GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

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REPORTS FOR THE UNITS

Pure Mathematics

Chief Examiner Report

Multiple attempts at questions.

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

Graph Paper

At the request of a number of centres, graph paper is no longer being listed under additional materials on question papers. Graph paper should be available for candidates if they request it, but it will only be listed on the front of a paper if it is required. Examiners would like to stress that ‘sketch’ does not mean a graph drawn on graph paper; sketches should be drawn in the answer booklet, as they are not required to be plotted accurately.

Legacy units

Centres are reminded that **under no circumstances** can ‘legacy’ units be used in the ‘new’ specification from January 2007 onwards. The only units that are acceptable for this specification are units 4721 – 4737.

4721: Core Mathematics 1

General Comments

This question paper proved to be accessible to the vast majority of candidates, with many scoring high marks. It contained a couple of quite challenging part questions, however, and only a very few candidates gained all 72 marks. Most candidates attempted all questions in the order in which they were set.

There were four places within the paper which required the solution of a quadratic equation. Candidates who were unable to perform this task effectively lost many marks. Too many candidates also lost marks by failing to simplify fractions or surds correctly. This was disappointing given that these techniques are first learned at GCSE level. It should be stressed to candidates that many marks are available for knowledge and application of such topics as well as for those parts of the specification met for the first time at AS level.

There was some evidence that a small minority of candidates ran out of time and failed to finish Q9.

Comments on Individual Questions

1) Part (i) proved to be a straightforward opening question and most candidates scored both marks. It was rare to see an incorrect formula for gradient, which was encouraging, although there were a few arithmetical slips.

   By contrast, part (ii) was poorly answered by many candidates. The most common wrong solution involved simply working out the y-value when $x = 3$ and not using differentiation at all.

2) Only the very best candidates scored full marks on this question.

   Part (i) proved especially challenging. While some candidates knew that $27^{\frac{2}{3}}$ was equal to 9, they did not know how to deal with the minus sign. Other candidates understood that a reciprocal was involved but could not process the fractional power. There were few fully correct answers and they were mostly produced by the strongest candidates. However, it was clear that some centres had taught this topic extremely effectively, as almost all their candidates scored both marks here.

   In part (ii), many candidates stated that the required power was $\frac{1}{5}$, misinterpreting the question. $\frac{5}{2}$ was also commonly seen, while many candidates simply left out this part.

   All but the weakest candidates started part (iii) successfully, 2 marks out of 3 being the modal mark. However, having reached $\frac{8-4\sqrt{5}}{4}$, many were unable to cancel correctly, the most common final answer being $2-4\sqrt{5}$. This was disappointing after earlier good work in dealing with the surds. The minority of candidates who did not start off correctly generally multiplied by either $3+\sqrt{5}$ or $\sqrt{5}$.

3) As in previous papers, candidates had difficulty in reaching the correct value for c in part (i). The values for a and b were almost always correct (with the occasional $b = 6$ seen) but then candidates subtracted b^2 rather than $2b^2$ from 13. It was pleasing to see a few candidates checking their work by multiplying out their final expression, some then adjusting their value for c.

Few candidates used their completed square form in part (ii), preferring to start afresh using the quadratic formula. There were far too many instances where an incorrect formula was stated. In this case, all 3 marks were forfeited. Those candidates who did recall the formula correctly sometimes made careless errors in calculating $b^2 - 4ac$, others made a mistake similar to that in Q2(ii) by cancelling $\frac{-12 \pm 2\sqrt{10}}{4}$ to $-3 \pm 2\sqrt{10}$.

4) Most candidates scored quite well on this question as a whole.

The vast majority of responses to part (i) were fully correct. Some candidates, rather than first expanding two brackets, preferred to multiply sets of three terms together. This method tended to be less successful but did gain all 3 marks if the written working was well organised, with no incorrect terms seen.

As in previous sessions, examiners were very disappointed with the standard of graph sketching. In part (ii), too many candidates failed to see the significance of the factorised expression in part (i) and resorted to working out y-values for a selection of x-values. This approach rarely led to a good curve, although marks could be gained for correct intercepts. Too many curves had completely horizontal sections, particularly between (0, 12) and (1, 12) or between (3, 0) and (4, 0). Weaker candidates also failed to show the curve crossing the x-axis at three distinct points.

In part (iii), follow-through marks could be gained for a correct reflection of the curve sketched previously. Candidates often failed to label their curves which caused difficulties in marking. Although many candidates correctly reflected their curve, a significant proportion reflected in the y-axis rather than in the x-axis and others reflected in both.

5) Solutions to this question demonstrated a somewhat confused approach to solving inequalities.

Many candidates appeared to have no idea how to tackle part (i). Dealing with only one side of the inequality was common, leading to statements such as $1 < 4x < 14$. Some candidates ignored one of the constants and found a single solution set. Others failed to simplify the fractions in their final answer.

Many candidates scored better on part (ii). They usually gained the first 3 marks by correctly finding the roots of the quadratic equation but then, as in previous papers, appeared to have no method for dealing with the inequality. Even candidates who clearly knew how to solve a quadratic inequality 'wrapped' their final answer, giving $-1 \geq y \geq 5$ and losing the final mark.

6) Part (i) was generally well answered with candidates either factorising the quartic expression straightaway as $(x^2 - 5)(x^2 - 5)$ or realising that they needed to use a substitution to form a quadratic equation. It was most encouraging that only a tiny minority of candidates took the square root of each individual term. Candidates who used the letter x and substituted x for x^2 often finished with the answer $x = 5$ and forgot to take the square root. A very large number of candidates forgot to give the negative square root and thus lost the final mark. There was also a small minority of candidates who clearly had no idea how to tackle this question and did not attempt it.

The differentiation in part (ii) was done very well by most and a good number of candidates spotted the connection with the expression in part (i).

Part (iii) was also tackled well, with some candidates solving the quartic equation successfully here, having failed to do so in part (i). Some tried to use the discriminant without quoting a quadratic equation and then stated '1 solution'. This did not merit a method mark as the solution it gives is for x^2 and there was no suggestion that the candidates understood this.

7) The first two parts of this question were extremely well answered. Even the weakest candidates were able to make progress and often scored all of the first 5 marks.

In part (i), candidates who decided to eliminate y generally combined the equations successfully and then solved the ensuing quadratic equation correctly. Unfortunately, many candidates of all abilities then forgot to work out the y values and lost a mark here. Candidates who chose to eliminate x often forgot the '$y =$' on the left hand side of their equation and ended up solving $y^2 - 3y = 0$ rather than $y^2 - 4y = 0$.

Almost all candidates stated correctly that there were 2 intersections in part (ii), some drawing a small sketch to demonstrate.

Part (iii) proved to be one of the most demanding parts of this paper. Many candidates had no idea how to tackle this part of the question. Those who made an attempt often correctly differentiated to get $\frac{dy}{dx} = 2x - 5$ but then either substituted in an x value (often one of the values found in part (i)) or solved $2x - 5 = 0$. Others used a gradient of either 5 or –1 without explaining why. Working was often very muddled. However, there was a pleasing number of completely correct solutions, some using the equal gradient approach, others setting the discriminant of $x^2 - 6x + (4 - c) = 0$ equal to zero and solving.

8) It was obvious from the responses to part (i) that the majority of candidates did not really understand what is required when they are asked to 'show' a result. Explanations lacked words and/or details and '$8 \times 4 = 32$' was commonly seen in weaker attempts, without any reasoning.

Part (ii) was more straightforward and most candidates did it well, although $\frac{32}{x} = x^{-32}$ was a common misconception.

Candidates found part (iii) difficult. Even those who correctly set $\frac{dA}{dx} = 0$ and then solved to get $x = 2$ seemed to have little idea how to show that this gave a minimum surface area. Some 'justified' their answer by stating that $2x^2 + \frac{32}{x}$ was a positive quadratic so its turning point was a minimum. Many candidates managed to get the correct value for x by substituting values into the expression for A but, this being a non-calculator paper, they tried only integer values and often made computational errors, rendering their answer invalid.

A large number of candidates scored only the 3 marks for part (ii) in this question.

9) Many candidates of all abilities scored well on this final question.

Part (i) was done very well. An incorrect formula was seen in only a few cases, with (3, 4) being the most common wrong answer.

The technique for finding the length of a line in part (ii) was also well understood, although far too many candidates did not read the question properly and found AB instead of AC.

If candidates could recall the general equation for a circle, part (iii) was done well. However, there were large numbers of candidates who seemed to be unfamiliar with this part of the specification, some quoting formulae involving π. Many candidates failed to see the link with the previous parts of the question and calculated the length of the radius, not realising that they had already found it in part (ii).

The final part of the question was done well by most, the vast majority finding the gradient of *AB* and progressing correctly from this. Some centres had candidates who tried to differentiate the equation of the circle implicitly. This was rarely correct, although the very best candidates succeeded. It was pleasing to note that most candidates gave their final answer in the form specified, thus earning the final mark.

4722: Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and overall the standard was good. There were several straightforward questions where candidates who had mastered routine concepts could demonstrate their knowledge, but other questions had aspects that challenged even the most able candidates. However, there was a significant number of candidates who struggled with even the most basic of concepts and could make little or no progress with any of the questions.

Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and decipher answers given. This was especially true if a candidate made a second attempt at a question. When asked for a sketch graph, candidates must appreciate that there is no need for graph paper, and that its use can sometimes be counter-productive. A simple sketch on the lined paper is all that is required, but they must still ensure that the intention is clear, such as showing the asymptotic behaviour of an exponential curve. There were two questions that required candidates to prove a given result. It is very important in this type of question that sufficient detail is provided to convince the examiner that the principle has been fully understood. Candidates must also ensure that any intermediate values in their working are accurate enough to justify the final answer to the specified degree of accuracy. A significant number failed to gain full marks due to using a rounded value from a previous part of the question.

Most candidates showed a basic understanding of logarithms, but only the most able candidates could manipulate them accurately. Solving trigonometric equations posed problems for many, and Q5 was particularly poorly done. Once again, some candidates lost marks through a lack of mastery of basic skills, such as algebraic manipulation, use of indices and solving quadratic equations. In topics where a variety of approaches can be used, it is important that candidates appreciate which is the most efficient and effective in a given situation. This was particularly true of Q8, using the factor and reminder theorems, and Q7, finding the area and perimeter of a segment.

Comments on Individual Questions

1) Most candidates could attempt the binomial expansion, though a number chose to expand the four brackets and were rarely successful. Whilst this is a valid approach, it would not be helpful in a question with a larger index. When using the binomial expansion, a few made errors such as ${}^4C_2$ becoming ${}^4/_2$ or the three components of a term being added rather than multiplied, but most could attempt a correct expansion. The most common error was a failure to deal with powers of $3x$ correctly. As in previous sessions, the more successful methods involved effective use of brackets, but some of the candidates who did this then subsequently ignored the brackets. Some candidates attempted to remove a common factor and then use the expansion of $(1+x)^n$, but this was rarely successful and usually only gained 1 mark. Candidates should use the technique most appropriate to the examination being sat. The vast majority of candidates could gain some credit on this question, usually from $-96x + 16$ along with three other terms, but it was a minority that gained full marks.

2) (i) This question was very well answered, with the majority of candidates gaining both of the marks available, though a few used $u_n = 1 - n$ instead of the given sequence. Some of the weaker candidates failed to get the correct answer for u_3, as they could not correctly evaluate $1 - (-1)$.

(ii) This part of the question was done much less successfully, with only the most able candidates gaining all 3 marks. Whilst most candidates could identify the general meaning of sigma notation, most then attempted to find S_{100} for an AP, or sometimes a GP, rather than looking at the pattern of numbers that had emerged in part (i). Some candidates attempted to find the sum of the first 100 natural numbers, rather than appreciating that they should be summing the first 100 terms of the sequence, and some attempted the 100th term instead. Candidates seemed to have little, if any, experience with sequences other than arithmetic or geometric.

3) This was a straightforward, routine question that was answered well by many of the candidates. Most realised that integration was required and could make an attempt at this, though dividing by ½ caused problems for some. Many then used (4, 5) to find the value of the constant, though some candidates failed to gain the final mark by writing 'equation = $4x^{1/2} - 3$', rather than $y = 4x^{1/2} - 3$. A number of candidates saw the word gradient and attempted to use $y = mx + c$, or an equivalent form. Sometimes this was used to find the constant of integration, but a significant minority tried to treat the entire question in this way, with either an algebraic or numerical gradient.

4) (i) The values for x were often found correctly, but many candidates then wasted valuable time by finding the y coordinates, which had not been requested. It was worrying to see that some candidates lacked the necessary skills to solve the resulting quadratic equation. A minority of candidates ignored the straight line, and simply found the two points of intersection of the curve with the x-axis.

(ii) The majority of candidates understood the concept of definite integration, and could integrate the given functions correctly and then attempt to use limits. However, there was a number of arithmetical errors made when evaluating the integral, involving indices and negative signs. In many cases, the choice of limits seemed somewhat random, and they were often not the values found in part (i). There were the usual errors in the order in which the limits were used, and whether the results were then added or subtracted.

There were roughly equal proportions of candidates who chose to carry out a subtraction first, and those who calculated the area under the curve and the line separately. In the first method, the common errors were an incorrect order of subtraction (or just using the quadratic derived in part (i)) and a lack of brackets when subtracting, resulting in a constant term of 6 not 2. In the second method, only a few candidates were sufficiently astute to use ½bh to find the area of the triangle. Most used integration, often with different limits from those used when integrating the curve. When a negative area was obtained for the area, most candidates appreciated that this was impossible, but then tried to fudge their answer rather than re-evaluate the strategy used. Whilst virtually all candidates gained some credit on this question, there were few fully correct solutions.

5) (i) Most candidates recognised the need to use $\cos^2 x + \sin^2 x \equiv 1$, but there were some slips on substitution. A number of candidates used $\cos x + \sin x \equiv 1$, and some candidates went round in circles and never actually reached a quadratic in $\cos x$ only. Of those who did obtain a quadratic, a worrying number then struggled to solve it. Some felt that ½ and –1 were the final solutions and made no attempt at the actual angles, an approach that was not helped by using the substitution $x = \cos x$ to solve the quadratic equation. However, a number of candidates did find the correct two angles, though this was sometimes spoilt by the additional solutions of 120° and/or 0°.

(ii) This was found to be harder than part (i), and all but the most able candidates struggled with this question. Some candidates attempted to use the same identity as in part (i) and some attempted to use double angle formulae; both approaches rarely gained any credit. A number of candidates betrayed a lack of understanding of trigonometric functions, with $\sin 2x + \cos 2x$ being 'factorised' to $2(\sin x + \cos x)$, or even $2x(\sin + \cos)$. Of those candidates who recognised the need to use the identity for tangent, the common mistake was also to cancel the 2s, resulting in $\tan x = -1$. Only a minority obtained $\tan 2x$ and this was often equal to something other than –1, with 0 being the most common alternative. Many then struggled to use the correct solution method, with dividing by 2 often coming before an attempt to find inverse tan. Completely correct solutions were few and far between, and many candidates failed to gain any credit at all.

6) (i)(a) This question was generally well done, with many candidates obtaining the correct answer. Those who used the relevant formula for the n^{th} term of an AP were nearly always correct; however those who attempted a more informal evaluation usually obtained a final answer of 1300, from $100 + (240 \times 5)$.

(i)(b) Again, this question was usually done well, with the majority of candidates attempting to use either one of the two formulae for the sum of an AP. A small minority actually wrote down the correct calculation but could not then evaluate it accurately. A few attempted a more informal method; this was rarely successful.

(ii) Some candidates thought that this part was still about APs, but most could make a correct initial statement involving r. A number of candidates then struggled to make further progress in solving this equation. A common error was for 1500 ÷ 100 to become 150. Attempts to use logarithms were rarely successful as they could make no further progress beyond log $r = k$. The second problem was with premature approximation. A large majority of candidates seemed to have no appreciation that using $r = 1.01$ would not produce a final answer accurate to even 3 significant figures.

7) This question was reasonably well attempted, and a valuable source of marks to candidates who had been struggling elsewhere on the paper. There was the usual minority of candidates who chose to work in degrees and not radians, oblivious to the extra work that this created, and some who believed that 0.8 radians meant an angle of 0.8π. Premature approximations often led to inaccurate final answers. There were several errors that were common to both parts (ii) and (iii) of the question. These included ignoring the given value for *AC* and of using an incorrect value, assuming that *AB* and *DC* were parallel, and not realising that *AC* and *AD* were the same length. In addition, a number of candidates seemed either unfamiliar with the definitions of sector and arc or were unclear which formula pertained to which. This resulted in the length of the arc being found in part (ii) and/or the area of the sector being found in part (iii), an approach that gained no credit, even when done correctly.

(i) Most candidates could attempt to find the length of *AC* by using the correct cosine rule, though some then simplified it to 9cos0.8. Some candidates assumed that angle *ACB* was a right angle. The question requested candidates to 'show that' the length was 7.90cm – examiners expected to see sufficient intermediary steps in the proof, and also expect to see working that was accurate enough to justify this answer. Stating $AC = 7.89 = 7.90$ lost the final accuracy mark, as did giving $AC = 7.9$.

(ii) Most candidates could correctly find the area of the sector, though some went no further than this. Whilst many candidates did appreciate the need to then find the area of the triangle *DAC*, many did not use the most efficient method and instead resorted to right-angled triangles, often incorrectly.

(iii) The majority of candidates correctly calculated the length of the arc, but some then left this as their final answer and others found the perimeter of the sector not the segment. However, most appreciated the need to find the length of *DC* and attempted to do so, usually accurately.

8) (i) The responses to this question were very varied, with a number of concise and accurate solutions seen, but also some very lengthy solutions that gained little or no credit. A number of candidates seemed familiar with the factor and remainder theorems and could attempt the two equations. A surprisingly large minority was happy to use one theorem, but then used a different method to obtain the second equation. Of those who managed to get two correct equations, a number then struggled to solve these basic simultaneous equations. In addition, a significant proportion of the candidates chose to use much more cumbersome and

time-consuming methods, such as long division and matching coefficients. When matching coefficients, the problem was exacerbated by a choice of non-unique letters. In each case they seemed familiar with the standard methods but struggled to apply it to this situation, and were rarely successful. Whichever of these methods they used, candidates had to obtain equations involving just a and b which rarely happened. Whilst it is important for candidates to be familiar with a variety of methods, it is also important that they appreciate the most efficient and accurate method for a given situation. This is a comment that has been made in previous reports, yet there has been little change since previous examination sessions.

(ii) This part of the question was generally very well done, though incorrect coefficients from part (i) prevented full marks being gained. The majority attempted long division and were often successful with their cubic, though there were odd slips on the way. Some candidates appeared to have been taught more intricate methods for division, which offered ample scope for error, as candidates seemed unsure of the actual method. Other successful methods included inspection and matching coefficients, though those using the latter method needed to state their quotient explicitly. Candidates using either of the latter two methods sometimes forgot to include a possible remainder. A number of candidates used f(-2) to find the remainder, though some then made no attempt at a quotient. Candidates were not penalised if the quotient and remainder were not explicitly labelled as such, and indeed incorrect labels were ignored.

9) (i) Most sketches were done well, though the asymptotic behaviour of the curve was not always clear – either through the curve not being extended far enough or the curve appearing to drift into the x-axis. Graph paper was not required for this question yet many candidates used it. In some cases this was actually counterproductive as, in plotting a limited number of points only, they failed to demonstrate the general shape of the curve. In addition, a number of candidates failed to extend the curve sufficiently into the second quadrant. Most correctly identified the point of intersection with the y-axis, though some did not make this explicit as requested.

(ii) Most candidates seemed familiar with the trapezium rule and could attempt the question, though errors were common. A surprising number did not evaluate the integral between the requested limits; the common errors were using limits of 0.5 and 2, 1 and 4, and 2 and 4. There were the usual mistakes of using x-coordinates not y-coordinates and using an incorrect value for h. A lack of care when writing out the formula often resulted in brackets being omitted and hence the formula was then used incorrectly. Once again, many candidates failed to work to the required degree of accuracy, with intermediate values for the y-coordinates only being used to 1 or 2 significant figures, resulting in an inaccurate final answer that was penalised.

(iii) This question proved to be challenging for all but the most able candidates. Most set up a correct equation and then introduced logarithms. Dropping the power of x to derive a linear equation was all that was required to gain the first 2 marks, and many candidates managed this. Only a few then realised the need to split log 6 into log 2 + log 3 in order to reach the final result, but some varied and elegant proofs were then seen. Most candidates resorted instead to a decimal 'proof', an approach that gained no further credit. Some candidates started the question by stating that $x = \log_{½} {}^{1}/_{6}$. Whilst this was a correct statement, it gained no credit unless candidates could show how they intended to make further progress with it, as change of base of a logarithm is not on the specification for C2.

4723: Core Mathematics 3

General Comments

This paper proved to be a suitable test. A significant number of candidates recorded full marks, displaying a sound grasp of all the topics assessed. There were a few candidates who struggled to record many marks but, in general, there were several questions which were accessible to the vast majority of candidates. The time allowed was sufficient to enable candidates to complete the paper.

Qs 5, 6, 7 and 9 included aspects which challenged all candidates. But the question which caused most concern was Q4. Candidates were led through the steps to be taken but many candidates struggled. This seemed to be a part of the specification which was unfamiliar to many candidates.

Accurate use of notation and attention to detail are features to be expected in mathematics at this level. Some candidates do indeed show a fine appreciation of mathematical notation and use it with precision and care. However, many other candidates are less assured in these aspects of their work. Often there is no penalty for slight errors of notation; some candidates used an angle of θ throughout their attempt at Q5 whilst, in Q9, the volume of revolution about the y-axis was often indicated as $\int \pi x^2 \, dx$ or as $\int \pi x^2$. However, there were two questions where the use and understanding of correct notation were crucial. In Q4, $\frac{dx}{dy}$ and $\frac{dy}{dx}$ represent quite different things but, for many candidates, they seemed indistinguishable, each merely an invitation to differentiate. In Q9(ii), more candidates would surely have succeeded had they stated $\frac{dp}{dt} = 0.2$ and appreciated that it was the value of $\frac{dV}{dt}$ that they were to find.

Comments on Individual Questions

1) This question enabled many candidates to make a confident and successful start to the paper. The differentiation was carried out accurately in most cases and candidates knew the process for forming the equation, invariably presenting the final answer in an acceptable form involving three non-zero terms. In a number of cases, candidates produced an equation in which the gradient was retained as an expression in x. A few other candidates reached the correct value of $\frac{2}{3}$ for the gradient, and then stopped.

2) The method adopted by the majority of candidates involved squaring both sides of the inequality. There were some slips in simplification but, in most cases, the correct critical values of $\frac{2}{3}$ and 4 were obtained. Similarly, those candidates dealing with a pair of linear equations or inequalities generally reached the values $\frac{2}{3}$ and 4 without difficulty.

   However, attempts then to find the solution of the inequality revealed considerable uncertainty. For some candidates, of course, the step from

$(3x-2)(x-4)<4$ to the correct answer was no problem at all, whether they used a sketch to illustrate or not. In many other cases, though, the conclusion was the incorrect $x < \frac{2}{3}$, $x < 4$. Precision was expected in the form of the final answer so that, for example, an answer of '$x > \frac{2}{3}$, $x < 4$' did not earn full credit. Those candidates who had reached the critical values by way of linear equations or inequalities needed a careful sketch to enable them to conclude correctly; some succeeded but, often, the process seemed haphazard and lacked conviction.

3) This question was answered very well and the majority of candidates earned all six marks. In part (i), a few candidates did not know what to do; there were occasional slips in the substitution on other scripts. A brief comment drawing attention to the change of sign was required in conclusion and most candidates did provide this.

   The iteration process in part (ii) was managed successfully and the correct answer was usually reached. In a significant number of cases, candidates gave the intermediate results to 2 decimal places. Doubtless they were approximating values shown on calculators but better practice would have been to have given this evidence of working correct to 3 or more decimal places. Doubts about candidates' grasp of the topic were raised when a 'suitable starting value' for the iteration was taken to be 0 or 1 or, in several cases, 10. To earn full credit in part (ii), candidates had to show that they appreciated the significance of the result from part (i) by using a starting value between 2 and 3 inclusive.

4) This question presented a considerable challenge and, whilst most candidates could manage at least a mark or two, full marks were obtained only by those candidates aware of the connection between $\frac{dy}{dx}$ and $\frac{dx}{dy}$ and also able to differentiate accurately and present their work carefully.

   In part (i), it was very common for the statement $\ln y = \ln 5^{x-1}$ to be followed by $\ln y = x - 1 \ln 5$; the tendency to omit essential brackets has been mentioned before in reports and this was another unfortunate example. With the expression for x given in the question, solutions were expected to be clear and comprehensive but, all too often, this was not the case. The differentiation in part (ii) presented more problems. In many cases $\ln 5$ was differentiated to give $\frac{1}{5}$ and many candidates unnecessarily used the quotient rule. Other candidates ignored the result from part (i), returned to $y = 5^{x-1}$ and claimed a derivative such as $(x-1)5^{x-2}$. It was clear from responses to part (iii) that many candidates thought that substitution in their expression for $\frac{dx}{dy}$ would provide the required gradient.

5) The vast majority of candidates earned the mark in part (i) for the identity although, in a few cases, candidates had to use the identity for $\sin(A + B)$ as a starting point.

   Part (ii) was not done well. The common approach was to use a calculator to find a value for α and then compare decimal approximations of sin 2α and $\frac{1}{8}\sqrt{15}$. No matter how many decimal places are exhibited, this is not a valid method. Correct approaches involved either the use of the identity $\cos^2\alpha = 1 - \sin^2\alpha$ or of a right-angled triangle. When so many attempts were poor, it was pleasing to note the occasional solution in which the choice of the positive square root of $\frac{15}{16}$ was justified.

Responses to part (iii) were better. Most candidates recognised the need to replace sec β by $\frac{1}{\cos\beta}$ but a few then moved from $\frac{\sin 2\beta}{\cos\beta}$ to $\tan 2\beta$ or $\tan\beta$. Usually the double-angle identity was used and the correct answer obtained. Some candidates seemed reluctant to provide only one answer and offered 72.5° as well.

6) The vast majority of candidates knew the procedure for composition of functions. Those who first evaluated f(–3) were usually successful. Those who produced an expression for ff(x) fared slightly less well, usually because errors occurred in attempts to simplify their expression before they substituted.

A procedure for finding an inverse function was well known but it was rare for all three marks to be earned in part (ii). The common answer was $\sqrt{2-x}$ whereas the given domain of f meant that it was the negative root of $2 - x$ that was required.

The sketch in part (iii) was not done well. Some candidates attempted a graph based on their incorrect inverse function from part (ii) and gained no credit. Others took the expected route of attempting a reflection of the given graph in the line $y = x$ but many of these attempts were not impressive. Curves drifted into the first quadrant or showed a minimum point in the third quadrant; often the intercept on the negative y-axis was labelled as $\sqrt{2}$. It seemed as if many candidates thought that the term 'sketch' gave them licence to produce a scrappy attempt that was no more than a rough approximation to the required curve.

7) The integration in part (a) was generally answered well and many candidates reached the correct, exact answer without difficulty. A natural logarithm appeared in some attempts, and $-8(4x-1)^{-1}$ and $-2(4x-1)^{-1}$ were other wrong integrals sometimes seen. Some candidates used a substitution approach, which worked well, although there were some instances when incorrect limits were applied.

A few candidates were perplexed by part (b), not even realising that integration was involved. Most candidates made some progress though complete success needed an appropriate strategy, correct limits and confidence in using logarithm properties. Some attempted to use the wrong rectangle, using the rectangle two sides of which are PR and RQ. There was uncertainty about the limits for the integration with $\frac{1}{2a}$ and $\frac{1}{a}$ sometimes appearing as x-values. It was common for solutions to include $\ln 2a - \ln a = \ln a$ or, more surprisingly,

$\ln 2a - \ln a = \ln \frac{2a}{a} = \ln a$. Nevertheless, a good number of candidates did proceed correctly to reach $1 - \ln 2$ but a convincing final step to convert this to $\ln(\frac{1}{2}e)$ eluded most.

8) The routine request of part (i) presented no problems to most candidates. The only error to occur with any frequency resulted from confusion between sin α and cos α and led to the incorrect value 22.6˚. Examiners were not impressed by the appearance of cos α = 5 and sin α = 12 in some solutions.

Some candidates ignored part (i) in answering part (ii) and offered a stretch by scale factor 5 with a translation of magnitude $12 \sin x$. Some of those candidates correctly using the form obtained in part (i) were able to give the necessary details concisely, accurately and using appropriate terminology. But too many candidates gave imprecise, unmathematical descriptions; in particular, the translation was often referred to as a shift, a move or even a transformation. The correct term 'translation' or 'translate' was expected.

A few candidates did not realise the significance of part (i) to the solution required in part (iii) but the vast majority made sensible progress and succeeded in finding one correct answer. Many then failed to adopt the correct procedure for finding the second answer; usually such candidates just subtracted their first answer from 360˚.

9) A pleasing number of candidates earned full marks on this question but the accumulation of various skills proved too demanding for many. There were some doomed attempts to integrate the square of $2\ln(x-1)$, whilst expressing x in terms of y presented problems for others; sadly, $x = \frac{y}{2\ln} + 1$ was an attempt seen several times. Many candidates did correctly reach $\int \pi(e^{\frac{1}{2}y} + 1)^2 \, dy$; some then tried to integrate immediately and others went wrong in their attempts to expand. Because the formula for V was given, necessary detail was required in solutions but this was not always present.

Many candidates did not recognise part (ii) as requiring an application of the chain rule. Some merely substituted p = 4 in the expression for V or in their attempt at $\frac{dV}{dp}$. The factor π often vanished from attempts at $\frac{dV}{dp}$. It was also surprising to note that, whereas the integrals of e^y and $e^{\frac{1}{2}y}$ had been correctly executed in part (i), attempts at the differentiation of e^p and $4e^{\frac{1}{2}p}$ sometimes involved errors such as pe^p and $2pe^{\frac{1}{2}p-1}$. Where the significance of the chain rule was recognised, appropriate multiplication by 0.2 usually followed.

Nevertheless, there was some impressive work seen in answering both parts of this final question. Whilst no one step in the solution was particularly demanding, sustained accuracy and necessary detail were needed. It was thus a pleasure to note solutions which showed a sure grasp of correct notation throughout, care in verifying the formula for V and the necessary knowledge concerning connected rates of change in part (ii).

4724: Core Mathematics 4

General Comments

This paper produced a very wide range of responses; a good number of candidates produced a fully correct paper but it was disappointing to see a not insignificant group obtaining fewer than 5 marks. Solutions to questions set in standard ways were generally well produced but any deviation from a standard approach caused widespread difficulty.

There were six occasions when the answer was given; the correct answer nearly always appeared, irrespective of the previous working. Candidates should be aware that, in such cases, every aspect of working is carefully scrutinised and every necessary step of working is expected to be shown. There were two occasions when the lower limit of a definite integral was 0; too many candidates automatically assumed that, when substituted, 0 would always produce 0.

The most common error seen was an inability to cope with directed numbers, particularly when brackets were involved. There were also too many instances of omission of necessary brackets.

Comments on Individual Questions

1) The vast majority were fully aware that this question concerned implicit differentiation. Most obtained $\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right) = 2y\,\frac{\mathrm{d}y}{\mathrm{d}x}$; many fewer were able to deal adequately with $\frac{\mathrm{d}}{\mathrm{d}x}(2xy)$, the ‘12’ was often overlooked and ‘$\frac{\mathrm{d}y}{\mathrm{d}x} =$’ appeared frequently at the beginning of the work. It was a pity that so few substituted (1, 2) as soon as they had differentiated – the subsequent algebra tended to cause fewer mistakes than finding the general expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and then substituting (1, 2).

2) Part (i) was generally done well though the third term was seen in a variety of guises; obviously $27x^2$ was the most common form but the $(-3x)^2$ part was not infrequently shown as $-3x^2$ or $-9x^2$.

   Part (ii) was not well done; no doubt, if the word ‘Hence’ had been included, more candidates would have changed the given expression into $(1+2x)^2(1-3x)^{-2}$. Unfortunately many wrote the expression as $\frac{1+4x+4x^2}{1-6x+9x^2}$ and gave the coefficient of x^2 as $\frac{4}{9}$, showing a lamentable understanding of division. Only a handful of candidates attempted the actual division - but not successfully. Whichever way was attempted, the expansion of $(1+2x)^2$ was required; rather than using the normal squaring procedure, the binomial expansion was often used and it was quite surprising how many errors were made.

3) Almost without exception, the partial fractions were correctly produced. In part (ii), $\int \frac{1}{3-x}\,\mathrm{d}x$ was often stated to be $\ln(3-x)$ though this seemed to cause little problem in showing that the value of the complete definite integral was 0; the expression (ln 2 – ln 1) – (ln 1 – ln 2) looked plausibly like 0 and was usually stated to be so. Relatively few candidates had any idea about part (iii); the most common answers were "There is no area", "The graph lies on the *x*-axis so it's a straight line" (have candidates seen many instances of this in their mathematical lives?) and "There is a turning point". The examiners were relatively lenient here and allowed any indication that the graph crossed the *x*-axis between $x = 1$ and $x = 2$ (or there was a root) or that areas on the upper half and lower half were equal.

4) Most candidates showed competence in using the scalar product method of finding the angle between 2 vectors, and were duly awarded marks – but many found the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and/or $\overrightarrow{OA}$ and $\overrightarrow{OC}$. There were often careless errors in finding $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ($-4--3=-7$ and $-5--3=-8$ for instance).

In part (ii), the '$\frac{1}{2}$' was often omitted from the formula $\frac{1}{2}ab\sin C$ and ab was frequently misinterpreted as the scalar product $\underset{\sim}{a}.\underset{\sim}{b}$ Occasional evidences of the use of the vector product $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$ was seen, but it was expected that some facility in the use of this formula would have been shown.

5) This was the question which caused most problems. In part (i), the rate at which the area was increasing was shown as $\frac{\mathrm{d}A}{\mathrm{d}t}$, $\frac{\mathrm{d}t}{\mathrm{d}A}$ or just A; the proportionality to A^2 was shown as kA^2, cA^2, tA^2 or A^2.

In part (ii), clear separation of variables into $\int k\,\mathrm{d}t = \int \frac{1}{A^2}\,\mathrm{d}A$ (or equivalent) was essential; this was quite often seen but $\int \frac{1}{A^2}\,\mathrm{d}A$ caused difficulty, one of the most common answers being $\ln A^2$. The introduction of an arbitrary integration constant (or use of suitable definite limits) was also needed if much progress was to be possible. Although two boundary conditions had been stated, a few candidates also attempted to introduce the situation at $t=0$; they ran into problems and were unable to produce the correct answer but due allowance was made and they were able to achieve most of the marks.

6) In part (i) it was good to see so few candidates just convert $\mathrm{d}x$ into $\mathrm{d}u$, but quite a few had difficulty in converting $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$ into $\mathrm{d}x = \frac{\mathrm{d}u}{\mathrm{e}^x}$ (or equivalent). There was much algebraic manipulation of the e^{2x} term; $\mathrm{e}^{2x} = 2(u\text{-}1)$ was common and the final correct answer was still forthcoming – but, of course, received no credit.

Part (ii) proved impossible for many; it had been hoped that $\frac{u-1}{u}$ would have been converted into $1-\frac{1}{u}$ and then integrated but integration by parts proved to be an equally popular way. There were many cases of $\int \frac{u-1}{u}\,\mathrm{d}u$ being stated to be $(u-1)\ln u$ or (without brackets) $u-1\ln u$, this latter one being close to the truth. Other answers of $\left(\frac{u^2}{2}-u\right)\ln u$ and $\frac{u.1-(u-1).1}{u^2}$ were seen, the latter showing confusion with the derivative of a quotient. The change to new limits was generally well done, although some then spoiled the effect by re-substituting and returning to an expression in x. Those who worked competently through this second part generally completed satisfactorily though there was occasional manipulation to secure the given answer.

7) In general this question was done well, the vast majority making a sensible effort – spoiled, just occasionally, by careless misreading of the vectors or taking the hard way through solving the three simultaneous equations. But, all in all, this was probably the most successful question on the paper.

8) The given answer was a big help in completing part (i) of the question, and some, no doubt, tried differentiating the right hand side in their heads. Although this was hardly ever seen in actual writing, there was no reason why the part could not have been completed in this manner. If the derivative of the right-hand side had been worked out and shown to be $\cos^2 6x$ then, provided a satisfactory reversal of the statement had been stated, all would have been satisfactory. However, in all cases of given answers, no unsatisfactory statements are allowed anywhere. A few candidates tried a substitution $6x = u$, but a number became confused as to whether this was a calculus substitution or a straightforward algebraic one.

In part (ii), there were two not uncommon misreadings. Some wrote the upper limit as $\frac{1}{2}\pi$ instead of $\frac{1}{12}\pi$ and this was easily catered for. Others decided to work with $\int_0^{\frac{1}{12}\pi} \cos^2 6x\,\mathrm{d}x$ instead of $\int_0^{\frac{1}{12}\pi} x\cos^2 6x\,\mathrm{d}x$; it was surprising that these candidates did not realise that 6 marks for just using limits on part (i) was an unlikely reward. The use of integration by parts on the given integral was usually well accomplished though careless omission of brackets (or attempting to do too much all at once) often caused problems with the sign associated with the integral of sin $12x$. The substitution of both 0 and $\frac{1}{12}\pi$ sometimes caused problems at the cos $12x$ part, cos 0 being quoted as 0 and cos π as either 0 or 1.

9) The idea of parametric differentiation was generally successful in part (i), hardly anybody attempting to convert to the cartesian equation. There were occasional problems with the signs of the derivatives, and even the derivative of $4\cos t$ was seen on more than one occasion to be $-4t \sin t$.

In part (ii), those who used the version $y - y_1 = m(x - x_1)$ were generally more successful than those who used $y = mx + c$ and then substituted in the coordinates of the point; the reason for this was in the clearing of fractions when the '*c*' tended to remain as '*c*'.

Parts (iii) and (iv) were often omitted, possibly through lack of time. Some of those attempting part (iii) took $x = 0$ and $y = 0$ but used them in the original parametric equations rather than in the tangent equation. Use of the tangent equation often produced $S = 4y \sin p - 12$ and $R = 3x \cos p - 12$. Those attempting part (iv) were often successful although $p = 45°$ was fairly common.

4725: Further Pure Mathematics 1

General comments

Few candidates appeared to be under time pressure, with most making an attempt at all questions. The majority of candidates worked sequentially through the paper and correct solutions to all questions were seen.

While a good number of candidates were able to score high marks, there were more candidates this session who made algebraic errors at early stages of their solutions, which result in a considerable proportion of the marks being lost. This particularly applied in Qs 4 and 9 where candidates did not use the given answer to check back when it was obvious that an error had occurred.

In general the presentation of work from candidates was of a high standard.

Comments on Individual Questions

1) This proved to be a straightforward question, with the majority of candidates scoring full marks. The most common error was to find **A – B** correctly, but not to state the value of k.

2) This question proved to be more demanding, as a large number of candidates drew a sketch of a shear in the positive x-direction and then stated the matrix for this shear. A significant number drew a shear in the y-direction and too many candidates drew a sketch with no indication of scale on either axis. A significant number gave the matrix representing S as a 2 x 4 matrix consisting of the image of all of the vertices of the unit square.

3) In (i) most were able to write down the conjugate root. There were two methods that were used frequently in (ii), most usually successfully. Either the sum and product of roots was used directly or a quadratic equation was expanded to find the required values. The most common error in the second approach was to expand $(x + 2 - 3\mathrm{i})(x + 2 + 3\mathrm{i}) = 0$, thus not appreciating how the roots were related to the corresponding equation.

   Some candidates substituted one or both of the roots into the given equation and then solved to find p and q, which meant that considerably more working was required and so the likelihood of an error occurring increased.

4) Most candidates were able to score full marks on this question. However, a small but significant number showed little or no working in going from the unsimplified answer derived from the given results to the given answer and so an “omission of essential working” penalty was frequently applied.

5) A good proportion of the marks for this question were gained by most candidates, with many scoring full marks. However, the following errors were seen quite frequently:
(i) $6 - 4i - (6 + 3i) = -i$,
(ii) $(2 + 3i)^2 = -5 + 6i$,
(iii) $(2 + i)(2 - i) = 3$.

6) As in Q2, too many candidates gave no indication of scale on their sketch, nor a clear indication of which sketch represented C_1 and which C_2. The most frequent error in C_2 was to draw the whole line, rather than the half line. A significant number of candidates tried to find the intersection by first finding the equation of each locus and solving a pair of simultaneous equations, rather than using a trigonometrical approach. Some candidates gave their answer as a coordinate pair, even though the question asked for a complex number in the form $x + iy$.

7) Parts (i) and (ii) were generally answered correctly, but the induction proof proved more demanding. Many candidates did not refer to the result for $n = 1$ (or that $n = 2$ had already been established). Many tried to establish that $A^n + A = A^{n+1}$, while a large number of candidates did not give a clear explanation of the final induction conclusion.

8) This question proved to be quite demanding. Most knew how to find the determinant of a 3×3 matrix, but algebraic errors in finding the cofactors were very common, with some candidates giving the reciprocal of the determinant, confusing the method of finding the inverse matrix with the evaluation of the determinant. The majority knew that singularity meant that $\det\mathbf{M} = 0$ and solved their determinant correctly, but the solution $a = 0$ was frequently omitted. A number of candidates solved a quadratic equation which resulted in complex roots, but did not seem to think that an earlier error had occurred.

Part (iii) proved very testing. Too many solved the equations when $a = 3$, which was not required and often made an error in the process. Those who established that $a = 3$ had unique solutions, usually thought that $a = 2$ must have no solutions and did not investigate the actual situation.

9) Many candidates did not use the method of differences to establish the result in part (i) but tried to use standard results. Most candidates were able to establish the result in part (ii). The most common error in part (iii) was to take $\sum_{r=1}^{n} 1$ to be 1 rather than n. As in Q4, too many candidates did not show sufficient working in establishing the given answer.

10) The value of the product of the roots was frequently given as 4 rather than –4, but most then went on to use their results to establish the values of p and q from

the sum and product of the new roots. Again, sign errors were quite common. Some tried to substitute $x = u - 1$, which is a correct method of solving the question, but then substituted into the new equation rather than the original equation, which meant that the solution was invalid

4726: Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible and were able to pick up marks in all questions. There was some evidence that some candidates had to rush through the final parts of Q9, but the majority completed the paper, answering questions in the order set. The early questions enabled candidates to make a good start, and only Q6 and the last part of Q8 caused widespread problems. As previously, indifferent algebraic manipulation and simplification caused further problems, including that of time allocation. However, there were few very poor scripts, and candidates appeared to be well-prepared for most of the paper. It was nevertheless noted that the spread of marks in many centres tended to be small, even from centres with larger entries.

Many marks were lost in answering question such as Qs 3, 4, 6 and 9, in which answers to be proved were given in the question. Candidates should understand that in such cases the answer should be fully and clearly justified. Candidates should be aware of the information in the *List of Formulae* booklet, and they should gauge the length of their answers to the marks awarded for each question. Nevertheless, many candidates were able to demonstrate their knowledge and to produce excellent and well-constructed scripts with some original solutions.

Comments on Individual Questions

1) Candidates who quoted (possibly directly from the formulae booklet) the Maclaurin expansion for sin$x$ and who multiplied this by $(1 + x)$ quickly gained the marks. Candidates who opted to obtain the series by repeated differentiation and substituting $x = 0$ were often equally successful, but they lost time. With only the first three terms required, it was surprising how many candidates produced numerous other terms. Others left 3! in their answer, but they were not penalised in this case. It was disappointing to see candidates attempting the Maclaurin expansion for $(1 + x)$ and eventually getting $(1 + x)$ or, in some cases, an apparent quadratic equivalent!

2) Candidates generally found this question straightforward. Most candidates used $1+\tan^2 y=\sec^2 y$ successfully and only a minority started part (i) with $\tan^{-1} x=\sin^{-1} x/\cos^{-1} x$. Candidates who needed to use other trigonometric identities again wasted some time.

   In part (ii), a significant minority did not appreciate that "verify" meant that a direct substitution of $\mathrm{d}y/\mathrm{d}x$ and $\mathrm{d}^2y/\mathrm{d}x^2$ was sufficient, and there were some attempts to solve the differential equation. Even amongst those candidates using the correct method, many arrived at the conclusion "0=0", instead of substituting in the left-hand side of the equation an producing 0 from that working.

3) (i) Many candidates could not go beyond "$x^2 + 3 = 0$" in an attempt at the asymptotes, and other produced answers such as $\sqrt{3}$ or even $\sqrt{3}\mathrm{i}$. It appeared that relatively few expected a horizontal asymptote and, as candidates could not divide out, the asymptote was often not found.

   (ii) Candidates were spilt into two main groups. The first re-wrote the equation as a quadratic in x and used "$b^2 - 4ac \geq 0$", mostly without saying why or explaining its relevance to the question. The quadratic inequality was usually factorised correctly (or solutions for $b^2 - 4ac = 0$ were found), but, with the answer being given in the question, the final mark was often lost as the final inequalities for y were not justified. Candidates who drew a quick sketch or used another value of y to check the range required were most successful. Most candidates using this method scored a minimum of 4 marks. Conditions such as "$b^2 - 4ac > 0$" or even "$b^2 - 4ac < 0$" were accepted, provided they were set in the context of the question.

The second group employed differentiation, with many candidates producing the correct coordinates of the turning points. Many of these candidates then believed that this was sufficient, gaining 3 marks. Others who justified which was a maximum and which a minimum gained an extra mark, but very few then justified the given inequality. Again, a quick sketch using, for example, part (i) and the obvious points proved the most successful.

4) (i) This part was generally well done, with most candidates starting with the right-hand side and deriving the left-hand side. Only a few candidates ignored the instructions and "derived" the left-hand side by quoting (without proof) other hyperbolic identities.

(ii) The majority of candidates used part (i) successfully to produce a quadratic in cosh x. As the formulae booklet gives only one equivalent logarithmic answer for $\cosh^{-1} x$, any single correct answer was accepted for full marks, although many candidates produced two solutions, either by knowledge of the symmetry of $\cosh^{-1} x$ or by reverting to the exponential definition. Again, a mark was lost by those candidates who merely ignored or rejected without a reason the second root of the original quadratic equation.

5) There were many better solutions to this type of question than in previous years, although many candidates lost the last 2 marks by not realising that they could use the formulae booklet to quote a $\tan^{-1}$ solution. Others used various substitutions with varying amounts of success at this stage. Some time was lost by candidates who derived the formulae for sin x and dx in terms of t. It is acceptable to use the formulae booklet for sin x and to quote the result for dx.

6) This proved to be the most difficult question, with a modal mark of 2 from the last part. This part was generously marked and enabled the majority of candidates to pick up some marks.

The first two parts were badly done in general. It was disappointing that so few candidates had the confidence to write down the areas of the first few rectangles, taken one at a time, for 1 mark. Even those who did often failed to recognise a G.P. The candidates who were inexact as to the area of the final rectangle (often given as $h3^{nh}$ in part (i)) were not penalised if they correctly used the sum of a G.P. thereafter.

There were some excellent solutions, particularly from those candidates who noted that the difference between the areas of the first set of rectangles and the second was that the first rectangle was omitted (area h) and the last rectangle included (area $3h$) – such candidates then added $2h$ to their answer for the total area in part (i).

7) This was the best done of the longer questions, with the majority of the candidates picking up half marks at least.

(i) Most candidates solved $r = 0$ and found the equation of the required tangent, although some gave more than one answer and others went for a cartesian equation.

(ii) Candidates who "stated" that $r = \sqrt{3}+1, \theta = \frac{1}{4}\pi$ (or even 45°) gained the 2 marks at once, without any working needed.

(iii) Although it was not necessary to show the rays $\theta = -\frac{1}{3}\pi$ and $\theta = \frac{1}{4}\pi$, the better candidates did so and were able to use parts (i) and (ii), with a clear tangent at $\theta = -\frac{1}{3}\pi$ and r increasing throughout to a maximum at $\theta = \frac{1}{4}\pi$. The best solutions came from a small table of values (or at least the two ends and $\theta = 0$, say), with a clearly drawn sketch. However, rough sketches and those full sketches taken from a graphical calculator could still gain full marks.

(iv) Most candidates defined the correct integral and then made a reasonable attempt, although $\tan^2\theta = 1+\sec^2\theta$ and $\tan^2\theta = 1-\sec^2\theta$ were often seen. Even so, most candidates gained at least 3 marks. The value of $\ln\sec\theta$ at $\theta = \frac{1}{4}\pi$ caused some problems at the end.

8) (i) The given equation was often found using the quotient or product rule. As long as the initial steps were clear, marks were not lost by those candidates who thought it "obvious" from an early stage. Many candidates <u>were</u> careful in the derivation of the given equation.

(ii) A substantial minority of candidates gained no marks as they used the original $y = \sinh x / x^2$ in the Newton-Raphson process. Others who wrote the equation as $x = 2\tanh x$ and who used a mixture of the iteration $x_{n+1} = \mathrm{F}(x_n)$ and Newton-Raphson could gain a couple of marks. Those who used the correct f(x) often gained full marks, making good use of their graphic calculator to produce the required approximations.

(iii) Although a similar question appeared on the Specimen Paper, it was disappointing how relatively few candidates could define e_1 and e_2 and use the quadratic convergence of Newton-Raphson. Credit was given for reasonable e_1 and e_2 which were then used correctly to find e_3, and credit was also given to those who had covered errors involving the second differential of f(x). There was some evidence of premature approximation in e_1 and e_2 which lost some candidates the accuracy mark at the end. Candidates who used $|e_1|$ and $|e_2|$ could also gain full marks if accurate and consistent.

9) (i) A mark was lost when, in $\mathrm{e}^y = x \pm \sqrt{x^2+1}$ or $y = \ln\left(x \pm \sqrt{x^2+1}\right)$, the negative sign disappeared without explanation. Most candidates using the exponential definition of sinh y gained 2 marks. Others who quoted $e^y = \sinh y + \cosh y$ were equally successful as the $\cosh^2 y = 1 + \sinh^2 y$ leading to $\cosh y = +\sqrt{1+\sinh^2 y}$ only was often glossed over.

(ii) The majority of candidates could start this part and gain some marks for an early attempt to integrate by parts, although this was not always accurate. The evaluation of $\sinh^{n-1}\theta\cosh\theta$ between the given limits was often just written down as the $\sqrt{2}$ given in the question. The best efforts came from candidates using $\cosh\theta = \sqrt{1+\sinh^2\theta}$ (as in part (i)) or by the longer method of using the exponential definition. A few candidates started with

$$\sinh^{n-2}\theta\sinh^2\theta = \sinh^{n-2}\theta\left(\cosh^2\theta - 1\right) = \left(\sinh^{n-2}\theta\cosh\theta\right)\cosh\theta - I_{n-2}$$

and then successfully used parts on the first part of this. They are to be congratulated!

(iii) Problems arose for candidates who evaluated I_2 instead of I_0. Despite this, many gained at least 3 marks. There was some evidence of some candidates rushing through this part and making careless errors in signs. Full marks could be gained from correct if unsimplified solutions.

4727: Further Pure Mathematics 3

General Comments

Although the majority of candidates showed familiarity with all of the topics tested in this paper, there were three questions which caused some difficulty: these were Qs 4, 5(iii) and 7. Good progress was generally made with all other questions. The presentation of candidates' answers was usually good. The majority of candidates completed the paper in the time allocated, but a small number appeared to find the paper a little long. This was usually because longer methods than necessary had been used in some earlier questions, in particular Q1, and perhaps some time had also been spent attempting Q7. In a few cases it was evident that considerable amounts of rough working had been done; candidates should not spend time making trial attempts at questions which are then written out neatly.

Comments on Individual Questions

1) (a) Although many candidates scored well on this part, it was very common to see that a good deal of time had been spent working out the answers. For both the identity and the inverse it had been expected that answers would simply be written down, as 1 or $1+0\mathrm{i}$ and as $\dfrac{1}{1+2\mathrm{i}}$, followed by a small amount of working to put the inverse in the form requested. Instead, a majority started from the definitions of identity and inverse and solved simultaneous equations to obtain their answers. This took longer and was one of the reasons for a few candidates finding that they had run out of time later on.

(b) This was another request where answers only were required, but here less time was spent unnecessarily in obtaining them. However it was very common to see the identity element given as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and the inverse stated to be non-existent, or to be $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & 0 \end{pmatrix}$. Candidates who gave such answers had overlooked the fact that the group operation was matrix addition.

2) (a) The product $z_1 z_2$ was almost always correct, but for the quotient many answers stopped at $\frac{2}{3}\mathrm{e}^{-\frac{1}{12}\pi\mathrm{i}}$, as the required range of values of θ had not been noticed. For those who did notice the range, most added 2π correctly, but a few added π instead.

(b) De Moivre's theorem was well known and many candidates gained the first 2 marks. As in part (a) the error in not giving θ in the range from 0 to 2π was very common, so that the final mark was not obtained. Some answers did not progress beyond $\frac{1}{32(\cos\frac{5}{8}\pi + \mathrm{i}\sin\frac{5}{8}\pi)}$, an answer of $\frac{1}{32}\left(\cos\frac{5}{8}\pi + \mathrm{i}\sin\frac{11}{8}\pi\right)$ was seen occasionally and sometimes w^{-5} was confused with $w^{\frac{1}{5}}$.

3) As usual with vector questions, a variety of methods was possible, and many candidates obtained full marks. The neatest solution for this standard problem uses a vector product and this was often seen. One of the alternative methods, that of stating the perpendicular vector in terms of a parameter and then using a scalar product, was also quite common. A smaller number of answers used a scalar product to find an angle and then trigonometry for the distance. A fairly common error was seen in answers which used a scalar product presumably instead of a vector product, and stopped at $\sqrt{109}$. This value is the projection, not the perpendicular distance, and to complete the calculation correctly requires the use of Pythagoras; this last step was seldom carried out.

4) Almost all candidates recognised this differential equation as one which required an integrating factor. In some cases the negative sign was omitted, but most answers obtained at least 2 of the first 3 marks, for finding the integrating factor and multiplying through by it. Unfortunately, simplification of the right hand side, which was usually of the form $\int x^2(1+x^3)^k\,\mathrm{d}x$, proved too difficult for most candidates, who often attempted integration by parts. Although a simple substitution may be made, it had been expected that candidates at this level would do the integral at sight, but only the best were able to do this. Most answers did not make any further progress, but a few were able to earn 2 more marks for evaluating the arbitrary constant in their general solution.

5) (i) The majority of candidates obtained full marks for this part without any difficulty. A vector product was usually used and evaluated accurately and then one of the given points was substituted to find the constant in the equation. Occasionally answers using two scalar products equated to 0 were seen, which is quite correct although it takes slightly longer to do.

(ii) This part was often answered correctly, although some gave only $2t\,\mathbf{i} + 3t\,\mathbf{j} + 5t\,\mathbf{k}$.

(iii) Many candidates made no progress with this part, despite the guidance in part (ii). For those who realised that they needed to use a parametric form for the direction of l_2 and equate the scalar product of this and the vector $2\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ to zero for perpendicularity, the working was straightforward. A common error was to assume that the direction of the line l_2 was the same as the direction of the normal to the plane in part (i). Some gained a mark by equating the scalar product of the unknown direction $p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$ and the vector $2\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ to zero, but this did not usually lead on to the solution. Very occasionally other successful methods were seen, which obtained full credit if carried out correctly.

6) It is pleasing to report that many answers to this question scored full marks.

(i) The particular integral was almost always found correctly. Some used the general form $p\sin x + q\cos x$, while others used only the term $p\sin x$, which is correct in this case. However, examiners were very surprised to see many incorrect solutions to the auxiliary equation, which had usually been written down correctly. $m = \pm 2$ was the most common wrong answer, but $m = \pm 4\mathrm{i}$ or $2\mathrm{i}$ only were also seen. If $2\mathrm{i}$ was followed by the correct complementary function, no penalty was applied. The complementary function was sometimes left in the form $A\mathrm{e}^{\mathrm{i}x} + B\mathrm{e}^{-\mathrm{i}x}$, which was penalised once here and also in the final solution in part (ii).

(ii) Most candidates gained at least 2 marks here, for substituting the given values for x and y and for differentiating the general solution and substituting values. Full credit was available only for those answers which had a correct form of the general solution.

7) This question was found demanding, and many answers scored only the first mark of part (i).

(i) Most candidates were able to form the series $C + \mathrm{i}S$ and to use de Moivre's theorem to express it in exponential form, but few recognised the fact that it was now a geometrical progression. For a paper at this level, this apparent difficulty had not been anticipated. For those who did recognise that the formula for the sum of a G.P. was required, many used $n = 5$, rather than $n = 6$. Again, some difficulty was found in extracting factors in the numerator and denominator to obtain the given answer.

(ii) Those who had been unable to complete part (i) often did not attempt this part, despite the fact that the supplied answer meant that full credit could be earned. The marks for C were obtained for expressing the numerator and denominator in terms of sines; this was done either directly or by using $e^{i\phi} = \cos\phi + i\sin\phi$ four times. Examiners were generous in overlooking the factors of 2 and i which might have been cancelled mentally. When the correct form for C had been derived, S was usually correctly stated.

(iii) This part could only be attempted successfully if S had been found in part (ii), and most such answers obtained some of the values of θ. Those which followed $\tan\frac{5}{2}\theta = 1$ or its equivalent were often correct, but it was rare to see any consideration of the possibility that $\sin 3\theta = 0$, which led to two more solutions.

8) This question earned a good number of marks for the majority of candidates. It was found quite accessible, despite the fact that the group table was probably unfamiliar.

(i) Many candidates spotted the one pair of products in the table which demonstrated that D was not commutative, namely $r^4.a$ and $a.r^4$. A few answers assumed commutativity to obtain a contradiction such as $r^3 = e$, which was fine, but those which appealed generally to the table not being symmetrical about the leading diagonal or to $ar \neq ra$ were not accepted without further details being given.

(ii) Most answers were correct, although a few included 10.

(iii) Most candidates wrote down both of the subgroups correctly.

(iv) Many wrote down the order of r^3 as 5 correctly, but in order to determine the orders of ar and ar^2 the majority showed no appreciation that the group D was not commutative, and used simplifications such as $(ar)^2 = a^2r^2$, usually leading to order 10 for both elements. Such answers earned a method mark if it was clear that an attempt was being made to reach the identity element e.

(v) Some candidates showed no working for this part while others did some or a considerable amount of calculation. It had been expected that the results of part (iv) would be used to write two of the entries in the leading diagonal as *e*, and then to use the defining relationships of the group, in particular $r^4a = ar$, and perhaps the Latin square property, to complete the table. In fairness to all, marks were awarded only for entries in the table, which rewarded suitably those answers which contained mistakes. Those who had answered part (iv) correctly were usually successful in this part also. By far the most common incorrect section of the group table was one which had *e* in the diagonal from bottom left to top right and which was identical with the section of the given table showing products of the elements r, r^2, r^3 and r^4. This was produced by ignoring the non-commutativity of the group and writing products such as $(ar)^2 = a^2r^2 = r^2$.

Mechanics

Chief Examiner Report

A high standard of work was produced by candidates sitting mechanics examinations during the summer 2006 season. The use of basic principles rather than standard "recipes" was more evident than in some previous sessions.

Candidates could be more aware of how often marks are lost through errors in mathematical manipulations (rather than mechanics), and that clear annotated diagrams will assist in gaining credit for solutions which are partly correct.

One problem, not unique to mechanics, is the insertion of loose supplementary sheets into answer booklets, when they should be *tied on* at the end of the booklet.

<u>Multiple attempts at questions.</u>

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

'If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.'

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

<u>Graph Paper</u>

At the request of a number of centres, graph paper is no longer being listed under additional materials on question papers. Graph paper should be available for candidates if they request it, but it will only be listed on the front of a paper if it is required. Examiners would like to stress that 'sketch' does not mean a graph drawn on graph paper; sketches should be drawn in the answer booklet, as they are not required to be plotted accurately.

<u>Legacy units</u>

Centres are reminded that **under no circumstances** can 'legacy' units be used in the 'new' specification from January 2007 onwards. The only units that are acceptable for this specification are units 4721 – 4737.

4728: Mechanics 1

General Comments

The general quality of candidates' work was very good, with few poor scripts being seen. The commonest outcome for most questions was the award of full marks. Nevertheless, some candidates would have found their ability better rewarded by:

- thinking more clearly about the forces acting on an object when using Newton's Second Law (Q6, Q7)
- showing how numerical values (given in the question paper) are used, *particularly when a printed answer is given* (Qs 2, 4, and 6)
- being careful not to make errors in GCSE mathematics techniques.

Specific references to these matters are made in the comments on individual questions below.

It was also apparent that in Q3 many candidates felt obliged to plot a graph carefully on graph paper. This wasted valuable time. It was acceptable for axes and graph lines to be drawn in the answer booklet, *with the aid of a ruler*. Significant points on the sketch should be indicated either by numbers at the appropriate places on the axes, or by pairs of co-ordinates adjacent to the points. Though no exact scale is expected, the proportions of the sketched graph should be appropriate.

Comments on Individual Questions

1) Most candidates correctly answered this question. The commonest errors related to signs. Some scripts showed the addition of the "before" momentums (rather than their difference), or different directions for positive velocity before and after the collision. $1200m$ was often seen as the mass of the heavier wagon, and weakness in using/expanding brackets was also noted.

2) (i) This part was well answered, although the use of the sine rule was often seen. Whatever method was chosen it was essential for candidates to show their working clearly when obtaining a given answer.

(ii) The second part of the question was frequently completed correctly. Sometimes Pythagoras' Theorem was used, sometimes the cosine rule, but most often (and successfully) simple resolution was employed.

3) (i) This was well attempted and almost all candidates scored full marks. However, graphs (properly drawn on graph paper) were usually seen instead of sketches. It was pleasing that straight lines were drawn with the aid of a ruler, and sudden changes in speed were indicated by discontinuities in the gradients of the lines. These are expected in a sketch as in a formal graph. Most candidates who made a sketch understood that the first line segment had a longer time interval than the second, and that the first and third line segments (where the speed is the same) would be

approximately parallel.

(ii) This portion of the sketch was almost always added appropriately.

(iii) Candidates who had drawn a graph understandably used it to find the required time, ignoring the instruction "Calculate". The calculations seen frequently drew on knowledge of co-ordinate geometry methods. Other, informal, methods were used (after 80 s the woman is 60 m ahead, but the man closes the gap at 2.25 ms^{-1}) and could receive full marks. "Trial and refinement" was attempted but not usually in a methodical fashion.

4) (i) This part of the question was well answered, the most common error being use of $s = (u+v)t/2$ with $u = 0$.

(ii) Again almost all candidates were successful in integrating the velocity to find the displacement, but many could not obtain the integration constant. Candidates were expected to show how the coefficients of the printed answer were obtained from those in the velocity expression.

(iii) The correct method for finding the acceleration was usually seen. The evaluation of the time often included trivial errors of accuracy in solving the equation $0.6 = 0.06t - 0.3$.

(iv) Though some candidates were unable to evaluate the displacement from the correct value of t, this part of the question was generally completed successfully.

5) (i) Nearly all candidates began this question with a correct evaluation of m, though $R = m$ was sometimes assumed.

(ii) This part of the question was well answered by candidates who understood how the changed forces were inter-related. The common error was to assume the normal component of reaction continued to be 25 N. Many promising solutions ended at the step where candidates were required to solve the simultaneous equations $P\sin\alpha = 5$ and $P\cos\alpha = 6$.

6) Candidates found this an awkward question. However, many entirely correct solutions were seen, their authors handling the varying situations with aplomb.

(i) The expectation was that the explicit summation of the three resisting forces would be shown.

(ii) Most candidates were able to answer this part of the question correctly.

(iii) When considering the motion of the engine, the most common error was assuming a resisting force of 15000 N applied to the engine.

(iv) Though many candidates could find the acceleration of the train correctly, mistakes were then made in using the acceleration to find the mass of *B*. The converse error was to find an incorrect acceleration for the train. In this case credit was given to candidates who adopted the correct method for finding the mass of *B*.

(v) When the question paper gives a printed answer, candidates must show explicitly how it is obtained from given or calculated values. In particular, care must be taken with the definition of a positive direction for vector quantities.

7) (i) Many candidates experienced difficulty understanding the significance of their quantity F. Some used $F = ma$, and immediately assumed that the magnitude of the friction force had been found. Others used this value of F to convert the question into a statics problem. Some candidates assumed that the frictional force F was equal to the component of the weight.

(ii) The essence of this part of the question was sometimes not perceived. A comparison of the magnitude of the frictional force with the component of weight was expected. Those candidates who had found in (i) a frictional force greater than the weight component did not appreciate that this would prevent the particle sliding down the plane.

(iii) The final part of the question was well answered, with candidates able to gain significant credit even when the values calculated earlier were incorrect, *if their method of working was clearly shown*.

4729: Mechanics 2

General Comments

A large number of very able candidates were entered for this paper. There were only a few candidates who were incorrectly entered and showed minimal understanding. Generally, the very able candidates who lost marks often did this through sign errors, premature approximation or simple arithmetic errors in Qs 2, 7 and 8 respectively. As before, examiners advise candidates to be spatially aware, to use clearly labelled diagrams and to take care with basic geometry and trigonometry. The majority of candidates appeared to have sufficient time to answer all the questions.

Comments on individual questions

1) The majority of candidates scored full marks. Some incorrectly added kinetic energy to the work done.

2) A large number of candidates lost the final two marks in this question as they failed to appreciate the change in direction of the sphere.

3) The two most common errors in part (i) were using the incorrect formula for the centre of mass of a solid hemisphere (often choosing a lamina) and poor trigonometry in calculating the horizontal distance. Candidates also encountered problems with geometry and trigonometry when taking moments in part (ii). However, there were many perfect solutions to this question.

4) The majority of candidates scored full marks for the first two parts. Marks were lost in parts (iii) and (iv) when candidates used a total mass of 900 kg and/or failed to understand the meaning of the word "retardation". Some candidates omitted the use of "g" in calculating the weight component down the slope.

5) The most common problems encountered were in locating the position of the centre of mass of the triangular section *BCD* and calculating the area of this triangle. Some candidates treated the problem as a set of five rods. A surprising number of candidates had their (x , y) coordinates the wrong way round but they were not penalised for this. A large number of candidates calculated the value of the - coordinate when they were expected to simply state the value by observing the symmetry. The geometric principle used in answering part (ii) was well understood. A small number of candidates used an incorrect trigonometric ratio or found the angle between *BD* and the horizontal.

6) A small number of candidates confused angular speed and speed. In part (i), the request was to calculate the tension in the string. The answer 4.9 N is expected and not 0.5g. Problems with components were encountered in parts (ii) and (iii)

and the length *AP* was sometimes taken to be 1m hence leading to an incorrect angle.

7) Some candidates took 15m to be the height gained in part (i). Part (ii) was generally well understood although the majority of candidates lost one mark through premature approximation. The height of *B* above the ground was found successfully by a variety of methods including the use of the equation of the trajectory. In part (iii) some candidates thought that the vertical component of velocity was the speed of the ball before hitting the net.

8) There was a high incidence of random errors in all three parts. For example in part (i) $10 = 4 + mx$ leading to $mx = 2.5$ and in part (iii) correct simultaneous equations were frequently poorly solved ($-3x = 3$ led to $x = -3$ in several cases). In part (i) examiners would have appreciated greater clarity in realising *B'* s minimum speed of 2 ms^{-1}. Diagrams with directions were scarce in part (iii) and some candidates used incorrect formulae for the calculation of kinetic energy, for example the lack of "½" or failing to square the velocity.

4730: Mechanics 3

General Comments

Candidates were very well prepared for examination and most demonstrated a good working knowledge of the topics examined. Qs 5 and 6, which collectively carry one third of the available marks, were particularly well attempted. Many candidates scored full marks in each case.

Comments on Individual Questions

1) This question proved surprisingly difficult for one that was intended as a straightforward opener. Many candidates showed (in a diagram) the direction of the impulse to be at an acute angle to the direction of the original motion (usually denoted by θ). Clearly the ball could not be deflected through 90° if this was a correct representation of the situation. Almost all such candidates found θ to be acute and equal to 60°, instead of 120°.

   A significant proportion of candidates failed to treat the equation connecting impulse and change of momentum as a vector equation, giving $20 = 0.4(v - 25)$.

   Candidates who considered the change of momentum in directions parallel to and perpendicular to the impulse often wrote $20 = 0.4(v\cos\alpha - 25\sin\alpha)$ instead of $20 = 0.4(v\cos\alpha + 25\sin\alpha)$.

2) This question was very well attempted. Almost all candidates used $a = v\frac{dv}{dx}$; those who didn't scored very few marks for the question. The most common error seen was missing a minus sign in applying Newton's second law.

3) This was generally well attempted although a significant minority of candidates found difficulties and scored very few marks. Errors included the omission of the term representing the tension, from equations obtained by taking moments for *AB* or *AC*. Some candidates failed to recognise the absence of friction at *B* and at *C*, and consequently could not find either the tension or the horizontal component of the force on *AB* at *A*. Considerable confusion arose from clumsy methods involving the unnecessary calculation of the angle that each rod makes with the ground, and the length of each rod.

4) Part (i) was well answered but in a significant minority of cases the significant minus sign was absent. In some cases $\ddot{x} \approx -g\theta$ was obtained but $\ddot{x} = 2.45\ddot{\theta}$ was not subsequently used. The equation $2.45\ddot{\theta} = -g\sin(0.01)$ was also frequently seen, and said to be evidence for SHM.

   Parts (ii) and (iii) were poorly attempted, common errors being the use of 0.04

radians instead of 0.06, and the use of $n = \pi$ instead of $n = 2$ in the formulae $\dot{\theta}^2 = n^2(\vartheta_{max}{}^2 - \theta^2)$ and $\theta = A\cos nt$. The formula for $\dot{\theta}^2$ was frequently quoted with v^2 as its subject and in such cases there were three categories – one in which the units of the resultant v were given as ms^{-1}, one in which units were absent and one in which units were shown as rad s^{-1}. Only in the last of these cases was v assumed to represent the candidate's answer for the angular speed. The assumption that in the first two cases the candidate intended v to represent linear speed, was often confirmed when the candidate divided his value of v, usually 0.16 or 0.183, by 2.45.

5) Almost all of the candidates adopted a correct procedure and full marks for the question was common. However there were errors, mainly of sign and of the omission of the masses from the 'after' impact terms of the momentum equation.

6) The topic of the question was clearly well understood by candidates and completely correct solutions were common.

7) In part (i) candidates usually employed the principle of conservation of energy, but many mistakes were made. Candidates found difficulty with the loss of potential energy of *Q*, and many candidates had the gain in kinetic energy of *P* and of *Q* on opposite sides of the equation.

   In part (ii) finding the value of θ corresponding to the maximum contact force was often omitted, or given without any calculation as $\pi / 2$.

   Many candidates found part (iii) very difficult and scored no marks. Some candidates applied Newton's second law tangentially to *P*, but then substituted $T = 0.4$g.

4731: Mechanics 4

General Comments

There was a wide range of performance on this paper, with 10% of candidates scoring more than 60 marks (out of 72), and about 20% scoring less than 30 marks. The only topic which caused widespread difficulty was relative velocity (Q6).

Comments on Individual Questions

1) (Centre of mass for a rod of variable density)
   The method was very well understood, and about 70% of the candidates scored full marks on this question.

2) (Constant angular acceleration, and angular momentum)
   This question was well answered, with about 70% of the candidates scoring full marks.
   In part (i) most candidates found the angular acceleration first, and this almost always led to the correct answer. Several considered the work done by the couple, but some of these candidates equated it to the increase in angular momentum instead of kinetic energy.
   In part (ii) the conservation of angular momentum was almost always applied correctly.

3) (Moment of inertia of a lamina)
   There was a lot of good work on this question, and about 40% of the candidates scored full marks. Minor errors included finding the mass per unit area as $\text{mass} \times \text{area}$ instead of $\text{mass} \div \text{area}$, and failing to integrate $1/x^6$ correctly (typically obtaining $-5/x^5$ or a multiple of $1/x^7$). More serious errors usually led to the integration of a multiple of $1/x^4$; about 10% of the candidates scored no marks on this question.

4) (Energy approach to equilibrium)
   The methods were generally understood, and about 30% of the candidates scored full marks. Common errors included manipulative slips in algebra, trigonometry and calculus, and taking the extension of the string to be $RC - a$ instead of just RC.

5) (Rotation, and energy principle)
   About a quarter of the candidates scored full marks on this question.
   Parts (i) and (ii) were usually answered correctly, but in part (iii) very many candidates ignored the work done against the frictional couple when forming the energy equation.

6) (Relative velocity)
This was by far the worst answered question, with an average mark of about 5 out of 13. Many candidates had no idea what to do, and about a quarter of the candidates scored no marks at all.
Parts (ii) and (iii) were quite often answered correctly. In part (i), finding the other possible bearing caused a lot of difficulty, and correct answers to parts (iv) and (v) were very rare.

7) (Rotation, and force acting at the axis)
The average mark on this question was about 9 out of 17.
Part (i) was usually answered correctly.
In part (ii) almost all candidates used the conservation of energy. Because the answer is given, candidates need to do more than obtain the right answer; very many lost marks for not indicating clearly how they had used the initial position $\theta = \frac{1}{3}\pi$. Beginning by writing $\frac{1}{2}I\omega^2 = mg(a - 2a\cos\theta)$, without any explanation, was not sufficient.
In part (iii) the method for finding *R* and *S* was reasonably well known, but sign errors were extremely common.
There were some excellent solutions to part (iv), by a variety of methods, but most candidates did not get beyond substituting the given value of θ into their expressions for *R* and *S*.

Probability and Statistics

Chief Examiner Report

There continues to be a pleasingly large quantity of excellent work seen on the Statistics units. However, this year all examiners have noted an increase in the number of solutions obtained from a calculator without showing sufficient detail of the working. In general, correct solutions will gain full marks even if no working is shown, but if the answer is only slightly incorrect it is very hard to award method marks unless Examiners can see clearly where the mistake has arisen. Omission of such essential working could result in the loss of as much as 5 or 6 marks.

As mentioned in last year's Chief Examiner's Report, Centres are particularly asked to note the following points which have been agreed by the Examiners responsible for these papers in order to encourage good practice.

- Answers given to an excessive number of significant figures (such as "probability = 0.11853315"), which have not in the past lost marks, may in future be penalised.
- Hypothesis tests are likely no longer to include the explicit instruction "stating your hypotheses clearly"; any answer to a question involving hypothesis tests should include a statement of hypotheses unless they are already given in the question.
- Likewise, questions that involve critical regions (particularly for significance tests using discrete distributions) may not ask explicitly for the relevant probabilities to be quoted from tables, but candidates should always write down the values of these probabilities.
- Conclusions to hypothesis tests should be stated in terms that acknowledge the uncertainty involved. Thus "the mean height is 1.8" is too assertive and may not gain full credit; a statement such as "there is insufficient evidence that the mean height is not 1.8" is much to be preferred.

<u>Multiple attempts at questions.</u>

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

'If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.'

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

<u>Graph Paper</u>

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Legacy units

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4732: Probability and Statistics 1

General comments

Most candidates showed a good understanding of much of the mathematics in this paper. There were some very good scripts. Almost all candidates scored some marks on each question and there was a wide range of total marks.

This year, very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in some cases, marks were lost through premature rounding of intermediate answers. It is worth noting that in almost all cases an exact answer, expressed as a fraction, is the best form of answer. Many candidates unnecessarily converted their fractional answers to decimals. Some candidates were not fluent in using either the fraction or power keys on their calculator; $(1^1/_4)^2$ caused difficulty.

In questions requiring written answers, candidates commonly failed to gain the marks because they gave only responses in technical language, without interpretation and/or reference to the relevant context.

This paper did not require much algebraic understanding, but in those questions which did so (particularly Qs 1(iii) and 5(ii)), responses frequently showed poor algebraic technique. Arithmetic was often poor in Q 7.

A few candidates appeared to run out of time, although the majority finished comfortably.

Many candidates failed to fill in the question numbers on the front page of their answer booklet.

Very few candidates scored full marks. This was due mainly to the existence of a few marks for which marking criteria were particularly tight (Qs 6(i)(b), 7(i) and 8(i)).

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

The formula booklet MF1 was useful in Qs 5(ii), 6(i) and 8(iii). However, many candidates appeared to be unaware of the existence of MF1. Some candidates tried to use the given formulae, but clearly did not understand how to use them properly (e.g. $\sum x^2p$ was misinterpreted as $(\sum x^2)p$ or $\sum xp)^2$). A few candidates used the less convenient version from the formula booklet, $\sum\ (x - \sum\ )^2p$. The volume of arithmetic involved in this version led to errors in most cases. When finding the variance, many candidates found $\sum x^2p$ correctly but went on to divide by the number of values of x.

Candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1. They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

The formulae for the mean and standard deviation of a frequency distribution are not given in MF1. Many candidates quoted them incorrectly, sometimes omitting the "f" or, more seriously, attempting $\Sigma x^2 f / \Sigma xf$ or $\Sigma x/n$ (where n is the number of classes). Others quoted them correctly but misunderstood them, calculating, for example, $\Sigma (xf)^2$ or $\Sigma x \times \Sigma f$ and $\Sigma x^2 \times \Sigma f$. A few tried $\Sigma (x - \bar{x})f / \Sigma f$, and got lost in the arithmetic. When finding the standard deviation, many candidates divided by the number of classes, either instead of, or in addition to, dividing by the total frequency.

Some candidates' use of the binomial tables showed misunderstanding.

Comments on Individual Questions

1) (i) Most candidates saw the point, and some gave excellent answers referring either to gradients or regression coefficients. However, many failed to explain correctly. Statements such as "The x-value and the y-value are negative" were common. Some candidates thought that the two negatives resulted in the correlation being positive. Some gave correct but lengthy explanations such as "negative, because as x increases, y decreases". A few did not recognise any connection between the equations of the regression lines and the type of correlation.

(ii) Most candidates answered correctly. A few chose the wrong regression line. Others used both equations but did not choose one answer above the other.

(iii) The response to this part was disappointing. Large numbers of candidates appeared not to appreciate that the mean lies on both regression lines. Others attempted to solve the two equations simultaneously, but could not handle the decimals without making errors. Some introduced new letters "a" and "b" representing the two constants in the general equation of a regression line. These candidates almost all failed to make progress.

2) (i) The majority of candidates misread this question as "the second disc is black AND the first disc was black". Of those who understood the question correctly, most used the formula $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$. Some of these obtained the correct answer, gaining the mark. Only a few (perhaps prompted by the tariff of only one mark) realised that the answer could be written down immediately. Candidates need to be aware that the phrase "given that" may well indicate a conditional probability that can be written down immediately. The Specification says "calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram". Formulae such as the above are therefore not required. Further guidance on the interpretation of this

part of the Specification can be obtained by consulting past papers.

(ii) Many candidates answered this part correctly. Some failed to change one denominator from 8 to 7. Others confused red with black. Many gave only one route, either BB or RB but not both.

(iii) Many candidates answered this part correctly.

3) (i) This part was well answered. Common errors were 7!, $^7P_5$ and $^7P_3$.

(ii) Many candidates realised that there are five objects to be arranged and therefore that 5! is involved. However, many just gave 5! or multiplied it by 3! or 4!, perhaps dividing by 2! also. A few used 7!.

(iii) Candidates who used direct probabilities tended to be more successful than those who used combinations. However, very few used the most efficient method ($1 - {}^4/_7 \times {}^3/_6$). A few attempted to list all the possibilities in the form
(D, I), (D, E), (D, V) etc instead of the simpler (D, not D) etc. Those who used combinations often gave one or two incorrect combinations (e.g. $^3C_1 \times {}^4C_2 / {}^7C_2$ or $^4C_1 / {}^7C_2$) or used only one route, eg (D, not D). A few candidates attempted a binomial method.

4) (i) Misunderstanding of the tables was common, with candidates looking up $P(X \le 5)$ instead of $P(X \le 4)$. Some omitted to subtract from 1. A few treated the probabilities in the tables as individual rather than cumulative.

(ii) This was generally well answered. Some candidates omitted the combination. Others found $P(Y \le 3)$.

(iii) This type of question appeared to be new to most candidates. Instead of $P(Z \ge 1)$, many attempted to answer the question for $P(Z = 1)$, which is more difficult. Some understood the question, but proceeded to go through the full rigmarole of writing $1 - {}^nC_0 \times 0.27^0 \times 0.73^n > 0.95$. The apparently complex nature of this inequality prevented some from proceeding further. A few candidates went straight to $0.73^n < 0.05$. Many of these candidates succeeded, using either trial or improvement or logarithms. Those who used logs sometimes failed to change the direction of the inequality when dividing by log 0.73. Candidates who used "=" rather than "<" avoided this problem. They were then helped by some good luck. The value of n (9.52 to 3 sig figs) happens to round *up* to 10 (which was the correct answer), rather than rounding down to 9.

5) (i)(a) Most candidates wrote down a correct equation, involving p and q, using the given fact that $E(X) = 1{}^1/_4$. Some then guessed the values of p and q. Only some of the candidates wrote down the other equation using the

fact that the total probability equals 1. As in question 1(iii), the solution of the simultaneous equations revealed disappointingly poor algebraic skills. A few candidates assumed that $p + q = 1$.

(ii) The errors mentioned in the introduction, above, were fairly common. However, most candidates knew how to begin the calculation of standard deviation. Sadly, many candidates who correctly evaluated $\Sigma x^2 p$, then proceeded to divide by 4. A few calculated the mean, despite its being given in the question. As long as correct use of the formula $\Sigma x^2 p$ was made, candidates were able to score the final mark for taking a square root, even though they may have made arithmetical errors. Some candidates attempted to use some form of a formula for the standard deviation of a frequency distribution. *np* and *npq* were also sometimes seen.

6) (i)(a) Almost all candidates succeeded in finding r_s correctly. A few made arithmetical slips. Some omitted the "6" or the "1 –" from the formula. Others ranked the two data sets in opposite orders. A few found the differences for the original data rather than for ranks.

(i)(b) Comments were frequently muddled. Many candidates appeared not to understand the significance of a near-zero value as opposed to a value near to –1. Many candidates referred to the lack of "correlation" without interpreting this word in the context. A comment such as "There is little relationship between Distance travelled and Commission" was required to gain the mark.

(i)(c) Almost all candidates answered this part well. A few made the change in Commission rather than in Distance travelled.

(ii) In both parts (a) and (b), many candidates referred to correlation rather than to the value of the coefficient. Many others suggested that r_s was negative, but that r was positive!

(ii)(a) Of those who did refer to the value of r_s, most candidates stated that r_s was close to –1, rather than equal to –1.

(ii)(b) Many candidates answered this part correctly. Some stated that the correlation was strong and therefore r would be close to 1.

7) (i) This part was poorly answered. Some incorrect midpoints were seen, particularly 150 for the last class, which was not surprising, but also 2 for the first class. Candidates who made small errors in the midpoints and/or small arithmetical errors in calculation could score 5 out of 6 marks. However, it was common for candidates to lose some or all of the method marks also. Many candidates used class width or 0.5 x class width or upper class boundary for the values of x. Other common errors were $\Sigma x / \Sigma f$, $\Sigma x^2 f / \Sigma xf$, $\Sigma xf^2 / \Sigma f$, $\Sigma xf)^2 / \Sigma f \Sigma x / 5$ and $\Sigma xf / 240$.

(ii) It was clear that most candidates had not recently (if ever) used interpolation in a grouped frequency table. In view of this unfamiliarity, marking was generous. Some candidates drew a cumulative frequency graph. Many of these found the correct answer, although some plotted at midpoints or labelled the x-axis "1-4", "5-14" etc. Candidates who interpolated had varying success. A variety of methods were accepted, including those which ignored the complications due to the data being discrete. A wide range of answers was also accepted and even candidates who just guessed the locations of the two quartiles in the two relevant classes were able to score full marks. Those who used the midpoints of the two relevant classed could score 2 out of 4 marks. One mark could be gained merely by identifying the two classes within which the upper and lower quartiles lay.

(iii) Many candidates understood these parts and gave correct answers. No explanation was required, so it is impossible to know the reasoning behind the incorrect answers given by many candidates. However, some candidates did give their explanations, and some misunderstandings of the properties of these three statistics were evident. Some candidates thought that either or both of mean and standard deviation would decrease because there would be division by a larger number. Others suggested that the increase in the range would cause an increase in the interquartile range. A few stated that the standard deviation would not change because it does not take into account extreme values.

8) This question was well answered on the whole, with many weaker candidates scoring more than half the available marks.

(i) Almost all candidates gave the answer "Geometric". Few gave the other modelling assumption. Most quoted some property of the geometric distribution such as that attempts are repeated until a success occurs. Those who did understand what was necessary often failed to score the mark because they wrote about independent "events" or "trials" rather than giving an answer in context.

(ii)(a) This part was answered well, although some candidates gave $(^2/_3)^4$ x $^1/_3$. Others subtracted from 1. A few tried to find a binomial probability. A small minority of candidates found q to be $^3/_4$.

(ii)(b) A minority of candidates used the most efficient method of $1 - (^2/_3)^3$. Some omitted the "1-". Others gave $1 - (^2/_3)^4$. Others tried the efficient method but, realising that they were dealing with a geometric distribution, multiplied $(^2/_3)^3$ by $^1/_3$ before subtracting from 1. Many candidates used the long method – adding several terms. Some of these candidates added an extra term, either$(^2/_3)^3 \times {^1/_3}$ or $(^2/_3)^{-1} \times {^1/_3}$ (!). Some confused the two methods, adding the correct three terms but then subtracting from 1. A few multiplied three terms.

(iii) Most candidates gave the correct answer. Some wrote $1 / {^1/_3} = {^1/_3}$. A few tried some form of np. Others gave the formula for the standard deviation of the geometric distribution (which is not required by the Specification). Candidates should be aware that the relevant formula is given in MF1.

(iv) This part was found to be difficult and not many candidates answered it correctly. Many had the correct form of the calculation (q^2p), but with $p = {}^1/_3$ or $p = {}^8/_{81}$ (from (ii)(a)). Others recognised the correct p (${}^{19}/_{27}$, from (ii)(b)), but found p^3 or p^2q or q^3 or $3p$. Some attempted a binomial calculation with various values of p.

4733: Probability and Statistics 2

General Comments

This was a hard paper, although many excellent scripts were seen. Although many candidates showed ample details of their working, there seemed to be a substantial increase in the number of candidates who answered questions, particularly on the normal distribution, with insufficient or, indeed, no working. This is a very high-risk strategy, especially as examiners need to see details such as whether the appropriate continuity correction has been used (if necessary) and whether the divisor is the standard deviation or the variance, in order to award method marks.
It cannot be too strongly emphasised that full working must always be shown.
In any case, whenever statistical tables are used, *all* the relevant probabilities need to be quoted – for instance, in Qs 2.

Centres are reminded that, in future, questions may not carry instructions to "show all relevant probabilities" or "state your hypotheses clearly"; these things should be done without specific reminders.

Weaker candidates seem to be unable to handle any distribution other than the normal distribution and they convert everything to normal, regardless of its validity. Use of an invalid distribution may obtain no marks at all on a particular section, and in this paper there was a premium on selection of the correct distribution and approximation.

Comments on individual questions

1) For many this was a straightforward start to the paper, although even some good candidates forgot altogether about the mean. Standards of integration and numerical work were high. The subtraction $E(X^2) - [E(X)]^2$ is "ill-matched", the two quantities being very similar, and a wide spread of answers was acceptable; nevertheless it was very pleasing to see so many answers that were correct to 3 or 4 significant figures.

2) As usual the question on hypothesis testing using a discrete distribution was the least well done. The first part of this question is relatively straightforward, but was poorly answered. Many tried to use a normal approximation, which is unacceptable as *np* is too small. Others did not realise that the right-hand tail was needed (they did not seem to realise that calculating $P(R \leq 6)$ is absurd); still others thought that $P(R \geq 6) = 0$. However, a pleasing number of candidates did make a clear comparison with 0.025.

   Part (ii) is relatively unusual and was found quite hard. However, even allowing for the unfamiliarity, it was disappointing that so many candidates used $P(R = 1)$ rather than $P(R \leq 1)$. In order to score full marks here it was necessary to show $P(R \leq 1)$ for two values of n, one giving an answer less than 0.025 and one giving an answer greater than 0.025.

3) A completely standard question on finding the parameters of a normal distribution was perhaps less well done than usual. Too many omitted the negative sign needed in the equation for $P(X < 140) = 0.01$. Many made surprisingly heavy weather of solving the two simultaneous equations; substitution is usually inefficient. This is such a standard question that it is surprising that more candidates are not used to writing down two equations in the form $140 - \mu = -2.326\sigma$, $300 - \mu = 0.842\sigma$, and subtracting.

   In part (ii) many tried to justify their answer by doing calculations based on a normal model. Although they should probably have realised that the information invalidates such a model, they could still obtain full credit. Some unexpectedly confused slow times with a lower mean.

4) Most good candidates knew what was meant by a random sample; essentially, its properties have to conform to the modelling assumptions for a binomial distribution. Some candidates were confused by the reference to a population numbered with digits starting at 000, although this is the usual way in which random numbers are used; they could nevertheless score most of the subsequent marks even when using values of p such as 499/799.

   Most correctly used a binomial distribution in part (ii), but in part (iii) the number of mistakes with continuity corrections seemed even higher than usual.

5) This was found to be the hardest question. Some candidates did not realise that they had to deal with 303 customers (302 or 301 in part (iii)), and many were confused as to whether they were calculating probabilities for the number of no-show passengers (right) or the number of "show" passengers (wrong). Too many used a normal approximation instead of a Poisson; and in part (iii) candidates found it hard to see that they had to use a distribution with n less than 303. Even in part (ii), where the answer was given, there were many who found $P(R = 1\ 2, 3)$ instead of $P(R = 0, 1, 2)$. Nevertheless, roughly one candidate in six scored full marks on this question, and of course if you could see how to do it the calculations were simple enough.

6) Most could score one mark for stating that customers had to arrive independently. As usual, an answer referring only to unspecified "events" did not score the mark.

   Most adequately prepared candidates got part (ii) right, although some found $1 - P(< 2)$ rather than $1 - P(\le 2)$, and some looked up the tables for $\lambda = 0.04$. The normal approximation to Poisson was generally well done apart from the errors with the continuity correction, again perhaps even more frequent this year. However, this was a question on which some candidates showed far too little working.

   Part (iv) has appeared in similar form on several recent examinations. Some coped with it well, although few who used trial and improvement obtained a sufficiently accurate result, and as in previous years many found the hardest part to be converting the parameter of 3.91 to a time in seconds, or minutes and seconds.

7) This question was on the whole well done and for many it was a substantial source of marks. In part (i) the value 4014 was often seen, although sometimes the inequality for the critical *region* was wrong or omitted. It was particularly pleasing that many candidates were able to do something like the right calculation in part (ii). The comments at the end served their purpose of differentiating at the level of grade A; many argued parts (iii) and (iv) well. However, the Central Limit Theorem is poorly understood in many quarters, and some even said that its use was necessary because it was *not* stated in the question that the distribution was normal. The extent of the prevailing confusions over not just this theorem but basic logic is demonstrated by the large number of answers that read something like "It is necessary to use the Central Limit Theorem because n is greater than 30".

4734: Probability and Statistics 3

General Comments

Many found little difficulty in the majority of questions and showed a good understanding of the statistical concepts. Computational facility was good, and there was evidence that many used a graphical calculator efficiently. As mentioned elsewhere, it should be remembered that there should be sufficient detail of exactly how and where the calculator is being used.

The paper emphasised statistical tests but only a few candidates did not show familiarity with them. The examiners were pleased with the manner in which conclusions were given contextually.

Presentation on the whole was satisfactory but there seemed to be more scripts where candidates made inappropriate remarks. This practice should be discouraged.

Comments on Individual Questions

1) This was very well answered by a large majority of candidates. The most common error was to use Po(8.5), indicating that the theory was known but the question was not read carefully enough.

2) The relevant χ^2 test was usually carried out accurately but some tried to show that two proportions, and not always the largest and smallest, differed significantly. On many scripts, the null hypothesis was not stated or was stated incorrectly.

3) Although the principles were usually known only the better candidates could score high marks. Only a minority of candidates gave the correct assumption, many believing that each population needed to be normal – a sufficient but not a necessary condition.

   The hypotheses were often expressed in *x*s or *d*s or in words such as "times not improved". There were problems over the standard error, some using $\sqrt{\{0.548(^1/_{10} + {}^1/_{10})\}}$ leading to $\upsilon = 18$. Conclusions, however, were usually consistent.

4) Various solutions were offered. Some involved poor integration but did have a correct initial statement. In part (i) many tried to find F(x) but often were wrong with the upper range. Very few worked down from the upper end.

   Part (ii) was more often correct than part (i) but several thought E(X^2) involved $x(\mathrm{f}(x))^2$.

5) Parts (i) and (ii) were almost always correct and (iii) was well done although

some wasted time in calculating the value of the test statistics which was given in the question. Part (iv) usually elicited the choice of Friday but the reason often referred to the high expected value.

6) Part (i) was well answered, most being aware of the relevance of the confidence intervals.

For part (ii), a several methods were seen. Many used $p_1q_1/80 + p_2q_2/85$ for the estimating the variance and others used the value $p = 0.43$. In some cases candidates went back to the given confidence intervals to obtain this variance.

7) (i) Where candidates used the distribution of $T_1 - ½T_2$ (or equivalent) the variance gave the most difficulty, but careful candidates found this to be a straightforward question.

(ii) A common misinterpretation led to the use of $T_1 + 5T_2$ and many thought the variance of $T_1 + (T_2 + T_2 + T_2 + T_2)$ was $0.75^2 + 16(0.7^2)$. Standard deviation and variance was often confused in both parts. However, more were successful in this part than in part (i).

(iii) Only a minority could see where independence was required in part (i). Many thought it was required for calculating Var($½T_2$).

8) (i) Most calculated $s_B{}^2$ correctly, but some used a biased estimate. There were also instances of $\sigma_A{}^2$ being "corrected" by multiplying by 40/39. A pooled estimate of variance was often seen.

(ii) Many knew to subtract 0.025 from $\bar{x}_B - \bar{x}_A$ but some subtracted from $\bar{x}_A - \bar{x}_B$. Only the best were able to obtain an inequality for α.

(iii) Part (a) was mostly known but misunderstanding of the Central Limit Theorem led to unacceptable responses to (b).

4735: Probability and Statistics 4

General Comments

In the significantly smaller entry almost all were able to attain at least an e grade. There were some very good scripts but most lost some marks, often through lack of care. All candidates appeared familiar with most of the principles involved and were able to apply them confidently.

Comments on Individual Questions

1) This proved to be an easy start and almost all were successful in part (i). The formula for Var($aX + bY$) appears in the formula booklet but was not always applied carefully.

2) This straightforward question was usually answered well. There were some unacceptable hypotheses (which should relate to population medians), and conclusions where "the first boy is taller than the second" was often seen despite it clearly being incorrect.

3) This was generally found to be easy, with many scoring full marks. It was pleasing to see the notation being used correctly but there were some who did not realise than P($F \mid C'$) was required for part (iii).

4) This was well answered in all parts. In part (iii), however, where candidates were asked to show that the two moment generating functions were the same, it was necessary that sufficient detail be given. This was not always forthcoming. Almost all knew the consequences of the equality.

5) Many could answer the first two parts, if without all 7 rankings in part (ii).

   Very few then used the probability found to answer part (iii). Most resorted to the table of critical values which could yield only a significance level of less than or equal to 2% and not less than 2%, as required.

6) This was another high-scoring question with several recognising the given probability generating function. Only the final part caused some problems although most recognised the relevance of the pgf.

7) This proved to be a discriminating question, although there were some excellent solutions to all parts. Not all were able to give E(X), several using integration. Others quoted from the formula book, unfortunately the continuous distribution, which can give the correct value, but the incorrect variance should have alerted

the candidate.

There were many good solutions to part (ii) but the probability in part (iii) was found difficult.

The principles for part (iii) were usually known and there were several good attempts at the final part.

Decision Mathematics

Chief Examiner Report

<u>Multiple attempts at questions.</u>

In recent sessions examiners have noted an increasing number of candidates making two, or more, attempts at a question, and leaving the examiner to choose which attempt to mark. Examiners have been given this instruction.

‘If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt, and ignore the others.’

Please inform candidates that it is in their best interest to make sure that, when they have a second attempt at a question, they make it clear to the examiner which attempt is to be marked. The obvious way for candidates to do this is to make sure they cross out any attempts which they do not regard as their best attempt at the question.

<u>Graph Paper</u>

At the request of a number of centres, graph paper is no longer being listed under additional materials on question papers. Graph paper should be available for candidates if they request it, but it will only be listed on the front of a paper if it is required. Examiners would like to stress that ‘sketch’ does not mean a graph drawn on graph paper; sketches should be drawn in the answer booklet, as they are not required to be plotted accurately.

<u>Legacy units</u>

Centres are reminded that **under no circumstances** can ‘legacy’ units be used in the ‘new’ specification from January 2007 onwards. The only units that are acceptable for this specification are units 4721 – 4737.

4736: Decision Mathematics 1

General Comments

Most candidates were able to attempt every question and gain marks on all the questions they tried. Candidates were usually able to deal with standard situations and straightforward applications of the algorithms, but they were less certain about the underlying concepts and principles.

The presentation of candidates' solutions generally showed an improvement on previous sessions with more candidates setting out their work clearly and fewer instances of poor handwriting or overwriting of numerical answers.

Comments on Individual Questions

1) Most candidates were able to have a sensible attempt at this question, although several did not apply the first-fit algorithm correctly in part (i) through failing to go back and fill in spaces in the first box once they had started on the second. A few candidates sorted the list into increasing order instead of decreasing order in part (ii) and some ignored the statement that it was not necessary to use an algorithm to sort the list into decreasing order.

   There were several correct solutions to part (iii) but also some candidates who used linear, cubic or quartic order and several who just squared the time with no reference to the change in the size of the problem. If an algorithm is of quadratic order then for large problems the run time is approximately proportional to the square of the size of the problem, scaling the problem size by a factor of 2 will scale the run time by a factor of 2^2 approximately, so the run time for the larger problem is approximately $2^2 \times 15$ seconds = 60 seconds.

2) In part (i) most candidates were able to produce one graph that satisfied the requirements but some were not able to find any other mathematically different (topologically different) solutions. A few candidates imposed the additional restriction that the graphs needed to be connected, although often these candidates did succeed in finding three suitable graphs.

   There were some very good answers to part (ii) and others that were vague or did little more than give the statement that an Eulerian graph has all even nodes. An Eulerian graph is a connected graph in which all the nodes are of even order and so Eulerian graphs have no odd nodes, the graphs described in part (i) have all their nodes of odd order and so cannot be Eulerian.

3) Most candidates scored well on this question. In part (i) candidates were required to state that this is a travelling salesperson problem. Some candidates described the problem (shortest Hamiltonian cycle) without giving it its name and many named other standard network problems or algorithms.
   In part (ii), most candidates were able to apply the nearest neighbour method

correctly. A few missed out *C*, going directly from *D* to *A* or going from *D* to *A* and then to *C* from there, the nearest neighbour method says to choose the 'least weight arc to a vertex that has not already been visited'. Some candidates did not complete the cycle by connecting the final vertex back to *A*. Some candidates showed their solution as a diagram; in this case they must remember to indicate the direction of travel. The nearest neighbour algorithm gives an upper bound to the shortest possible Hamiltonian cycle, in some problems this may be the shortest solution but in other cases there may be a shorter tour. Several candidates claimed that the shortest tour must be less than the solution found, rather than less than or equal to the length of the nearest neighbour tour, and many omitted the units from their answer.

Despite being instructed not to represent the network as a matrix, some candidates seem to think that they are not using Prim unless they set out the solution in this way. A number of candidates used Kruskal's algorithm on the network instead of Prim's. A significant number of candidates constructed the tree correctly and gave its weight but did not use this answer to calculate a lower bound for the journey time by adding on the two lowest weight arcs from vertex *A*. Again, some candidates omitted to give units on their final answer.

There were many correct answers for part (iv) although some candidates gave routes that did not satisfy the original requirements.

4) This should have been an easy question for most of the candidates, but in fact was often the one they had the most problems with. In part (i) the feasible region has four edges and so four inequalities are needed to define it. Some candidates gave equality constraints, or had the strict inequalities or had the inequality signs reversed (gave the inequalities that defined the shaded region). Several omitted at least one inequality or gave redundant information.

Most candidates were able to read off the coordinates of three of the four vertices from the graph; those who tried to read off the coordinates of the fourth vertex, rather than calculating them, generally incurred loss of accuracy. Some candidates did not know the difference between (2, 1) and (1, 2).

In part (iii), the majority of the candidates calculated the value of P at each of their vertices and then looked for the maximum value, while others used a sliding profit line on the printed graph. In either case it is necessary to explicitly state the values of x and y at the optimum point and the resulting maximum value of P, as asked in the question.

The change to a minimisation in part (iv) caused confusion for some candidates. Some candidates ignored the word 'minimise' and found the maximum value of Q, some negated the Q expression to make a maximisation problem but forgot to change the sign back again at the end, but the majority proceeded as in part (iii) by either checking vertices or using a sliding profit line. The difference here was that two vertices resulted in the same minimum value and the set of feasible points resulting in this value was this pair of points and all points on the line segment joining them.

Some candidates did not attempt part (v). Of those who did, the majority just gave the value of P and the value of Q at each vertex and noted that they were different in every case. This was not enough as equality could have occurred at an interior point, or set of points, although those who realised that P was strictly greater than Q at each vertex were usually able to complete the argument. Some candidates considered what would happen if P equalled Q and realised that this led to $x = 3y$, a few then went on to explain how they knew that this line did not cross the feasible region. The candidates who used sketch graphs were generally more successful than those who gave an argument using inequalities. Some candidates seemed to think that if P and Q were equal then they would both have to be 0, this was clearly not true and rarely led to any useful reasoning.

5) Most candidates were able to use the slack variables to eliminate the inequality signs in the non-trivial constraints although some added in the slack variables but kept the inequality signs intact. The majority of the candidates were then able to set up an appropriate representation of the problem as a Simplex tableau. A few candidates did not rearrange the objective into an equation with all the variables on one side before setting up the tableau, resulting in sign errors in the coefficients of x, y and z in the objective row, and some omitted the objective row altogether. Some candidates invented a coefficient for y in the second constraint, rather than setting it to be zero, and some put the 3 as the coefficient of y and 0 as the coefficient of z, instead of the other way round.

Parts (iii) and (iv) of the question were structured to encourage candidates to show their methods more clearly, in most cases this was successful and resulted in a better standard of solutions than in previous sessions.

Many candidates said that the pivot should be chosen from the x-column because it had the most negative coefficient in the objective row, in fact it was the only column with a negative coefficient in the objective row. When the Simplex algorithm is run on a computer, this is usually the criterion applied, but in fact any column with a negative value in the objective row is suitable (subject to there being a pivot choice when the ratios are considered) and although choosing the column with the most negative value in the objective row will usually lead to the greatest improvement in the value of P this is not always the case.

To choose the pivot the ratios $10 \div 2$ and $30 \div 2$ needed to be seen. Some candidates used the reciprocals of these ratios, which although equivalent is not what the method says, and some just described the calculations in algebraic terms without showing the actual values in this case. A few candidates also included the ratio for the objective row, this was incorrect as the ratio is only calculated for rows where the number in the denominator will be positive (not negative and not zero).

Having identified the pivot element, some candidates then proceeded to use a different element to demonstrate the first iteration of the tableau. Examiners attempted to credit correct methods even when they did not follow from earlier working. Most candidates coded their pivoting operations appropriately, although some did not explain what they had done to the pivot row.

The pivoting operations must each be of the form 'current row $\pm$ multiple of pivot row' to produce the value 1 in the position that previously contained the pivot element and the value 0 in all other positions in the pivot column. This should give a tableau with the appropriate basis columns (columns consisting of 0s and a single 1) with non-negative values in the column representing the right hand side of the equations and an increase (or at least no decrease) in the value of P.

Once an iteration has been completed, the resulting values for x, y, z and P can be read off directly from the tableau. There is no need to convert back to equations and work algebraically. Most candidates knew that the maximum value of the objective had not been achieved as there was still a negative value in the objective row. A few assumed, incorrectly, that the objective could not be achieved while any of x, y and z were still 0. Some candidates did not appreciate the difference between 'negative' and 'non-positive'.

6) Several candidates achieved full marks on this question, with the insert seeming to help candidates to make fewer arithmetic mistakes than they might otherwise have done.

In part (a), most candidates were able to find the route and length of the shortest path correctly even if there were small errors in the application of Dijkstra's algorithm. Some candidates did not record the values of the permanent labels and some did not show their working values (temporary labels) or obliterated them when an improvement was found. A few candidates recorded temporary labels that were larger than the current value, this is not what Dijkstra's algorithm says, and some unnecessarily recorded the vertex from which they had travelled to achieve each value.

In part (b)(i), the only odd nodes were A and J, and the length of the shortest path joining these nodes has already been found in part (a). Some candidates used the nearest neighbour method to find a tour through all the vertices but not one that covered every arc, and some candidates carefully added up all the arc weights to give the value 300 that had already been given in the question.

In part (b)(ii), there were four odd nodes so six shortest distances needed to be considered, three of which had already been found in part (a). The six shortest distances led to three pairings of odd nodes and the shortest of these needed to be chosen. Candidates needed to show the six shortest distances as well as picking the pairing with the smallest total. Several candidates then added this total to 300, instead of the reduced value of 270, having removed arc BH from the original network.

4737: Decision Mathematics 2

General Comments

Most candidates were able to complete this paper in the time allowed. As in previous sessions, some candidates did not seem to be ready to sit this paper, using approaches that were more appropriate for an AS paper than for an A2 paper. Candidates should take care when reading the questions and over the presentation of their answers. The best candidates explained their reasoning and gave full, accurate and efficient solutions.

Comments on Individual Questions

1) In part (i), many candidates calculated the capacity of the given cut correctly to get an answer of 29 litres per second. Some candidates calculated the capacity of the cut α shown on the diagram, but they usually realised their error when they reached part (ii). A small number of candidates did not seem to be familiar with the notation for describing a cut using a partition into sets.

   The majority of the candidates were able to show that the capacity of the cut α is 12 litres per second, although candidates should note that when an answer is given in the question they need to show all their working to get the credit. A few candidates realised how to adapt the method to finding the minimum possible flow across the cut, using the minimum flows from *S* to *T* and the maximum flows from *T* to *S*, although it was unusual to see a completely correct solution to this part of the question. Most candidates ignored the reference to 'without regard to the remainder of the diagram' and either found the minimum flow for the entire network or identified the two arcs for which the upper and lower capacities were the same and drew a conclusion from this.

   There were some good explanations given in part (iii), although a few candidates thought that *SC* needed to have at least 5 litres per second flowing through it in order to supply *FI*. Some candidates considered the flow through *D*, rather than the flow through *A* as asked in the question, when considering the flow in *AD*, and this was regarded as being acceptable.

   Apart from candidates who did not understand the topic of network flows, most candidates were able to make a reasonable attempt at finding a suitable flow in part (iv). Candidates usually drew a diagram to show their flow, although some chose to list the flow routes or the flow in each arc. The lower capacities in some of the arcs were sometimes overlooked, in particular arc *AB* and the arcs attached to *H*.

   Many candidates answered part (v) by quoting 'maximum flow = minimum cut' and then saying that the maximum flow is 12 litres per second. It is known from part (ii) that there is a cut of capacity 12 litres per second, but it is not known whether or not this is the minimum cut, so the maximum flow must be less than or equal to 12 litres per second. However, from part (iv) it is also known that there is a feasible flow of 11 litres per second, and hence the maximum flow is at least 11 litres per second. Since all the capacities are integer valued, the maximum flow must be either 11 or 12 litres per second. It is, in fact, 11 litres per second, but this cannot be deduced purely from the previous answers without some extra working.

2) Few candidates gave good descriptions of what a maximin route is, usually resorting to saying that it 'maximises the minimum' but not specifying what the minimum referred to. Often there was confusion between a route (a path from the start to the finish) and the individual arcs making up a route. Some candidates described the maximum route and some gave descriptions involving finding the 'maximum weight for the minimum cost', or similar. To find the maximin route we need to consider the minimum weight on each route (from the start to the finish) and then find the route for which this minimum is greatest.

The setting up of the dynamic programming tabulation was much improved on previous sessions. Many candidates had the correct structure with columns for stage, state, action, working and maximin. The state column should show the state label of the current state, not a (stage; state) label, and the action column should show the state label of the state being moved into. For example, when moving from (2; 0) to (1; 2) the action is 2. This is necessary in order that the route of the solution can be read from the tabulation without needing to refer back to the original network.

Although the question had asked for two maximin routes, some candidates only wrote down one of the routes. Several candidates omitted the units when stating the maximum load that could be carried. There were several correct answers to part (iii).

3) Most candidates correctly identified that the greatest number of points that Colin can win when Rosie plays strategy *A* is 3 and that he achieves this by playing strategy *Y*. A few candidates forgot that the number of points won by Colin is the negative of the number won by Rosie and so gave 4 and *X* as their answers.

In part (ii), candidates needed to show that row *B* is dominant over row *C* and that column *Y* is dominant over column *Z*. Some just gave the definitions of dominance in an abstract context but most demonstrated each dominance by making four explicit comparisons and supporting them with a brief explanation. A few candidates deleted column *Y* instead of column *Z* in the reduced matrix, and some candidates did not label the rows and columns in the reduced matrix which led to all sorts of problems later on.

Most candidates knew how to find the play-safe strategies, although some only indicated the chosen row and columns on the matrix, without saying which strategies had been chosen, and some stated the values of the play-safe strategies rather than giving the strategies themselves. There were some candidates who seemed reluctant to give two play-safe strategies for Colin and who came up with reasons for preferring one choice over the other, this was not necessary. Most candidates who found the play-safe strategies were able to use them to show that this was not a stable game.

Parts (iv) and (v) were answered well, most candidates realised that 5 had been added throughout the matrix to make all the entries non-negative (although some candidates claimed to have made the entries positive) and that the given expression was the augmented pay-off for Rose when Colin chooses strategy *X*. Nearly all the candidates who attempted part (v) realised that they had to substitute the given probabilities into the expressions to find *m* and hence *M*. Several candidates only substituted into one expression, usually the one discussed in part (iv), and were fortunate that the three constraints all coincided at the optimum vertex rather than having to choose the minimum of the various values obtained.

4) Only a small number of candidates did not understand what was required in completing the immediate predecessors for the activities, and most candidates were able to fill in the blank column on the insert correctly. Several candidates successfully carried out the forwards pass and the backwards pass through the network. Most of the errors that occurred were where candidates did not deal with the dummy activities correctly, often just ignoring them for the purposes of the passes. Dummy activities may be regarded as activities of duration 0 for the purposes of carrying out the passes but do not need to be included in the list of critical activities.

Although activity *C* was not critical it only had 1 hour of float so a delay of 3 hours on activity *C* would delay the entire project by 2 hours. Most candidates were able to deduce this from the early and late event times on their activity network. There were several correct resource histograms but also graphs that seemed to be cascade charts with 'holes' in them or shaded cells that overhung empty spaces. A resource histogram shows the number of workers in use at each particular time and should resemble an ordinary histogram with no gaps in it. It is not necessary to label the activities in the histogram, although some candidates find this helpful.

5) There were many good answers to this question. Nearly all the candidates were able to draw a correct bipartite graph and write down the result of applying this particular alternating path. A few candidates left out one or more of the pairs who did not change their partners.

Although the information in part (iii) was quite involved, most candidates were able to complete the matrix with few or no errors. The Hungarian algorithm was then applied to find the minimum cost complete matchings. In this case it was only necessary to reduce the rows and columns to form a reduced cost matrix, no augmentation was required. Some candidates only indicated their matchings on the matrix and did not state who was paired with whom. Finally, many candidates were able to identify which people were paired with people that they had not named in each matching, although some candidates did not make it clear which matching they were referring to in their answers.

Advanced GCE Mathematics (3890, 3892, 7890)
June 2006 Assessment Series

Unit Threshold Marks

Unit		Maximum Mark	a	b	c	d	e	u
4721	Raw	72	56	48	40	33	26	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	53	45	37	29	22	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	60	52	44	37	30	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	60	52	44	37	30	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	50	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	59	51	43	36	29	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	58	50	43	36	29	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	51	44	37	30	23	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	52	44	36	29	22	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	53	45	38	31	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	31.0	46.3	61.2	73.5	84.2	100	12438
3891	0	0	0	100	100	100	1
3892	60.6	76.8	89.2	95.3	97.6	100	1109
7890	46.9	67.7	81.9	91.5	97.6	100	9525
7891	50.0	75.0	87.5	87.5	100	100	8
7892	59.9	80.2	89.4	95.5	98.6	100	1428

For a description of how UMS marks are calculated see;
www.ocr.org.uk/OCR/WebSite/docroot/understand/ums.jsp

Statistics are correct at the time of publication

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