the names for their second matching

[8]

				15

Advanced GCE Mathematics (3890, 3892, 7890)
June 2006 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	a	b	c	d	e	u
4721	Raw	72	56	48	40	33	26	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	53	45	37	29	22	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	60	52	44	37	30	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	60	52	44	37	30	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	50	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	59	51	43	36	29	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	51	44	37	30	23	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	52	44	36	29	22	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0

4737	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	31.0	46.3	61.2	73.5	84.2	100	12438
3891	0	0	0	100	100	100	1
3892	60.6	76.8	89.2	95.3	97.6	100	1109
7890	46.9	67.7	81.9	91.5	97.6	100	9525
7891	50.0	75.0	87.5	87.5	100	100	8
7892	59.9	80.2	89.4	95.5	98.6	100	1428

For a description of how UMS marks are calculated see;
www.ocr.org.uk/OCR/WebSite/docroot/understand/ums.jsp

Statistics are correct at the time of publication

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