RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7844, 7840, 7842, 7890, 7891, 7892**

Advanced Subsidiary GCE **AS 3841, 3844, 3840, 3842, 3842, 3890, 3891, 3892**

Combined Mark Schemes And Report on the Units

June 2005

3840-4/7840-4/MS/R/05
3890-2/7890-2/MS/R/05

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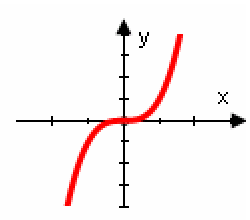
MARK SCHEMES FOR THE UNITS

REPORT ON THE UNITS

Mark Scheme 2631
June 2005

1	$x^2-6x-40\ge 0$ $(x+4)(x-10)\ge 0$ $x\le -4$, $x\ge 10$	M1 A1 M1 B1 4 **4**	$(x+4)(x-10)\ge 0$ Correct method to find roots $x=-4,10$ seen Correct method to solve quadratic inequality eg graph $x\le -4$, $x\ge 10$ (not wrapped)
2 (a) (b)	$2x^{\frac{2}{3}}\times 3x^{-1}$ $=6x^{-\frac{1}{3}}$ $2^{40}\times 4^{30}$ $=2^{40}\times 2^{60}$ $=2^{100}$	M1 A1 2 M1 A1 2 **4**	Adds indices $=6x^{-\frac{1}{3}}$ 2^{60} or 4^{20} 2^{100}
3	EITHER $3(x^2+4x)+7$ $3(x+2)^2-12+7$ $3(x+2)^2-5$ OR $3(x^2+2ax+a^2)+b$ $3x^2+6ax+3a^2+b$ $6a=12$ $a=2$ $3a^2+b=7$ $b=-5$	 M1 A1 M1 A1 4 **4**	Mark scheme is the same for EITHER and OR $a=\frac{12}{2 \text{ or } 6}$ $a=2$ $b=7-ka^2$ for some $k>0$ [$-6+7=$ or $12+7=b$ M0] $b=-5$

4	$x^2+(5-2x)^2=25$ $5x^2-20x=0$ $x=0\ ,\ 4$ $y=5\ ,\ \text{-}3$ OR $x=\dfrac{5-y}{2}$ $\dfrac{(5-y)^2}{4}+y^2=25$ $y^2-2y-15=0$ $y=5,-3$ $x=0,4$	M1 M1 M1 A1 A1 5 **<u>5</u>**	Substitute for x/y Obtain 2 or 3 term quadratic =0 following expansion of $(5-2x)^2$ to $ax^2+bx+c, b\neq 0$ Correct method to solve quadratic $x=0,4$ or $y=5,-3$ $y=5,-3$ or $x=0,4$
5 (i)		B1 1	Correct sketch
(ii)	Reflection in x-axis or Reflection in y-axis	B1 B1 2	Reflection In x-axis or y=0 or y-axis or x=0
(iii)	$y=(x-p)^3$	M1 A1 2 **<u>5</u>**	$y=(x\pm p)^3$ $y=(x-p)^3$
6	$8\times1+6\times\dfrac{1}{2}-2\times\dfrac{1}{\sqrt{2}}+10\times\dfrac{\sqrt{3}}{2}$ a=11 c=5 $-\dfrac{2}{\sqrt{2}}=-\sqrt{2}$ b=-1	B1 B1 B1 M1 A1 5 **<u>5</u>**	$\sin 45=\dfrac{1}{\sqrt{2}}$ or $\cos 30=\dfrac{\sqrt{3}}{2}$ seen a=11 c=5 Attempt to rationalise b=-1

7	$k = x^3$ $k^2 + 26k - 27 = 0$ $k = -27, 1$ $x = -3, 1$	*M1 A1 A1 dep*M1 A1 5 **<u>5</u>**	Attempt a substitution to obtain a quadratic. $k^2 + 26k - 27 = 0$ -27, 1 Attempt cube root. $x = -3, 1$ (no extras)
8 (i) (ii) (iii)	$\left[x^3 - \frac{x^2}{2}\right]_1^2$ $(8-2) - \left(1 - \frac{1}{2}\right)$ $5\frac{1}{2}$ $3x + \frac{1}{x} + c$ $y = 3x + \frac{1}{x} + c$ $y = 3x + \frac{1}{x} + 2$	B1 M1 A1 3 B1 B1 2 B1 ft M1 A1 3 **<u>8</u>**	$x^3 - \frac{x^2}{2}$ Substitute limits (top - bottom) $5\frac{1}{2}$ or $\frac{11}{2}$ $\frac{k}{x}$ seen $3x + \frac{1}{x} + c$ $y = 3x + \frac{1}{x} + c$ (must have c) Substitute x=1 and y=6 to find c c=2

9 (i)	$y = \frac{4}{3}x + \frac{5}{3}$ gradient = $\frac{4}{3}$	B1 1	$\frac{4}{3}$
(ii)	Gradient of $\perp^r = -\frac{3}{4}$ $y - 2 = -\frac{3}{4}(x-1)$	B1√	$-\frac{3}{4}$ or $-1/(4/3)$ seen or implied
	$4y + 3x = 11$	M1	Attempts equation of straight line through (1,2) any gradient but not *x*, *y* values interchanged.
		A1 3	$4y + 3x = 11$ o.e.
(iii)	$P\left(-\frac{5}{4}, 0\right)$ $Q\left(0, \frac{11}{4}\right)$	B1	$\left(-\frac{5}{4}, 0\right)$ seen or implied . Nothing incorrect seen.
		B1√	$\left(0, \frac{11}{4}\right)$ seen or implied (from a straight line equation in (ii))
	$\left(-\frac{5}{8}, \frac{11}{8}\right)$	B1√ 3	$\left(-\frac{5}{8}, \frac{11}{8}\right)$ aef. Nothing incorrect seen.
		<u>7</u>	

10 (i)	$\frac{dy}{dx} = x^2 - 9$	B1 B1 2	$x^2 - 9$ 1 term correct Both terms correct
(ii)	$x^2 - 9 = 0$ $x = 3, -3$ $y = -18, 18$	M1 A1 A1 3	Uses $\frac{dy}{dx} = 0$ $x = 3, -3$ $y = -18, 18$ [correct pair only A1, A0]
(iii)	$\frac{d^2y}{dx^2} = 2x$ $x = 3 \quad \frac{d^2y}{dx^2} = 6$ $x = -3 \quad \frac{d^2y}{dx^2} = -6$	M1 B1 B1 3	Looks at sign of $\frac{d^2y}{dx^2}$ or other correct method. E.g. says positive cubic. $x = 3$ minimum $x = -3$ maximum (nothing incorrect seen) If A0 A0 then allow one A1ft on correct conclusion for incorrect *x* and incorrect $\frac{d^2y}{dx^2}$
(iv)	Gradient of $24x + 3y + 2 = 0$ is -8 $x^2 - 9 = -8$ $x = \pm 1$ $x = 1, y = -8\frac{2}{3}$ $x = -1, y = 7\frac{1}{3}$ Check in $y = \frac{x^3}{3} - 9x$ $x = 1, y = -8\frac{2}{3}$ $x = -1, y = 8\frac{2}{3}$ $\therefore \; p = 1, q = -8\frac{2}{3}$	B1 M1 M1 A1 A1 5 **13**	Gradient = -8 $x^2 - 9 = -8$ x=1 or x=-1 substituted in line and curve $p = 1, q = -8\frac{2}{3}$ (dependent on previous M1) $\therefore$ accepted $p = -1, \quad q = 8\frac{2}{3}$ and $p = -1, \quad q = 7\frac{1}{3}$ (dependent on previous M1) $\therefore$ rejected

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1	(i)	Attempt formula of form $kr\theta$ for arc length	M1	[any constant k]
		Obtain 0.96 or $\frac{48}{50}$ or $\frac{24}{25}$	A1	**2** [or equiv such as 0.305π, 0.306π, 0.31π]
	(ii)	Obtain 55	B1	**1** [allow value rounding to 55.0]
2		Obtain $1 + 36x$	B1	[with terms simplified]
		Attempt at least one further term including attempt at binomial coefficient	M1	[coefficient must be ${}^9C_{..}$]
		Obtain $\ldots + 576x^2$	A1	
		Obtain $\ldots\ldots + 5376x^3$	A1	**4**
3		Attempt sum of arithmetic progression	M1	[using formula which is quadratic in n and linear in a and d or equiv]
		Obtain $\frac{1}{2}\times 80[2\times 1.71 + 79\times 0.02]$	A2	[or equiv such as 40(1.71 + 3.29)]
		State or imply $\frac{250}{1-r}$	B1	
		Attempt solution of equation of the form $\frac{k}{\pm 1 \pm r} = c$	M1	[showing valid algebraic technique]
		Obtain $-\frac{1}{4}$ or equiv	A1	**6** [or equiv]
4		Attempt correct application of modulus signs at least twice	M1	[maybe implied; only with first 3 values]
		Use at least four of the y-values 7, 6, 4, 0, 8, 24	A1	
		Attempt expression of form $k\{y + 2(y + \ldots) + y\}$	M1	[any k; or equiv using separate trapezia]
		Obtain $0.5\{7 + 2(6 + 4 + 0 + 8) + 24\}$ or equiv	M1	[condoning errors in y values]
		Obtain 33.5	A1	**5**

5	(i)	State $y > 4$	B1	**1** [using any notation; allow $y \geq 4$]
	(ii)	<u>Either</u> Attempt to find inverse function g^{-1}	M1	[producing or implying $e^{f(x)}$ or $e^{f(y)}$]
		Substitute 6 to obtain e^2	A1	**2** [or exact unsimplified equiv]
		<u>Or</u> Attempt solution of equation $3 \ln x = 6$	M1	
		Obtain e^2	A1	**(2)** [or exact unsimplified equiv]
	(iii)	Attempt process for composition of functions	M1	[must be gf attempt not fg]
		State $3\ln(x^2+4)$	A1	[maybe implied]
		Attempt use of the chain rule	M1	[applied to either gf or fg]
		Obtain $\frac{6x}{x^2+4}$	A1	**4** [or equiv]
6	(a)	Obtain integral of form $k(4x+3)^{\frac{1}{2}}$	M1	[any constant k]
		Obtain correct $\frac{1}{2}(4x+3)^{\frac{1}{2}}$	A1	[or equiv]
		Obtain $\frac{1}{2}(\sqrt{27}-\sqrt{3})$	A1	[or exact equiv]
		Simplify to obtain $\sqrt{3}$	A1	**4** [**AG**; necessary detail required]
	(b)	State $2 \ln x$	B1	
		Attempt correct use of at least one relevant logarithm property	M1	
		Obtain $\ln 225 - \ln 9$ or $2 \ln 5$ and hence $\ln 25$	A1	**3** [**AG**; necessary detail required]
7	(i)	<u>Either</u> Equate attempt at $P(\pm 6)$ to 4	M1	
		Obtain $216 - 4 \times 36 + 6a + 16 = 4$	A1	[or equiv]
		Confirm -14	A1	**3** [**AG**]

		Or Attempt complete division by $x - 6$	M1	
		Obtain remainder $16 + 6(a + 12)$	A1	[or equiv]
		Solve equation to confirm -14	A1	(3)
		Or (for method involving verification)		
		Attempt complete valid method	M1	
		Confirm accordingly with all details correct	A2	(3)
	(ii)	State 6 as a root	B1	[or determined otherwise]
		Attempt division or equiv to find quadratic factor	M1	
		Obtain $x^2 + 2x - 2$	A1	
		Use quadratic formula to find two further roots	M1	[allow if sign error(s); or equiv method]
		Obtain $-1 \pm \sqrt{3}$ or exact equivs	A1	**5**
8	**(i)**	Draw sketch showing increasing positive function	B1	[existing for negative and positive x]
		Draw any parabola with a maximum point	M1	
		Draw parabola which is symmetrical about y-axis and crossing y-axis above the exponential graph	A1	
		Indicate the two intersections	A1	**4** [with graphs more or less correct]
	(ii)	**(a)** Attempt to evaluate expression for both values	M1	
		Obtain 0.23 and -0.12 and refer to change of sign	A1	**2** [or equivs]
		(b) Rearrange equation to obtain $x = \frac{1}{4}\ln(100 - 3x^2)$	B1	**1** [**AG**; all details correct]
		(c) Obtain correct first iterate	B1	[… to 2 decimal places or more]

		Attempt iteration process at least twice in all	M1	
		Obtain one correct iterate after the first	A1	
		Conclude 1.14	A1	**4** [$1 \rightarrow 1.14368 \rightarrow 1.14128 \rightarrow 1.14133$ allow recovery after error]
9	**(i)**	State 270	B1	**1**
	(ii)	Differentiate to obtain $k\,\mathrm{e}^{-0.0077t}$	M1	[any constant k different from 600]
		Obtain correct $-4.62\,\mathrm{e}^{-0.0077t}$	A1	[or unsimplified equiv]
		Obtain 2.14	A1	**3** [accept + or −]
	(iii)	Attempt to find expression for M_Q involving e	M1	[or equiv involving $0.75^{\mathrm{f}(t)}$]
		Obtain $480\,\mathrm{e}^{-0.0032t}$	A1	[or $480 \times 0.75^{\frac{1}{90}t}$]
		Attempt solution of equation for t	M1	
		Attempt correct process for solution of equation of form $A\,\mathrm{e}^{ct} = 600\,\mathrm{e}^{-0.0077t}$	M1	[or equiv involving $0.75^{\mathrm{f}(t)}$]
		Obtain $\mathrm{e}^{0.0045t} = 1.25$ or equiv and hence 49.6	A1	**5** [accept $49.4 \le t \le 49.7$]

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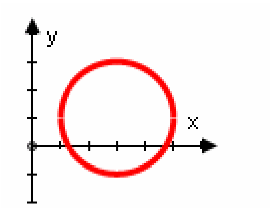
1 (i)	$(1+2x)e^{2x}$	M1 A1 2	Use of product rule to obtain 2-term form Correct answer SC $\left(1+\frac{1}{2}x\right)e^{2x}$ M1A0
(ii)	$\frac{dy}{dx}=\frac{(x^2+1).1-x.2x}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$	M1 A1 A1 3 **5**	For use of quotient (or equiv) rule For correct unsimplified expression For correct simplified answer
2 (i)	$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$	B1 1	For correct RHS Condone ${(\tan\theta+\tan\theta)}/{1-\tan^2\theta}$ and allow ${2t}/{1-t^2}$
(ii)	$\frac{2\tan\theta}{1-\tan^2\theta}=3\tan\theta\Rightarrow\tan\theta=0$ or $3\tan^2\theta=1$ Hence $\theta=0,\frac{1}{6}\pi,\frac{5}{6}\pi,\pi$	M1 A1 B1 A1 A1 5 **6**	For producing cubic or quadratic in $\tan\theta$ For $\tan^2\theta=\frac{1}{3}$ stated or implied For both values 0 and π For answer $\frac{1}{6}\pi$ For answer $\frac{5}{6}\pi$ (and no other in range) Ignore any references to angles not in $[0,\pi]$
3	$u=x-2,\quad du=dx$ $\frac{(x-2)^7}{7}+\frac{(x-2)^6}{3}+c$ BY PARTS: $I=\int x.d\left(\frac{(x-2)^6}{6}\right)$ $=\frac{x(x-2)^6}{6}-\int\frac{(x-2)^6}{6}dx$ $=\frac{x(x-2)^6}{6}-\frac{(x-2)^7}{42}$	B1 M1 A1 A1 ft A1 B1 M1A1 M1A1 **5**	Conversion of $dx\to du$ (may be implied) Substitution of $u=x-2, du$ for dx in integral $\int(u+2)\times u^5du$ Integrate to give $\frac{u^7}{7}+\frac{u^6}{3}(+c)$ Final form $\frac{(x-2)^7}{7}+\frac{(x-2)^6}{3}$ (cao)
4 (i)	$(1+2x)^{-1}\approx 1-2x+\frac{(-1).(-2)}{2!}\cdot(2x)^2$ $=1-2x+4x^2$	B1 B1 B2 4	$1-2x$ obtained Binomial coefficient for x^2 in a.c.f. $+4x^2$ obtained (B1 for $-4x^2$)

(ii)	$(1+x)\cdot(1+2x)^{-1}=(1+x)(1-2x+4x^2)$ $=(1-x+2x^2)$	M1 A1 A1 3	$(1+x)$ times their quadratic from (i) $1-x$ legitimately found $k=2$
(iii)	$\lvert 2x\rvert<1$ so $\lvert x\rvert<\frac{1}{2}$ or $-\frac{1}{2}<x<+\frac{1}{2}$	B1 1 **8**	Allow $\le$ for <
5 (i)	EITHER: $\int\cos^2 x\mathrm{d}x=\int\frac{1}{2}(1+\cos 2x)\mathrm{d}x$ $=\frac{1}{2}x+\frac{1}{4}\sin 2x+c$ OR: $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{RHS})=\frac{1}{2}+\frac{1}{2}\cos 2x$ $=\frac{1}{2}+\frac{1}{2}(2\cos^2 x-1)=\cos^2 x$	M1 A1 A1 B1 M1 A1 3	For relevant use of double angle formula For correct integrand in terms of $\cos 2x$ For showing given result correctly For correct differentiation For relevant use of double-angle formula For showing given result correctly
(ii)	$x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int(\frac{1}{2}x+\frac{1}{4}\sin 2x)dx$ $\frac{1}{2}x^2+\frac{1}{4}x\sin 2x-\frac{1}{4}x^2+\frac{1}{8}\cos 2x+c$ $\left[\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x\right]_0^{\frac{1}{2}\pi}=\frac{1}{16}\pi^2-\frac{1}{4}$	M1A1 M1A1 a.c.f. A1 5 **8**	For correct first step of integration by parts For second integration (ignore limits so far) For correct (exact) answer
6 (i)	$\left(\frac{x-3}{2}\right)^2+\left(\frac{y-1}{2}\right)^2=1$ Equation is $(x-3)^2+(y-1)^2=4$ 	M1 A1 B1 B1 B1 5	For substitution in the given identity For standard or expanded form correct For any recognition of a circle For sketch of circle with centre (3,1) For radius equal to 2 (Dimensions should look approximately in the correct proportions if dimensions not marked)
(ii)	*EITHER*: $\frac{\mathrm{d}x}{\mathrm{d}\theta}=-2\sin\theta,\ \frac{\mathrm{d}y}{\mathrm{d}\theta}=-2\cos\theta$ Hence $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\cos\theta}{-2\sin\theta}=-\frac{x-3}{y-1}$ *OR*: $2(x-3)+2(y-1)\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Hence $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x-3}{y-1}$	B1 M1 A1 M1 A1 A1 B1 M1 A1 3	For both derivatives correct For use of $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}/\frac{\mathrm{d}x}{\mathrm{d}\theta}$ For correct answer in terms of x and y For implicit differentiation of cartesian eqn For correct equation relating to x, y, and $\frac{\mathrm{d}y}{\mathrm{d}x}$ For correct answer For correct statement For use of perpendicular gradients For correct answer

	OR: Gradient of radius is $\frac{y-1}{x-3}$ So grad of tangent $= \frac{dy}{dx} = -\frac{x-3}{y-1}$	**8**	
7 (i)	$\overrightarrow{OP} = 2i + 3j$ $\overrightarrow{OQ} = \frac{1}{2}\left(\overrightarrow{OC} + \overrightarrow{OP}\right) = i + \frac{3}{2}j + \frac{5}{2}k$	B1 B1 2	$\overrightarrow{OP}$ correct $\overrightarrow{OQ}$ correct
(ii)	$\overrightarrow{BQ} = i - \frac{9}{2}j + \frac{5}{2}k$ $\therefore BQ$ is $r = 6j + \lambda(2i - 9j + 5k)$	M1 A1 ft A1 3	Equation in form $r = r_B + \lambda\overrightarrow{BQ}$ or equiv. A form for $\overrightarrow{BQ}$ based on <u>their</u> $\overrightarrow{OQ}$ Fully correct form c.a.o.
(iii)	$\overrightarrow{BQ} = \left(1, -\frac{9}{2}, \frac{5}{2}\right)$, $\overrightarrow{OP} = (2,3,0)$ $\therefore \cos\theta = \frac{\overrightarrow{BQ}.\overrightarrow{OP}}{BQ.OP} = \frac{\left(1,-\frac{9}{2},\frac{5}{2}\right).(2,3,0)}{\sqrt{1+\frac{81}{4}+\frac{25}{4}}.\sqrt{4+9}}$ $= \frac{-23}{\sqrt{110}.\sqrt{13}}$ $\cos\theta = -0.6082$ $\theta = 127.5$	M1 M1 dep A1 A1 4 **9**	$\overrightarrow{BQ}.\overrightarrow{OP}$ calculated $\cos\theta = \frac{\overrightarrow{BQ}.\overrightarrow{OP}}{\lvert\overrightarrow{BQ}\rvert.\lvert\overrightarrow{OP}\rvert}$ $\theta = 127.5^o$ (allow 127°/128° and also 52.5°)
8 (i)	$\frac{1}{p(1-p)} = \frac{1}{p} + \frac{1}{1-p}$	M1 M1A1 3	For correct form $\frac{A}{p} + \frac{B}{1-p}$ For correct partial fractions
(ii)	$\int\left(\frac{1}{p} + \frac{1}{1-p}\right)dp = \int k\,dt$ Hence $\ln p - \ln(1-p) = kt + c$	M1 A1 ft A1 ft B1 4	For separation and use of pfs to integrate For correct terms in p and kt For correct term $-\ln(1-p)$ For inclusion of one arbitrary constant
(iii)	$\ln 0.01 - \ln 0.99 = c$ $\ln 0.05 - \ln 0.95 = k + c$ Hence $t = \frac{\ln\left(\frac{0.95}{0.05}\right) - \ln\left(\frac{0.01}{0.99}\right)}{\ln\left(\frac{0.05}{0.95}\right) - \ln\left(\frac{0.01}{0.99}\right)} = 4.57$	M1 M1 M1 A1 4 **11**	For use of p = 0.01, t = 0 to find c For use of p = 0.05, t = 1 and c to find k For use of p = 0.95 and numerical c and k For correct answer for t

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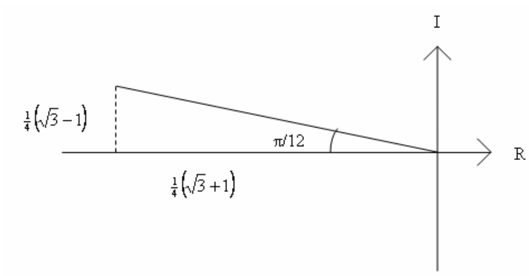
June 2005

1.		P.F. in the form	**B1**	S.R. A = 1 by cover-up B1
		$\frac{A}{x} + \frac{Bx + C}{x^2 + 1}$		i.e. $\frac{1}{x} + \frac{Bx + C}{x^2 + 1}$; may be implied
		Obtain $A(x^2 + 1) + x(Bx + C)$	**M1√**	If $Bx/(x^2 + 1)$
		Choose x-values and/or equate coeff.	**M1√**	Reasonable attempt at ALL their coefficients
		Obtain A = 1, B = -1, C = 3	**A1**	Coeff. only needed
2.	(i)	Obtain $k/\sqrt{(1 - (ax)^2)}$	**B1**	Allow ax^2; k constant
		Obtain $k = a$	**B1**	cao; AEEF SR Get $a/\cos y$ and a reasonable attempt to replace y in terms of x M1 cao; including $a/\cos(\sin^{-1} ax)$ A1
	(ii)	Attempt f(0) = 0 and f ′(0) = a	**M1√**	
		Obtain ax	**A1√**	From their (i); must be kx
3.		Attempt to solve denom. = 0	**M1**	
		Get $x = 1$ and $x = 2$ only	**A1**	
		Attempt to divide out / equate coeff.	**M1**	Allow here $y = x$
		Divide to get $x + k$, constant $k \neq 0$	**M1**	
		Get $k = 3$ in $y = x + 3$	**A1**	Must be equation
4.	(a)	Expand to $9\sum n^2 - 6\sum n + \sum 1$	**B1**	May be implied
		Use standard formulae	**M1**	At least two correct
		Attempt to simplify towards answer	**M1**	Mult. out and collect like terms
		Factorise to given answer in terms of n	**A1**	A.G. clearly seen SR n=1 for $2^2 = 4 = ½(6+3-1)$ B1 Assume for n=k AND add $(3k+2)^2$ M1 Clearly get to result for $(k+1)$ M1 Clearly bring together both parts A1
	(b)(i)	Obtain $1/(r + 1)(r + 2) - 1/(r(r + 1)) = \frac{r - (r + 2)}{r(r + 1)(r + 2)}$	**B1**	A.G.

	(ii)	Expand $\sum = k$ (f(1) – f(2) + f(2) –f(3) ..	M1	Attempt at difference method
		Cancel to or quote $\sum = k$(f(1) – f(n + 1))	M1	Sufficient terms seen / quoted; allow similar expressions
		Obtain $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$.	A1	cao; AEEF; must be n
5.	(i)	Diff. to $xe^x + e^x$ = A.G.	B1	
		Diff. to $e^x(x + 1) + e^x$	M1	Use of product rule
		Factorise to $e^x(x + 2)$	A1	
	(ii)	Conjecture on their results	B1	Reasonable from (i), including n
	(iii)	State result true for n = 1 (or 0)	B1	(√ on their conjecture)
		Use result for $n = k$ and product rule	M1	
		Obtain $e^x(x + (k+1))$	A1	
		Clearly bring together both parts	B1	Include or imply assumption for $n=k$
6.	(i)	Obtain $\frac{dy}{dx} = -z^{-2}\frac{dz}{dx}$	M1	Use Chain Rule (may be implied)
		Replace all y in diff. equation	M1	
		Clearly reach A.G.	A1	
	(ii)	Obtain $(e^{\int -2\,dx}) = e^{-2x}$ as I.F.	B1	Allow +/– 2
		Obtain LHS as perfect differential of their I.F. x z	M1	May be quoted
		RHS = $\int \pm xe^{-2x}$ dx by parts	M1√	Or $\int \pm xe^{2x}$ dx by parts
		Obtain $^1/_4\,(2xe^{-2x} + e^{-2x}) + c$	A1	Needs c
		Replace z by $1/y$	B1√	No need to write as y =
7.	(i)	Multiply by conjugate of z_2	B1	
		Multiply out top and bottom	M1√	
		Obtain $^1/_4((-\sqrt{3} - 1) + i(\sqrt{3} - 1))$	A1	SR Equate 1 – i to $(-\sqrt{3}+i)(x+iy)$ B1 Equate R and I parts M1 Solve for x and y A1
	(ii)	Attempt at modulus and argument e.g. diagram		
		Obtain $[\sqrt{2}, -^{\pi}/_4]$ and $[2, ^{5\pi}/_6]$ AEEF	B1	For any two values correct
			B2	For any three correct
			B3	For all four correct (not degrees)

	(iii)	Divide moduli, subtract arguments	M1	"Hence" only
		Obtain √2/2 and –13π/12	A1√	Or 11π/12; √ from (ii)
	(iv)		M1√	Argand diagram (√ from their (i) or (iii)) Clear attempt to use earlier answers (i) and (iii) M1
			A1	cao with lengths justified
8.	(i)	Use $r^2 = x^2 + y^2$, $x = r\cos\theta$, $y = r\sin\theta$	M1	Include $\tan\theta = y/x$
		Attempt to simplify	M1	
		Clear A.G.	A1	
	(ii)	Correct formula with correct r	B1	
		Expand to $1 + \tan^2\theta + k\tan\theta$ ($k\neq 0$) and use $\sec^2\theta = 1 + \tan^2\theta$	B1	
		Obtain $\tan\theta$	B1	
		Obtain $k\ln(\sec\theta)$	B1	AEEF
		Use given limits to $\frac{1}{2}(1 + \ln 2)$	B1	cao ; AEEF
	(iii)	Use $r = 0$ to get $\tan\theta = \pm 1$	M1	
		Get $\theta = -\frac{1}{4}\pi$	A1	cao; allow 7π/4, 0.785..
	(iv)	Recognise $r\cos\theta = x$	M1	
		Use $\theta \to \frac{1}{2}\pi$ to get $x = 1$	A1	

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1	(i)	Attempt to add $\frac{dy}{dx}(x=0, y=1)$ x 0.05 to 1	M1		
		0.95	A1	2	
	(ii)	Use [y value found in (i) or start again & produce new y value] and $x = 0.05$ to find $\frac{dy}{dx}(= x^2 - y)$ $\rightarrow$ class as "right hand gradient, g_2"	M1		
		Find average of g_2 and g_1 (from (i)), multiply by 0.05 and add to 1	M1		
		0.951 or better (0.9513125)	A1	3	5
2	(i)	State/imply (AE) $4m^2 + 5m + 1$ (=0)	M1		
		Obtain (CF as) $Ae^{-\frac{1}{4}t} + Be^{-t}$ (Ignore any label)	A1		
		Finish with θ = their CF [Must not finish with CF = …]	√A1	3	
	(ii)	Use $\theta = \frac{1}{4}\pi$ when $t = 0$	M1		
		Differentiate and use $\frac{d\theta}{dt} = 0$ when $t = 0$	M1		
		Either√ eqn in A & B [$\frac{1}{4}$A + B = 0 or A + 16B = $-\pi$ (A,B interch)]	√A1		
		[Follow through only from wrong GS, not from wrong differentiation]			
		$\theta = \frac{1}{12}\pi[4e^{-\frac{1}{4}t} - e^{-t}]$ c.a.o. AEF	A1	4	7
		S.R. –1 (once) from any A marks if variable(s) wrong at any stage.			
3	(i)	$x = 1$ & $x = 4$ drawn up to curve or better e.g. identify $\int_1^4 \frac{1}{x^4}\,dx$ as area under curve between $x = 1$ and $x = 4$	B1		
		Identify the 3 appropriate (not 4) rectangles clearly	B1		
		Statement re rectangles being under curve, or shading shown above rectangles or any other convincing demonstration **AG** (dep B2)	B1	3	
	(ii)	Extend to $\frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots + \frac{1}{n^4} < \int_1^n \frac{1}{x^4}\,dx$ or $\sum_1^n \frac{1}{r^4} < \int_1^n \frac{1}{x^4}\,dx + 1$	B1		
		$\int \frac{1}{x^4}\,dx = -\frac{1}{3x^3}$ AEF	M1		
		RHS of inequality = $\frac{1}{3} - \frac{1}{3n^3}$	A1		
		Add 1 to each side (if necessary) to produce result (convincingly) **AG**	A1	4	7
4	(i)	In both (i)(ii), a curve crossing x-axis & starting position on x-axis; then…			
		Ordinate + tangent (or tgt at point immed above starting point)	B1		
		+ ordinate + tangent (or ditto)	B1	2	

	(ii)	Any correct illustration showing divergence	B1	1	
	(iii)	State or use formula $x - \frac{x^3 - x - 2}{3x^2 - 1}$	M1		
		1st iteration correct wth starting value between 1 and 2 (inclusive) {1 → 2, 2 → 1.63(636363), 1.5 → 1.52(17391), check other values}	A1		
		Final answer **<u>1.521</u>**, <u>indep of previous A1</u>	*A1		
		3 d.p. justification [2 equal successive 3 d.p. or sign check]	dep*A1	4	**7**
5		$\alpha + \beta = -\frac{b}{a}$ <u>and</u> $\alpha\beta = \frac{c}{a}$ wherever seen	B1		
		Attempt $\alpha^2 + \beta^2$, from $(\alpha + \beta)^2 - 2\alpha\beta$ or subst α, β into eqn & add	M1		
		$\alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$ AEF	A1		
		Sum of new roots = $\frac{\alpha^2 + \beta^2}{\alpha\beta}$	M1		
		$= \frac{b^2 - 2ac}{ac}$ (= S)	A1		
		Product of new roots = 1 (= P)	B1		
		Eqn. is $x^2 - Sx + P$ (=0)	M1		
		$x^2 - \frac{b^2 - 2ac}{ac}x + 1 = 0$ AEF	A1	8	**8**
6	(i)	(i) $y = \tanh^{-1} x \rightarrow x = \tanh y \rightarrow x = \frac{e^y - e^{-y}}{e^y + e^{-y}}$	B1		
		$x e^{2y} + x = e^{2y} - 1$ AEF with e^{2y}	B1		
		$e^{2y} = \frac{1+x}{1-x}$ or $\frac{-1-x}{x-1}$	B1		
		$y = \frac{1}{2}\ln\frac{1+x}{1-x}$ **AG**	B1	4	
	(ii)	Change each inverse tanh into log format	M1		
		use e.g. ln a + ln b = ln ab	M1		
		$(1 + u)(1 + v) = 9(1 - u)(1 - v)$ AEF	A1		
		$v = \frac{5u-4}{4u-5}$ with at least 2 intermediate lines of working **AG**	A1	4	**8**
7	(i)	$\frac{dx}{dt} = -3\sin t + 3\sin 3t$ <u>or</u> $\frac{dy}{dt} = 3\cos t - 3\cos 3t$	B1		
		$9\sin^2 t + 9\cos^2 t + 9\sin^2 3t + 9\cos^2 3t = 18$ irresp of any prev sign errors	B1		
		'middle' term = $-18\sin t\sin 3t - 18\cos t\cos 3t$ √ f.t. on sign errors	B1		
		$18 - 18\cos(2t)$ <u>or</u> $18 - 18\cos(-2t)$ <u>or</u> $18 - 18(\cos^2 t - \sin^2 t)$	B1		

		$36\sin^2 t$ **WWW** **AG**	B1	5	
	(ii)	$\int_0^\pi 6\sin t\,dt$ { or $-\int_\pi^0 6\sin t\,dt$ } $-6\cos t$ {or $6\cos t$} **<u>12</u>**	M1 A1 A1	 3	 **8**
8	(i)	Indef int = ke^{-x^2} where $k = -\frac{1}{2}, \frac{1}{2}, -2$ or 2 [May use substitution] $I_1 = -\frac{1}{2e} + \frac{1}{2}$ AEF	M1 A1	 2	
	(ii)	(Indefinite) integration by parts with u = x^{n-1} and dv = xe^{-x^2} $-\frac{1}{2}x^{n-1}e^{-x^2} + \frac{1}{2}(n-1)\int x^{n-2}e^{-x^2}$ (dx) √their indef (i) integral (dep M2) Substituting limits $-\frac{1}{2e} + \frac{n-1}{2} I_{n-2}$ **AG** **WWW**	*M1 +M1 √A1 dep*M1 *A1 dep*A1	 6	
	(iii)	Attempt to use reduction formula once with $n = 3$ $I_3 = \frac{1}{2} - \frac{1}{e}$ or follow through $-\frac{1}{2e} + I_1$ (simplified where necessary)	M1 √ A1	 2	 **10**

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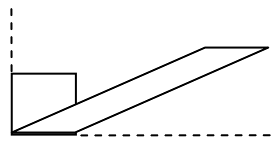
1 Attempt to find $[-2, 1, 2].[3, 1, 1]$ Obtain $(\pm)\,3$ State $\sqrt{9}$ and $\sqrt{11}$ Obtain $17.5°$ or 0.306 rad or 0.0975π rad	M1 A1 B1 A1	 **4**	 Allow greater accuracy Allow ± 1 error in 3rd sig.fig. **4**
2 State $(1+2\mathrm{i} =)\ R(\cos\alpha + \mathrm{i}\sin\alpha)$, where $R=\sqrt{5}=2.236,\ \alpha=\tan^{-1}2=1.107=63.4°$ $r\ (=\sqrt[6]{5}=5^{\frac{1}{6}})=1.31$ (1.30766 ...) Add $2(k)\pi$ to α and divide by 3 $\theta\ (=\frac{1}{3}\tan^{-1}2)=0.369\ (0.36904...)$, $2.46\ (2.46344...)$, $4.56\ (4.55783...)$	B1 B1 M1 B1 A1	 **5**	R and α may be in any form, or values implied by subsequent working Allow greater accuracy throughout Allow ± 1 error in 3rd sig.fig. throughout Allow $360k°$ For any one value of θ For the other two values of θ Allow 0.117π, 0.784π, 1.45π Angles in degrees score B0 A0 **5**
3 (i) $\mathbf{M}^{-1}=\frac{1}{2}\begin{pmatrix}2 & -3\\ 0 & 1\end{pmatrix}$	B1	**1**	
(ii) Multiply out $\mathbf{MUM}^{-1}$ Obtain $\begin{pmatrix}a & d\\ 0 & b\end{pmatrix}$ for any d Obtain $d=\frac{1}{2}(-3a+3b+c)$	M1 A1 A1	 **3**	 **AG**
(iii) $\left(\mathbf{D}^{-1}=\right)\left(\mathbf{MUM}^{-1}\right)^{-1}$ $= \mathbf{MU}^{-1}\mathbf{M}^{-1}$	M1 A1	 **2**	Attempt to use $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$ **6**
4 (a) table below	B2	**2**	Allow B1 for 1 or 2 errors
(b) (i) There are subgroup(s) *OR* element(s) of order 2 2 is not a factor of 5	B1 B1	 **2**	at least one example from $\{e, a\}$, $\{e, b\}$, $\{e, c\}$, $\{e, d\}$ *OR* a, b, c, d must be given Allow "all elements have order 2" Allow corollaries of Lagrange. e.g. order of element divides order of

Table for 4 (a):

	e	r	r^2	r^3	r^4
e	e	r	r^2	r^3	r^4
r	r	r^2	r^3	r^4	e
r^2	r^2	r^3	r^4	e	r
r^3	r^3	r^4	e	r	r^2
r^4	r^4	e	r	r^2	r^3

		group; prime order groups have no subgroups; every group of prime order is cyclic
(ii) Not associative	B1	
e.g. $(ab)d = a,\ a(bd) = d$ e.g. $(bc)d = b,\ b(cd) = d$	M1 A1 **3**	Attempt to find 3 elements which are not associative Table must be used correctly
		7
5 (i) Show closure: $(a+b\sqrt{3})+(c+d\sqrt{3}) =$ $=(a+c)+\sqrt{3}(b+d)$	B1	2 distinct elements must be seen
State identity = $0(+0\sqrt{3})$	B1	
State inverse $=-a-b\sqrt{3}$ *OR* $-(a+b\sqrt{3})$	B1	
Show associativity: $\left[(a+b\sqrt{3})+(c+d\sqrt{3})\right]+(f+g\sqrt{3})$	M1	3 distinct elements seen, bracketed 2+1 or 1+2
$=(a+b\sqrt{3})+\left[(c+d\sqrt{3})+(f+g\sqrt{3})\right]$	A1 **5**	**SR** State only that addition is associative B2
(ii) No identity *OR* no inverse in S' $\Rightarrow S'$ is not a (sub)group	B1* B1 (dep*) **2**	
		7
6 (i) $(\mathbf{A} =) \begin{pmatrix} 1 & 3 \\ 0 & \frac{3}{2} \end{pmatrix}$ seen	B1 **1**	may be part of a matrix equation
(ii) ***EITHER***: Sketch showing the unit square transformed to a parallelogram with vertices (0, 0), (1, 0), (4, $\frac{3}{2}$), (3, $\frac{3}{2}$)	M1	
Stretch and shear	B1	may be implied by next B1s or A1
Stretch, factor $\frac{3}{2}$, parallel to *y*-axis	B1	
Shear in *x*-direction, factor 2 or 3	B1	or equivalent description
EITHER: P = shear (3), Q = stretch	A1	
OR: P = stretch, Q = shear (2)	A1	
OR: Multiply 2 transformation matrices together to attempt to obtain **A**	M1	not necessarily stretch and shear
$\begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}$ and $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ seen	B1	

state Stretch, factor $\frac{3}{2}$, parallel to y-axis	B1	
state Shear in x-direction, factor 2 or 3	B1	
EITHER: P = shear (3), Q = stretch	A1	
OR: P = stretch, Q = shear (2)	A1 **5**	
(iii) Area scale factor of shear (P *OR* Q) = 1	B1	Allow "no effect"
Area scale factor of stretch (Q *OR* P) = $\frac{3}{2}$	B1√ **2**	f.t. from **(ii)**
		8
7 **(i)** *EITHER* Substitute $(1+2s, 2, -1+3s)$ into equation of plane *OR* check 2 points *OR* check 1 point and $[2, 0, 3].[3, -13, -2]$ (= 0)	M1	
Verify equation is satisfied	A1 **2**	
(ii) METHOD 1 Find $[2, 0, 3]\times[3, -13, -2]$	M2	For using l_2 is perpendicular to l_1 and to the normal to P
METHOD 2 For $\mathbf{b}=[x, y, z]$, $[x, y, z].[2, 0, 3]=0 \Rightarrow 2x+3z=0$	M1	
Substitute $[1, 2, -1]+t\mathbf{b}$ into P $\Rightarrow 3x-13y-2z=0$	M1	t may be numerical
METHOD 3 For $\mathbf{b}=[x, y, z]$, $[x, y, z].[2, 0, 3]=0 \Rightarrow 2x+3z=0$	M1	
$[x, y, z].[3, -13, -2]=0 \Rightarrow 3x-13y-2z=0$	M1	
METHOD 4 For $\mathbf{b}=[x, y, z]$, $[x, y, z]\times[2, 0, 3]=[3, -13, -2]$, evaluate vector product and equate components	M1	Allow multiples
$[x, y, z].[2, 0, 3]=0 \Rightarrow 2x+3z=0$	M1	
METHOD 5 For $\mathbf{b}=[x, y, z]$, $[x, y, z]\times[3, -13, -2]=[2, 0, 3]$, evaluate vector product and equate components	M1	Allow multiples
$[x, y, z].[3, -13, -2]=0 \Rightarrow 3x-13y-2z=0$	M1	
State $\mathbf{a}=[1, 2, -1]$	B1	
Obtain $\mathbf{b}=[3, 1, -2]$	A1 **4**	or any multiple
(iii) Plane Q has $\mathbf{n}=[3, 1, -2]$	M1	for using direction of **n** = direction of l_2, or for using any other valid method

Use any point to obtain $\mathbf{r}.[3, 1, -2] = 7$	A1 **2**	Allow multiples
(iv) $\mathbf{a} = [1, 2, -1]$ $\mathbf{b} = [3, -13, -2]$ $\mathbf{c} = [3, 1, -2]$	B1 B1 B1√ **3**	or other point in the plane **b** and **c** may be either way round f.t. from **(ii)** or **(iii)** Allow any 2 linear combinations of **b** and **c**, i.e. such that $\mathbf{b} \times \mathbf{c} = k[2, 0, 3]$, e.g. $[0, 1, 0]$ **SR** If B0 B0 for **b** and **c**, allow B1 for stating, in any form, that the normal to the plane is $[2, 0, 3]$ **11**
8 (i) Expand (real part of) $(c + is)^4$ to obtain $(\cos 4\theta =)\ c^4 - 6c^2s^2 + s^4$ Use $c^2 = 1 - s^2$ Obtain $8\sin^4\theta - 8\sin^2\theta + 1$	M1 M1 A1 **3**	at least 3 terms and 1 binomial coefficient needed **AG**
(ii) $\cos 4\theta = 0$ $\theta = \frac{1}{8}\pi \Rightarrow 8x^4 - 8x^2 + 1 = 0$ *OR* $8x^4 - 8x^2 + 1 = 0 \Rightarrow \theta = \frac{1}{8}\pi$ Obtain $(x =) \sin\frac{3}{8}\pi \left(= \sin\frac{5}{8}\pi\right)$ $\sin\frac{9}{8}\pi\left(= -\sin\frac{1}{8}\pi\right)$, $\sin\frac{11}{8}\pi\left(= -\sin\frac{3}{8}\pi\right)$	M1 B1 A1 A1 A1 **5**	Award at any stage in this part **AG** For one root For a second root For the third root, provided no incorrect roots are included **AEF** Ignore repeated roots e.g. $\sin\frac{7}{8}\pi$ **SR** If "sin" is omitted award maximum of A1 A1 A0 **SR** If n is an unqualified integer, award A1 for $\sin\frac{2n+1}{8}\pi$ **SR** Allow "otherwise", e.g. $x = \sin\theta \Rightarrow \sin 2\theta = \pm\frac{1}{\sqrt{2}}$
(iii) Solve $8x^4 - 8x^2 + 1 = 0$ for x^2 Obtain $(x =) \pm\frac{1}{4}\sqrt{8 \pm \sqrt{32}}$ *OR* $\pm\frac{1}{2}\sqrt{2 \pm \sqrt{2}}$ Identify $\sin\frac{1}{8}\pi$ as smallest (+ve) value $\frac{1}{2}\sqrt{2 - \sqrt{2}}$ Obtain $\sin\frac{11}{8}\pi = -\frac{1}{2}\sqrt{2 + \sqrt{2}}$	M1 A1 A1 B1 **4**	 **AEF** (all 4 values required) **AG** Justification is required (calculator allowed if figures seen) **AEF** Justification is not necessary **12**

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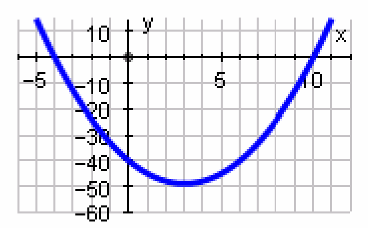
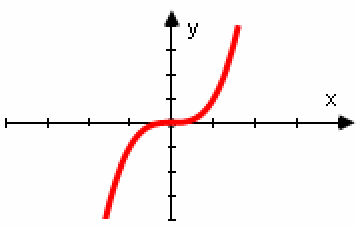
1	$x^2 - 6x - 40 \geq 0$ $(x+4)(x-10) \geq 0$  $x \leq -4, \quad x \geq 10$	M1 A1 M1 A1 4 **4**	Correct method to find roots -4, 10 Correct method to solve quadratic inequality e.g. +ve quadratic graph $x \leq -4, \quad x \geq 10$ (not wrapped, not strict inequalities, no 'and')
2(i)	EITHER $3(x^2+4x)+7$ $3(x+2)^2 - 12 + 7$ $3(x+2)^2 - 5$ OR $3(x^2+2ax+a^2)+b$ $3x^2 + 6ax + 3a^2 + b$ $6a = 12$ $a = 2$ $3a^2 + b = 7$ $b = -5$	M1 A1 M1 A1 4	$a = \frac{12}{6 \text{ or } 2}$ a = 2 $7 - a^2$ or $7 - 3a^2$ or $\frac{7}{3} - a^2$ (their a) b = –5
(ii)	$x = -2$	B1 ft 1 **5**	$x = -2$
3 (i)		B1 1	Correct sketch showing point of inflection at origin
(ii)	Reflection in *x*-axis or reflection in *y*-axis	B1 B1 2	Reflection In *x*-axis or *y*=0 or *y*-axis or *x*=0
(iii)	$y = (x-p)^3$	M1 A1 2 **5**	$y = (x \pm p)^3$ $y = (x-p)^3$

4	$k = x^3$ $k^2 + 26k - 27 = 0$ $k = -27, 1$ $x = -3, 1$	*M1 A1 A1 DM1 A1 5 **5**	Attempt a substitution to obtain a quadratic $k^2 + 26k - 27 = 0$ –27, 1 Attempt cube root x = –3, 1 (no extras) **(SR:** x = 1 seen www **B1** x = -3 seen www **B1)**
5 (a) (b) (c)	$2x^{\frac{2}{3}} \times 3x^{-1}$ $= 6x^{\frac{-1}{3}}$ $2^{40} \times 4^{30}$ $= 2^{40} \times 2^{60}$ $= 2^{100}$ $\frac{26\left(4+\sqrt{3}\right)}{\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right)}$ $= 8 + 2\sqrt{3}$	M1 A1 2 M1 A1 2 M1 A1 A1 3 **7**	Adds indices $6x^{\frac{-1}{3}}$ 2^{60} or 4^{20} 2^{100} Multiply top and bottom by $\left(4+\sqrt{3}\right)$ or $\left(-4-\sqrt{3}\right)$ $\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right) = 13$ $8 + 2\sqrt{3}$
6 (i) (ii) (iii)	$\left(x^2 + 2x + 1\right)\left(3x - 4\right)$ $= 3x^3 + 2x^2 - 5x - 4$ $9x^2 + 4x - 5$ $18x + 4$	M1 A1 A1 3 B1 ft B1 ft 2 M1 A1 ft 2 **7**	Expand 2 brackets to give an expression of the form $ax^2 + bx + c$ ($a \neq 0$, $b \neq 0$, $c \neq 0$) and attempt to multiply by third bracket $3x^3 + 2x^2 - 5x - 4$ 3 correct simplified terms Completely correct $9x^2 + 4x - 5$ 1 term correct Completely correct (3 terms) Attempt to differentiate their (ii) $18x + 4$ (2 terms) **(SR** (ii) $3ax^2 + 2bx + c$ **B1** (iii) $6ax + 2b$ **B1)**

7 (i)	b^2-4ac (a) 36 – 9 x 4 = 0 (b) 100 – 48 = 52 (c) 4 – 20 = -16	M1 A1 A1 3	Uses b^2-4ac 1 correct 3 correct **SR** All 3 values correct but √ used **B1**
(ii)	(a) Fig 3 (b) Fig 2 (c) Fig 5 (a) 1 root, touches *x*-axis once, line of symmetry *x*= -3 or root *x* =-3 (b) 2 roots, meets *x*-axis twice, line of symmetry *x*=5 (c) No real roots, does not meet *x*-axis	B1 B1 B1 B1 4 **7**	1 correct matching 3 correct matchings 1 correct comment relating roots to touching/crossing *x*-axis or about line of symmetry or vertex o.e. for one graph 2 further correct comments about roots, line of symmetry o.e. for the other 2 graphs
8 (i)	Circle, centre (0, 0), radius 5	B1 B1 2	Circle centre (0, 0) Radius 5
(ii)	$y=5-2x$ $x^2+(5-2x)^2=25$ $5x^2-20x=0$ OR $x=\frac{5-y}{2}$ $\frac{(5-y)^2}{4}+y^2=25$ $y^2-2y-15=0$ $x=0,4$ $y=5,-3$	M1 *M1 DM1 M1 A1 A1 6 **8**	Attempt to solve equations simultaneously Substitute for *x*/*y* or correct attempt at elimination of one variable (NOT for 2 linear equations) Obtain quadratic $ax^2+bx+c=0$ ($a\neq 0$, $b\neq 0$) Correct method to solve quadratic $x=0,4$ or $y=5,-3$ $y=5,-3$ or $x=0,4$ **SR** one correct pair **www** **B1** <u>SR</u> If solution by graphical methods: Drawing circle, centre (0,0) radius 5 B1 Drawing line B1 Looking for intersection M1 (0,5) correct A1 (4, –3) correct A2

9 (i)	$y=\frac{4}{3}x+\frac{5}{3}$ gradient $=\frac{4}{3}$	B1 1	$\frac{4}{3}$ or 1.33 or better
(ii)	gradient of $\perp^r=-\frac{3}{4}$	B1 ft	$-\frac{3}{4}$ seen or implied
	$y-2=-\frac{3}{4}(x-1)$	M1	Attempts equation of straight line through (1, 2) with any gradient
	$4y+3x=11$	A1	$y-2=-\frac{3}{4}(x-1)$
		A1 4	$3x+4y-11=0$ (not aef)
(iii)	$P\left(-\frac{5}{4},0\right)$	B1	$\left(-\frac{5}{4},0\right)$ seen or implied
	$Q\left(0,\frac{11}{4}\right)$	B1 ft	$\left(0,\frac{11}{4}\right)$ seen or implied (from a straight line equation in (ii))
	$\left(-\frac{5}{8},\frac{11}{8}\right)$	B1 ft 3	$\left(-\frac{5}{8},\frac{11}{8}\right)$ aef
(iv)	$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$	M1	Correct method to find line length using Pythagoras' theorem
	$\frac{\sqrt{146}}{4}$	A1	$\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{11}{4}\right)^2}$
		A1 3	$\frac{\sqrt{146}}{4}$
		11	

10 (i)	$\frac{dy}{dx} = x^2 - 9$	B1 B1 2	$x^2 - 9$ 1 term correct Both terms correct
(ii)	$x^2 - 9 = 0$ $x = 3, -3$ $y = -18, 18$	*M1 A1 A1 3	uses $\frac{dy}{dx} = 0$ $x = 3, -3$ $y = -18, 18$ (1 correct pair A1 A0)
(iii)	$\frac{d^2y}{dx^2} = 2x$ $x = 3 \quad \frac{d^2y}{dx^2} = 6$ $x = -3 \quad \frac{d^2y}{dx^2} = -6$	DM1 A1 A1 3	Looks at sign of $\frac{d^2y}{dx^2}$ or other correct method $x = 3$ minimum $x =$ -3 maximum (N.B. If no method shown but min and max correctly stated, award all 3 marks unless earlier incorrect working)
(iv)	gradient of $24x + 3y + 2 = 0$ is -8 $x^2 - 9 = -8$ $x = \pm 1$ For line $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 7\frac{1}{3}$ For curve $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 8\frac{2}{3}$ $\therefore p = 1,\ q = -8\frac{2}{3}$	B1 M1 M1 M1 A1 5 **13**	Gradient = – 8 $x^2 - 9 = -8$ one of their x values substituted in both line <u>and</u> curve second x value substituted in both line and curve **<u>or</u>** justification that first point is the correct one $p = 1,\ q = -8\frac{2}{3}$ seen <u>Alternative methods:</u> <u>Either:</u> Solve equations for curve and line simultaneously to get one solution (either x = 1 or x = -2) M1 Gradient of line = -8 B1 Substitution of one x value into their gradient formula and check for -8 M1 Substitution of other x value into gradient formula and check for -8 or justification as above M1 Correct q value A1 <u>Or:</u> Solve equations for curve and line simultaneously to get one solution M1 Factorise to $(x-1)^2(x+2)$ B1 State that a double root implies a tangent at x = 1 M2 Correct value for y A1

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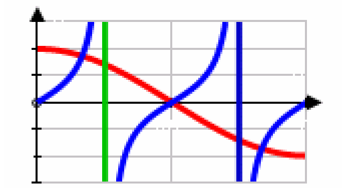
1	(i)	$u_1 = 2$, $u_2 = 5$, $u_3 = 8$ The sequence is an Arithmetic Progression	B1 B1 B1 3	For the correct value of u_1 For both correct values of u_2 and u_3 For a correct statement (any mention of arithmetic)
	(ii)	$\frac{1}{2} \times 100 \times (2 \times 2 + 99 \times 3) = 15050$	M1 M1 A1 3 **6**	For correct interpretation of Sigma notation – ie finding the sum of an AP or GP For use of correct $\frac{1}{2}n(2a + (n-1)d)$, or equiv, with n=100 and a & d not both =1 For correct value 15050
2	(i)	$r\theta = 12$, $\frac{1}{2}r^2\theta = 36$	B1 B1 2	For $r\theta = 12$ stated correctly at any point For $\frac{1}{2}r^2\theta = 36$ stated correctly at any point
	(ii)	$\frac{1}{2}r \times 12 = 36 \Rightarrow r = 6$ Hence $\theta = 2$	B1 B1 2	For showing given value correctly For correct value 2 (or 0.637π)
	(iii)	Segment area is $36 - \frac{1}{2} \times 6^2 \times \sin 2 = 19.6$ cm^2	M1* M1dep* A1 3 **7**	For use of $\Delta = \frac{1}{2}ab \sin C$, or equivalent For attempt at $36 - \Delta$ For correct value (rounding to) 19.6
3	(i)	$\int (2x^2 + 7x + 3)\,dx$ $= \frac{2}{3}x^3 + \frac{7}{2}x^2 + 3x + c$	M1 A1 A1 B1 4	For expanding and integration attempt For at least one term correct For all three terms correct For addition of arbitrary constant, and no $\int$ or dx
	(ii)	$\left[2x^{\frac{1}{2}}\right]_0^9$ = 6	M1 M1 A1 3 **7**	For integral of the form $kx^{\frac{1}{2}}$ For evaluating at least F(9), following attempt at integration For final answer of 6 only
4	(i)	$\cos BCA = \frac{5^2+6^2-9^2}{2\times5\times6} = -\frac{1}{3}$ So $\sin BCA = \frac{2}{3}\sqrt{2} \approx 0.9428 \ldots$	M1 M1 A1 B1 M1 M1 A1 B1 4	For relevant use of the correct cosine formula For attempt to rearrange correct formula For obtaining the given value correctly For correct answer for sin BCA in any form OR For substituting cos BCA = $-^1/_3$ For attempt at evaluation For full verification For correct answer for sin BCA in any form
	(ii)	Angles BCA and CAD are equal So $\sin ADC = \frac{5}{15}\sin CAD = \frac{1}{3} \times \frac{1}{3}\sqrt{8} = \frac{2}{9}\sqrt{2}$ $\Rightarrow ADC = 18.3°$	B1 M1 A1√ A1 4 **8**	For stating, using or implying the equal angles For correct use of the sine rule in Δ ADC (sides must be numerical, angles may still be in letters) For a correct equation from their value in (i) For correct answer, from correct working
5	(i)	f(-1) = 0 $\Rightarrow$ $-1 - a + b = 0$ f(3) = 16 $\Rightarrow$ $27 + 3a + b = 16$ Hence $a = -3$, $b = -2$	M1 A1 M1 A1 A1 5	For equating their attempt at f(-1) to 0, or equiv For the correct (unsimplified) equation For equating their attempt at f(3) to 16, or equiv For the correct (unsimplified) equation For both correct values – must follow two correct equations
	(ii)	f(2) = 8 – 6 – 2 = 0	B1	For the correct verification (from correct a &

			M1	b)
		Hence $f(x) = (x+1)^2(x-2)$		For recognition or use of two linear factors, or full division attempt by either $(x+1)$ or $(x-2)$
			A1 3	For correct third factor (repeated) of $(x+1)$, and full linear factorisation stated
			8	
6	(i)	$x^6 + 3x^3 + 3 + \frac{1}{x^3}$	M1	For 4 term binomial attempt or equiv
			A1	For any one (unsimplified) term correct
			A1	For any other (unsimplified) term correct
			A1 4	For full, simplified, expansion correct
	(ii)	$\frac{1}{7}x^7 + \frac{3}{4}x^4 + 3x - \frac{1}{2}x^{-2} + c$	M1	For any correct use of $\frac{x^{n+1}}{n+1}$
			A1√	For any two terms integrated correctly
			M1	For any correct use of x^{n+1} using a negative index
			A1√ 4	For all terms integrated correctly (must have at least 4 terms, including at least 1 negative index)
			8	[No penalty for omission of $+c$ in this part]
7	(i)	$\log_5\left(\frac{15\times20}{12}\right) = \log_5 25 = 2$	M1	For any relevant combination of $\log a \pm \log b$
			A1	For log 25 – must follow correct working only
			A1 3	For correct answer 2
	(ii)	Method A $\frac{1}{3}y = 10^{2x}$	M1	For correct division of both sides by 3
		Hence $2x = \log_{10}\left(\frac{1}{3}y\right)$	M1	For relevant use of $a = b^c \Leftrightarrow c = \log_b a$
			A1	For correct equation involving logs to base 10
		i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	A1 4	For correct answer for x
		Method B $\frac{1}{3}y = 10^{2x}$	M1	For correct division of both sides by 3
		$\log\frac{1}{3}y = \log 10^{2x}$	M1	For taking logs of both sides
		$\log\frac{1}{3}y = 2x\log 10$	A1	For correct linear equation involving logs
		i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$	A1 4	For correct answer for x
		Method C $y = 3\times10^{2x} \Rightarrow \log y = \log 3\times10^{2x}$	M1	For introducing logs throughout
			A1	For correct RHS $\log 3 + \log 10^{2x}$
		$\log y = \log 3 + \log 10^{2x}$	M1	For correct use of $\log a^b = b \log a$
		$\log y = \log 3 + 2x\log 10$	A1 4	For correct answer for x
		i.e. $x = \frac{1}{2}\log_{10}\left(\frac{1}{3}y\right)$		
		Method D	M1	For substituting for y, and separating RHS into at least 2 terms
		$x = a\log(b\times3\times10^{2x})$	M1	For attempting values for a and b
		$x = a\log 3b + a\log 10^{2x}$	A1	For obtaining $a = \frac{1}{2}$
		$x = 2ax\log 10 \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$	A1 4	For obtaining $b = \frac{1}{3}$
		$a\log 3b = 0 \Rightarrow 3b = 1 \Rightarrow b = \frac{1}{3}$	**7**	
8	(i)	$100\,000 \times 0.9^3 = 72900$	M1	For relevant use of ar^3 or equiv
			A1 2	For the correct answer 72900
	(ii)	$100\,000 \times 0.9^x = 5000$	B1	For a correct equation or inequality
		Hence $x\log 0.9 = \log 0.05$	M1	For complete solution method by logs or trial
		So x = 28.4, 28 or 29; or n = 29.4, 29 or 30	A1	For correct solution for **their** index – allow integer values either side
		i.e. 30th year / 30 years / year is 2030	A1√ 4	For correctly linking **their** index to date or

					number of years
	(iii)	Total is $\frac{100000(1-0.9^{30})}{1-0.9} = 957609$	M1 A1√ A1	 3 **9**	For relevant use of $\frac{a(1-r^n)}{1-r}$ For correct (unsimplified) statement for their integer n (if no n stated then use their year – 2000) For answer 958000 or better, including decimal
9	(a) (i)	$\cos \frac{1}{6}\pi = \frac{1}{2}\sqrt{3}$ $\tan \frac{1}{3}\pi = \sqrt{3}$ Hence $2\cos\frac{1}{6}\pi = 2 \times \frac{1}{2}\sqrt{3} = \tan\frac{1}{3}\pi$	B1 B1 B1	 3	For any correct exact value For any correct exact value For correct verification (allow via decimals)
	(ii)	 Other roots are $\pi/2$ and $5\pi/6$	B1 B1 B1 B1	 4	For correct sketch of either $y = \tan 2x$ or $y = 2\cos x$ For second correct sketch, with both graphs in proportion (ie 3 points of intersection) For one of $\pi/2$ or $5\pi/6$ (or equiv in degrees) For second correct value, and no others in range $0 \le x \le \pi$
	(b) (i)	0.05(0.1003 + 2(0.2027 + 0.3093) + 0.4228) = 0.0774	M1 M1 A1 A1	 4	State at least three of tan 0.1, tan 0.2, tan 0.3, tan 0.4 Substitute numerical values (must be attempt at y-coords, not x-coords) into correct trapezium rule, with h consistent with number of strips Obtain 0.05(tan 0.1 + 2(tan 0.2 + tan 0.3)+ tan 0.4) or equiv in decimals (SC – award A1 if values are now decimals from using degrees – gives final answer of 0.00131) Obtain 0.077 or better
	(ii)	Overestimate; tops of trapezia above the curve or equiv	B1	1 **12**	For correct statement and justification

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1	(i)	State $f(x) \le 10$	B1	**1** [Any equiv but must be or imply ≤]
	(ii)	Attempt correct process for composition of functions	M1	[whether algebraic or numerical]
		Obtain 6 or correct expression for $ff(x)$	A1	
		Obtain -71	A1	**3**
2		Either Obtain $x = 0$	B1	[ignoring errors in working]
		Form linear equation with signs of $6x$ and x different	M1	[ignoring other sign errors]
		State $6x - 1 = -x + 1$	A1	[or correct equiv with or without brackets]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	**4** [or exact equiv]
	Or	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$	B1	[or equiv]
		Attempt to solve quadratic equation	M1	[as far as factorisation or subn into formula]
		Obtain $\frac{2}{7}$ and no other non-zero value	A1	[or exact equiv]
		Obtain 0	B1	**(4)** [ignoring errors in working]
3	(i)	Attempt solution involving (natural) logarithm	M1	
		Obtain $-0.017t = \ln\frac{25}{180}$	A1	[or equiv]
		Obtain 116	A1	**3** [or greater accuracy rounding to 116]
	(ii)	Differentiate to obtain $k\,e^{-0.017t}$	M1	[any constant k different from 180; solution must involve differentiation]
		Obtain correct $-3.06e^{-0.017t}$	A1	[or unsimplified equiv; accept + or –]
		Obtain 1.2	A1	**3** [or greater accuracy; accept + or – answer]
4	(a)	State or imply $\int \pi y^2 \, dx$	B1	
		Integrate to obtain $k \ln x$	M1	[any constant k, involving π or not; or equiv such as $k \ln 4x$]
		Obtain $4\pi \ln x$ or $4 \ln x$	A1	[or equiv]
		Obtain $4\pi \ln 5$	A1	**4** [or similarly simplified equiv]

	(b)	Attempt calculation involving attempts at y values	M1	[with each of 1, 4, 2 present at least once as coefficients]
		Attempt $\frac{1}{3} \times 1(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$	M1	[with attempts at five y values]
		Obtain $\frac{1}{3}(\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26})$	A1	[or exact equiv or decimal equivs]
		Obtain 12.758	A1	**4** [or greater accuracy]
5	**(i)**	Obtain $R = \sqrt{13}$, or 3.6 or 3.61 or greater accuracy	B1	
		Attempt recognisable process for finding α	M1	[allow sine/cosine muddles]
		Obtain $\alpha = 33.7$	A1	**3** [or greater accuracy]
	(ii)	Attempt to find at least one value of $\theta + \alpha$	*M1	
		Obtain value rounding to 76 or 104	A1√	[following their R]
		Subtract their α from at least one value	M1	[dependent on ***M**]
		Obtain one value rounding to 42 or 43, or to 70	A1	
		Obtain other value 42.4 or 70.2	A1	**5** [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]
6	**(a)**	Attempt use of product rule	*M1	
		Obtain $\ln x + 1$	A1	[or unsimplified equiv]
		Equate attempt at first derivative to zero and obtain value involving e	M1	[dependent on ***M**]
		Obtain e^{-1}	A1	**4** [or exact equiv]
	(b)	Attempt use of quotient rule	M1	[or equiv using product rule or …]
		Obtain $\frac{(4x-c)4 - 4(4x+c)}{(4x-c)^2}$	A1	[or equiv]
		Show that first derivative cannot be zero	A1	**3** [**AG**; derivative must be correct]
7	**(i)**	State $2\cos^2 x - 1$	B1	**1**
	(ii)	Attempt to express left hand side in terms of $\cos x$	M1	[using expression of form $a\cos^2 x + b$]
		Identify $\frac{1}{\cos x}$ as $\sec x$	M1	[maybe implied]

		Confirm result	A1	**3** [**AG**; necessary detail required]
	(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$	B1	
		Attempt solution of quadratic equation in tan x	M1	[or equiv]
		Obtain $2\tan^2 x + 3\tan x - 9 = 0$ and hence tan $x = -3,\ \frac{3}{2}$	A1	
		Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of 0.313π, 1.31π, 0.602π, 1.60π)	A1	[allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi$, $1.89 + \pi$ allow degrees: 56, 236, 108, 288]
		Obtain all four solutions	A1	**5** [now with at least 3 s.f.; must be radians; no other solutions in the range 0 - 2π; ignore solutions outside range 0 - 2π]
8	**(i)**	Attempt relevant calculations with 5.2 and 5.3	M1	
		Obtain correct values	A1	x y_1 y_2 $y_1 - y_2$ 5.2 2.83 2.87 − 0.04 5.3 2.89 2.88 0.006
		Conclude appropriately	A1	**3** [**AG**; comparing y values or noting sign change in difference in y values or equiv]
	(ii)	Equate expressions and attempt rearrangement to $x =$	M1	
		Obtain $x = \frac{5}{3}\ln(3x+8)$	A1	**2** [**AG**; necessary detail required]
	(iii)	Obtain correct first iterate	B1	
		Carry out correct process to find at least two iterates in all	M1	
		Obtain 5.29	A1	**3** [must be exactly 2 decimal places; 5.2→5.2687→5.2832→5.2863→5.2869; 5.25→5.2793→5.2855→5.2868→5.2870; 5.3→5.2898→5.2877→5.2872→5.2871]
	(iv)	Obtain integral of form $k(3x+8)^{\frac{4}{3}}$	M1	
		Obtain integral of form $k\,e^{\frac{1}{5}x}$	M1	

		Obtain $\frac{1}{4}(3x+8)^{\frac{4}{3}} - 5e^{\frac{1}{5}x}$	A1	[or equiv]
		Apply limits 0 and their answer to **(iii)**	M1	[applied to difference of two integrals]
		Obtain 3.78	A1	**5** [or greater accuracy]
9	**(i)**	Indicate stretch and (at least one) translation	M1	[… in general terms]
		State translation by 7 units in negative x direction	A1	[or equiv; using correct terminology]
		State stretch in x direction with factor $1/m$	A1	[must follow the translation by 7; or equiv; using correct terminology]
		Indicate translation by 4 units in negative y direction	B1	**4** [or equiv; at any stage; the two translations may be combined]
	(ii)	Refer to each y value being image of unique x value	B1	[or equiv]
		Attempt correct process for finding inverse	M1	
		Obtain expression involving $(x+4)^2$ or $(y+4)^2$	M1	
		Obtain $\frac{(x+4)^2-7}{m}$	A1	**4** [or equiv]
	(iii)	Refer to fact that curves are reflections of each other in line $y = x$	B1	[or equiv]
		Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	M1	
		Apply discriminant to resulting quadratic equati on	M1	
		Obtain $(m-2)(m-14) < 0$	A1	[or equiv]
		Obtain $2 < m < 14$	A1	**5**

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1	(Quotient =) $x^2 + 2x + 2$ (Remainder =) $0x – 3$ Allow without working	B1 M1 A1 A1 **4**	For correct leading term x^2 in quotient For evidence of division/identity process For correct quotient For correct remainder. The '0*x*' need not be written but must be clearly derived. **4**
2	 $x \sin x – \int \sin x \, dx$ $(= x \sin x + \cos x)$ Answer = $\frac{1}{2}\pi – 1$	M1 A1 B1 M1 A1 **5**	For attempt at parts going correct way (u = x, dv = cos x and f(x) +/–∫g(x) (dx) For both terms correct Indic anywhere that $\int \sin x \, dx = – \cos x$ For correct method of limits For correct exact answer ISW **5**
3	(i) **r** = (2**i**-3**j**+**k** or –**i**–2**j**–4**k**) + t(3**i**–**j**+5**k**) (ii) *L*(2) (**r**) = 3**i**+2**j**–9**k**+s(4**i** – 4**j** + 5**k**) *L(1)&L(2)* must be of form **r** = **a** + t**b** 2+3t=3+4s, –3–t=2–4s,1+5t= – 9+5s or suitable equivalences (t,s) = (+/-3,2) or (–/+1,1) or (–/+9,-7) or (+/–4,2) or (0,1) or (–/+8,-7) Basic check other eqn & interp √	M1 A1 **2** M1 M1 M1 A1 B1 **5**	For (either point) + t(diff betw vectors) Completely correct including **r** =. AEF For point + (s or t) direction vector For 2/3 eqns with 2 different parameters For solving any relevant pair of eqns For both parameters correct **7**
4	(i) $dx = \sec^2\theta \, d\theta$ AEF Indefinite integral = $\int \cos^2\theta \, d\theta$ (ii) = $k\int$ +/– 1 +/– $\cos 2\theta \, d\theta$ $\frac{1}{2}[\theta + \frac{1}{2}\sin 2\theta]$ Limits = $\frac{1}{4}\pi$(accept 45) and 0 $(\pi + 2)/8$ AEF	M1 A1 A1 **3** M1 A1 M1 A1 **4**	Attempt to connect d*x*,dθ (not dx = dθ) For d*x* = sec²θ dθ or equiv correctly used With at least one intermed step **AG** "Satis" attempt to change to double angle Correct attempt + correct integration New limits for θ or resubstituting Ignore decimals after correct answer **7** Single 'parts' + sin²θ=1–cos²θ acceptable
5	(i)**OD**=**OA**+**AD** or **OB**+**BC**+**CD** AEF **AD** = **BC** or **CD** = **BA** (**a** + **c** – **b**) = 2**j** + **k** (ii) **AB.CB** = |**AB**||**CB**| cos θ Scalar product of <u>any</u> 2 vectors Magnitude of <u>any</u> vector 94°(94.386…) or 1.65 (1.647…)	M1 A1 A1 **3** M1 M1 M1 A1 **4**	Connect **OD** & 2/3/4 vectors in their diag Or similar ,from their diag [i.e.if diag mislabelled, M1A1A0 possible] Or **AB.BC** i.e.scalar prod for correct pair 2 + 3 – 6 = –1 is expected √19 or 3 expected Accept 86°(85.614…) or 1.49(424..) **7**
6	(i) For $d/dx\,(y^2) = 2y\,dy/dx$ Using d(uv) = u dv + v du $2xy\,dy/dx + y^2 = 2 + 3\,dy/dx$ $dy/dx = (2 – y^2)/(2xy – 3)$	B1 M1 A1 M1 A1 **5**	 Solving an equation,with at least 2 dy/dx terms, for d*y*/d*x*; d*y*/d*x* on one side, non d*y*/d*x* on other. **AG**

	(ii) Stating/using $2xy - 3 = 0$ Attempt to eliminate x or y $8x^2 = -9$ or $y^2 = -2$	B1 M1 A1 **3**	No use of $2 - y^2$ in this part. Between $2xy - 3 = 0$ & eqn of curve Together with suitable finish **8**
7	(i)$dy / dx = (dy/dt) / (dx/dt)$ $= (-1/t^2) / 2t$ as unsimplified expression $= -1 / 2t^3$ as simplified expression (ii) $(4, -1/2) \rightarrow t = -2$ <u>only</u> Satis attempt to find equation of tgt $x - 16y = 12$ <u>only</u> (iii) $t^3 - 12t - 16 = 0$ <u>or</u> $16y^3 + 12y^2 - 1 = 0$ <u>or</u> $x^3 - 24x^2 + 144x - 256 = 0$ $t = 4$ (only) ISW giving cartesian coords	M1 A1 A1 **3** B1 M1 A1 **3** M1 A1 B2 **4**	(S.R.Award M1 for attempt to change to cartesian eqn & differentiate + A1 for dy/dx or dx/dy in terms of x or y) Not $1/-2t^3$. Not in terms of x &/or y. Using $t = -2$ or 2 **AG** For substituting $(t^2, 1/t)$ into tgt eqn <u>or</u> solving simult tgt & their cartes eqns For simplified equiv non-fract cubic S.R. Award B1 for "4 or –2". S.R. If B0, award M1 for clear indic of method of soln of correct eqn. **10**
8	(i) $3x+4 \equiv A(2+x)^2+B(2+x)(1+x) + C(1+x)$ $A = 1$ $C = 2$ $A+B = 0$ or $4A+3B+C=3$ or $4A+2B+C = 4$ $B = -1$ (ii) $1 - x + x^2$ $1 - \frac{1}{2}x + \frac{1}{4}x^2$ $1 - x$ $+ \frac{3}{4}x^2$ $1 - 5/4\,x + 5/4\,x^2$ (iii) $-1 < x < 1$ AEF	M1 A/B1 A/B1 A1 A1 **5** B1 B1 B1 B1 B1 **5** B1 **1**	Accept ≡ or = If identity used, award 'A' mark, if cover-up rule used, award 'B' mark. <u>Any</u> correct eqn for B from identity Expansion of $(1 + x)^{-1}$ Expansion of $(1 + \frac{1}{2}x)^{-1}$ First 2 terms of $(1 + \frac{1}{2}x)^{-2}$ Third term of $(1 + \frac{1}{2}x)^{-2}$ Complete correct expansion <u>If partial fractions not used</u> Award B1 for expansion of $(1+x)^{-1}$ B1+B1 for expansion of $(1 + \frac{1}{2}x)^{-2}$, and B1 for $1 - 5/4x\ldots$ & B1 for $\ldots + 5/4x^2$ <u>Or</u> if denom expanded to give $a+bx+cx^2$ with a=4.b=8,c=5,award B1 Expansion of $[1+(b/a\}x+(c/a)x^2]^{-1} =$ $1 - (b/a)x + \ldots (-c/a + b^2/a^2)x^2$ B1+B1 Final ans = $(1 - 5/4x\ldots + 5/4x^2)$B1+B1 Other inequalities to be discarded. **11**
9	k = const of proportionality – = falling, $d\theta/dt$ = rate of change $\theta - 20$ = diff betw obj & surround temp (ii) $\int 1/(\theta - 20)\, d\theta = -k \int dt$ $\ln(\theta - 20) = -kt + c$ Subst $(\theta, t) = (100,0)$ or $(68,5)$	B2 **2** M1 A1A1 M1 A1	All 4 items (first two may be linked) S.R. Award B1 for any 2 items For separating variables For integ each side (c not essential) Dep on 'c' being involved [<u>or</u> M2 for limits (100,0) (68,5) + A1 for

	c = ln 80 k = 1/5 ln 5/3 $\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}$ (iii) Substitute θ = 68 – 32 *t* = 15.75 Extra time = 10.75, √their 15.75 – 5	A1 M1 A1 **8** M1 A1 B1 **3**	k] *AG* Subst into AEF of given eqn & solve Accept 15.7 or 15.8 f.t. only if θ = their (68 – 32) or 32 **13**

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1.	$6\Sigma r^2 + 2\Sigma r + \Sigma 1$	M1		Consider the sum of three separate terms
	$6\Sigma r^2 = n(n+1)(2n+1)$	A1		Correct formula stated
	$2\Sigma r = n(n+1)$	A1		Correct formula stated
	$\Sigma 1 = n$	A1		Correct term seen
	$n(2n^2 + 4n + 3)$	M1	6	Correct algebraic processes including factorisation and simplification
		A1	**6**	Obtain given answer correctly
2.	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1		Attempt to find A^2, 2 elements correct
		A1		All elements correct
	$4\mathbf{A} = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1		Use correct matrix $4\mathbf{A}$
	$\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$	A1	4	Obtain given answer correctly
	(ii)			
	$\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	M1	2	Multiply answer to (i) by $\mathbf{A}^{-1}$ or obtain $\mathbf{A}^{-1}$ or factorise $\mathbf{A}^2 - 4\mathbf{A}$
		A1	**6**	Obtain given answer correctly
3.	(i) $22 - 2i$	B1B1	2	Correct real and imaginary parts
	(ii) $z^* = 2 - 3i$	B1		Correct conjugate seen or implied
	$5 - 14i$	B1B1	3	Correct real and imaginary parts
	(iii) $\frac{4}{17} + \frac{1}{17}i$	M1		Attempt to use w^*
		A1	2	Obtain correct answer in any form
			7	

4.	$x^2 - y^2 = 21$ and $xy = -10$ $\pm(5 - 2\text{i})$	M1 A1A1 M1 M1 A1	 6 **6**	Attempt to equate real and imaginary parts of $(x + \text{i}y)^2$ and 21 –20i Obtain each result Eliminate to obtain a quadratic in x^2 or y^2 Solve to obtain $x = (\pm)\,5$ or $y = (\pm)\,2$ Obtain correct answers as complex numbers
5.	(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$ $\dfrac{1}{(r+1)(r+2)}$ (ii) *EITHER* $\dfrac{2}{3} - \dfrac{1}{2} + \dfrac{3}{4} - \dfrac{2}{3} \ldots \dfrac{n+1}{n+2} - \dfrac{n}{n+1}$ $\dfrac{n+1}{n+2} - \dfrac{1}{2}$ *OR* (iii) $\dfrac{1}{2}$	M1 A1 M1 A1 M1 A1 M2 A1A1 B1 ft	 2 4 1 **7**	Show correct process for subtracting fractions Obtain given answer correctly Express terms as differences using (i) At least first two and last term correct Show or imply that pairs of terms cancel Obtain correct answer in any form State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$ Each term correct Obtain value from their sum to n terms
6.	(i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope (ii) 0 or 0 +0i and 2 + 2i	B1 B1 B1 B1 B1 B1ftB1ft	 5 2 **7**	Sketch(s) showing correct features, each mark independent Obtain intersections as complex numbers
8.	(a) (i) $\alpha + \beta = 2 \quad \alpha\beta = 4$ (ii) *EITHER* $\alpha^2 + \beta^2 = -4$ *OR* (iii) $x^2 + 4x + 16 = 0$	B1B1 M1 A1 M1 A1 B1	2 2	Values stated Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly Find numeric values of roots, square and add Obtain given answer correctly State or use $\alpha^2\beta^2 = 16$

	(b) (i) $p = 2$	M1 A1	3	Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square Obtain $x^2 + 4x + 16 = 0$
		M1 A1	2	Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$ Obtain $p = 2$
	(ii) $a = 44$	M1 A1ft	2	Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation Obtain $11p^2$
			11	
9.	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	(iii) **M** = $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$	M1 A1	2	Attempt to find **DC** (not **CD**) Obtain given answer
	(iv)	B1		Explicit check for n = 1or n = 2
	$\mathbf{M}^k = \begin{pmatrix} 2^k & 3(2^k - 1) \\ 0 & 1 \end{pmatrix}$.	M1		Induction hypothesis that result is true for $\mathbf{M}^k$
		M1		Attempt to multiply $\mathbf{M}\mathbf{M}^k$ or vice versa
	$\begin{pmatrix} 2^{k+1} & 3(2^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$.	A1 A1		Element $3(2^{k+1} - 1)$ derived correctly All other elements correct
		A1	6	Explicit statement of induction conclusion
			12	

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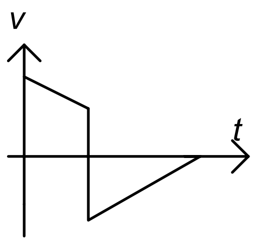
1	**(i)**	$2X\cos\theta = X$ $2\cos\theta = 1$ $\theta = 60$ **AG**	M1 A1 A1 3	For resolving forces in the **i** direction
	(ii)	$2X\sin\theta = 10$ $X = 5.77$	M1 A1 2	For resolving forces in the **j** direction or equivalent Allow $10/\sqrt{3}$ or $10\sqrt{3}/3$

2	**(i)**	$5^2 = 3^2 + 2a(3.2)$ Acceleration is 2.5 ms^{-2} $3.2 \div t = (3 + 5) \div 2$ Time taken is 0.8s	M1 A1 M1 A1 4	For using $v^2 = u^2 + 2as$ or any other complete method For using $s \div t = (u + v) \div 2$ or any other complete method
	(ii)	$2.5 = g\sin\alpha$ $\alpha = 14.8$	M1 A1 2	For using $(m)g\sin\alpha = (m)a$

3	**(i)**	$2T\cos 45^\circ = 0.3g$ or $T=0.3g \cos 45^\circ$ or $T^2 + T^2 = (0.3g)^2$ $T = 2.08$ **AG**	B1 B1 2	
	(ii)	$F = T\sin 45^\circ$ Frictional force is 1.47 N	M1 A1 2	For resolving forces on R_1 horizontally
	(iii)	$R = 0.3g + T\cos 45^\circ$ (= 4.41) $1.47 = \mu\, 4.41$ Coefficient is 1/3 (= 0.333)	B1 M1 A1 3	For using $F = \mu R$

4	**(i)**	$100u = (100 + 300)1.25$ Speed of B is 5 ms^{-1}	M1 A1 A1 3	For using the principle of conservation of momentum SR B1 for 5 ms^{-1} from use of weight
	(ii)		B1 B1 2	For line segment from t = 0 to t = 100 with negative slope and v +ve throughout. Values of v and t need not be shown. For line segment from t = 100 to t = 300 with positive slope and v everywhere –ve except at t = 300 where v must be zero. Values of v and t need not be shown.
	(iii)	$x(100) = ½ (1.25 + 0.75)100$ Distance is 100 m	M1 A1 2	For using the idea that area represents displacement
	(iv)	$x(300)$ = ans(iii) – ½ 200 × 1.0 $x(300) = 0$ ➔ Particle at origin (or equivalent)	M1 A1 2	For attempting to find the displacement at t = 300, or equivalent

5	(i)		M1	For applying Newton's 2nd law to either particle
		$0.29g - T = 0.29a$	A1	
		$T - 0.2g = 0.2a$	A1	
			M1	For eliminating T or a
		Acceleration is 1.8ms^{-2} and tension is 2.32 N	A1 5	
			Alternatively:	For using $(M + m)a = (M - m)g$ M1 Acceleration is 1.8ms^{-2} A1 For applying Newton's 2nd law to either particle M1 $0.29g - T = 0.29a$ or $T - 0.2g = 0.2a$ A1 Tension is 2.32 N A1
	(ii)	$H = ½\ 1.8 \times 1.3^2$	M1	For using $H = ½\ at^2$
		Height is 1.52(1) m	A1 ft 2	ft on a (0.845a)
	(iii)	Speed of Q is 2.34 ms^{-1}	B1	
		$-V = V - gt$ or $0 = V - g(t/2)$	M1	For using $v = u - gt$ to find the time for which the string is slack
		$t = 4.68/9.8 < 0.5$ (ie P is on the ground for less than 0.5 s)	A1 3	

6	(i)		M1	For resolving forces vertically or horizontally
		$V = 500\cos 30^o - 400$ (= 33.0)	A1	
		$H = 500\sin 30^o$ (= 250)	A1	
		$R = \sqrt{250^2 + 33.0^2}$	M1	For using $R = \sqrt{H^2 + V^2}$
		$R = 252$	A1 ft	ft consistent sin/cos mix (458)
		$\tan\theta = 33.0/250$	M1	For using $\tan\theta = V/H$
		$\theta = 7.5$	A1 7	
	(i) OR	For correct diagram	M1	
		For using cosine rule to find R or R^2	M1	
		$R^2 = 400^2 + 500^2 - 2\text{x}400\text{x}500\cos 30$	A1	
		$R = 252$	A1	
		Using sine rule to find an appropriate angle within the triangle	M1	
		$\frac{\sin 30}{R} = \frac{\sin\alpha}{400}$ or $\frac{\sin\beta}{500}$	A1	
		$\theta = 7.5$	A1 7	
	(ii)		M1	For using Newton's 2nd law
		$252 = (400/g)a$	A1 ft	ft on R
		Acceleration is 6.2 ms^{-2}	A1 ft 3	ft consistent sin/cos mix (11 ms^{-2})

7	(i)	$a = 0.2 + 0.02t$ $0.2 + 0.02\times30 = k$ $k = 0.8$	*M1 dep*M1 A1 3	For using $a = dv/dt$ For using $a(30) = k$
	(ii)	$v(30) = 15$ $25 = 15 + 0.8t$ ($t_{total} = 30 + 12.5$) Total time is 42.5 s	B1 B1 ft B1 3	 ft on k
	(iii)	$s = \int (0.2t + 0.01t^2)dt$ $s = 0.1t^2 + 0.01t^3/3$ (+C) $(0.1\times30^2 + 0.01\times30^3/3) - (0 + 0)$ Distance during 1st stage is 180m $25^2 = 15^2 + 2\times0.8s$ or $s = (15+25)12.5\div2$ ($s_{total} = 180 + 250$) Total distance is 430 m	*M1 A1 dep*M1 A1 M1 A1 A1 7	For using $s = \int vdt$ For using correct limits or equivalent For using $v^2 = u^2 + 2as$ or $s = (u + v)t\div2$

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1		centre of mass on AC	B1		Can be implied from M1	
		$\tan 32^o = 6/x$	M1			
		x= 9.60cm	A1	3		**3**

2	(i)	$T\cos\theta = 0.01 \times 9.8$	M1		resolving vertically	
		$8/10T = 0.01 \times 9.8$	A1		with $\cos\theta = 8/10$	
		T = 0.1225 N	A1	3	AG	
	(ii)	$T + T\sin\theta = ma$	M1		resolving horizontally	
		Use of $mr\omega^2$	M1			
		$\omega = 5.72$ rads^{-1}	A1	3		**6**

3	(i)	$\frac{1}{2} . 10^{-4} . v_1^2$	B1		K.E.	
		$10^{-4} . 9.8 . 1000$	B1		P.E.	
		KE=PE-1000x0.00097	M1			
		v_2= 14.1 ms^{-1}	A1	4	$v_1 = \sqrt{200}$ n.b. $\sqrt{220} = 14.8$ for $g = 9.81$	
		Alternatively				
		$a = 0.1$ from F=ma	(B1)			
		v_1 correct from formulae	(B1)	(2)	2 marks (out of 4) maximum	
	(ii)	$\frac{1}{2}mv_2^2 = \frac{1}{5} \times \frac{1}{2}m \times v_1^2$	M1			
		$\frac{1}{2}mv_2^2 = \frac{1}{5} \times \frac{1}{2}m \times 14.1^2$				
		$v_2 = 6.32$ ms^{-1}	A1 ft	3	Ft for 0.4472 x their v_1 (6.63 for $g = 9.81$)	**7**

4	(i)	5m = mu + 4m	M1		cons. of mom.	
		u = 1	A1			
		e = (2-1)/5	M1			
		e = 1/5	A1	4		
	(ii)	I = 4m	B1			
		→	B1	2	to the right	
	(iii)	4m = 5mv	M1			
		v = 4/5	A1			
		4/5 < 1	B1	3		**9**

5	(i)	½.700.20^2 or ½.700.15^2	B1		either K.E.	
		700x9.8x400sin5°	B1		correct P.E.	
		½.700.15^2+700.9.8.400sin5°= ½.700.20^2 + W.D.	M1		for 4 terms with W.D.	
		W.D. = 178,000 J	A1	4	or 178 kJ	
	(ii)	D=200 + 700.9.8sin5°	M1			
		D = 798 N	A1		may be implied	
		P = Dx15 = 12,000 = 12kW	A1	3	AG **(11,968W)**	
	(iii)	D' = 11,968÷20 = 598	M1			
		D'-700.9.8sin5°-200 = 700a	M1			
		a = -0.285 ms^{-2}	A1	3	allow –0.283 (from 12kW)	**10**
		Alternative for false assumption of constant acceleration				
	(i)	D-700 x 9.8sin5^o = 700a and 15^2 = 20^2 + 2a.400	(M1)		(D = 445, a = -0.21875)	
		W.D. = 400xD = 178,000	(A1)	(2)	2 marks (out of 4) maximum	

6	(i)	d' = 20	B1		d' = distance to line of action of W	
		40T' = 20.12.9.8	M1			
		T' = 58.8 N	A1	3		
	(ii)(a)	d = 48	B1		distance to line of action of T may be implied	
		Td = 12x9.8x40	M1		or moments about B	
		T = 98.0 N	A1	3		
	(b)	12. 9.8 = Y + Tsinθ	M1		resolving vertically (sinθ = 0.6)	
		Y = 58.8 N	A1			
		X = Tcosθ	M1		resolving horizontally (cosθ=0.8)	
		X = 78.4 N	A1			
		magnitude = 98.0 N	B1ft		On their T in (a)	
		angle = 36.9° to horizontal	B1ft	6	On their T I (a) or 53.1^o to vertical	**12**
or	(b)	60X = 40x9.8x12	(M1)		M(D)	
		X = 78.4 N	(A1)			
		80Y = 40x9.8x12	(M1)		M(C)	
		Y = 58.8 N	(A1)			
		magnitude = 98.0 N	(B1ft)		On their T in (a)	
		angle = 36.9°	(B1ft)	(6)	On their T I (a) or 53.1^o to vertical	
or	(b)	Recognise 3 concurrent forces	(M1)		Can be through diagram	

		Recognise symmetry	(M1)		Can be through diagram	
		Magnitude force at B same as T	(M1)			
		Magnitude force at B=98.0N	(A1ft)		ft their T	
		Angle is same as T's	(M1)			
		Angle = 36.9° to horizontal	(A1)	(6)	Or 53.1° to vertical	
or	(b)	Draw triangle of forces	(M1)			
		$R^2=117.6^2 + 98^2$ $- 2.117.6.98\cos 53.1°$	(M1) (A1)			
		R= 98.0 N	(A1ft)			
		Correct method for angle	(M1)			
		Angle = 36.9° to horizontal	(A1)	(6)		

7	(i)	$x = 49\cos\theta . t$	B1			
		$y=49\sin\theta.t - ½.9.8.t^2$	B1			
		$y=x\tan\theta - 4.9x^2/49^2.\cos^2\theta$	M1		aef (eliminating t)	
		$y=x\tan\theta – x^2(1+\tan^2\theta)/490$	A1	4	AG	
	(ii)	$30 = 70\tan\theta – 10(1+\tan^2\theta)$	M1			
		$\tan\theta=(70\pm\sqrt{3300})\div 20$	M1			
		81.1°	A1		θ_1 or θ_2	
		32.1°	A1	4	"	
	(iii)	$x^2\left(1+\tan^2\theta\right)/490 = x\tan\theta$	M1		Set $y = 0$	
		$x = 490\tan\theta/\left(1+\tan^2\theta\right)$	A1			
		$x = 75.0$	A1			
		$x = 220.6$	A1			
		$d = 146$m	A1ft	5		
						13
	(iii)	**Alternatively (1st 2 marks)**				
		$t = 49\sin\theta/4.9$ and $x=49\cos\theta.10\sin\theta$	M1		$s = ut + \frac{1}{2}at^2$ and $x = 49\cos\theta.t$ or $R = u^2\sin 2\theta/g$ (precise)	
		$490\sin\theta\cos\theta$ or $245\sin 2\theta$	A1		Sufficient for one angle θ	

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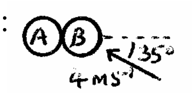
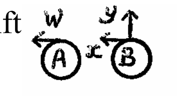
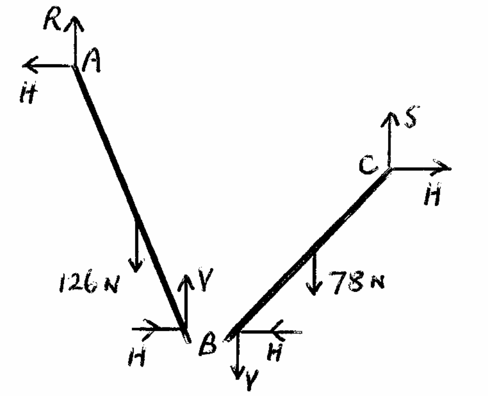
1 (i)	$0.7 \times 12 \sin 30$ $= 4.2 \text{ m s}^{-1}$	M1 A1 **2**	*SR* Treat $0.7 \times 12 \cos 30$ as a MR (unless there is evidence to contrary)
(ii)	$I = 0.15 \times 4.2 - (-0.15 \times 6)$ $= 1.53 \text{ N s}$	M1 A1 **2**	
2 (i)	$-(m)g\sin\theta = (m)\left(2.45\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right)$ $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -4\sin\theta \approx -4\theta$ Hence motion is approx SHM Period is $\frac{2\pi}{\sqrt{4}} = \pi = 3.14 \text{ s}$	B1 M1 A1 B1 **4**	Accept l or r instead of 2.45 For $\sin\theta \approx \theta$ Accept π
(ii)	$\theta = 0.08\sin 2t$ When $t = 1.2$, $\theta = 0.08\sin 2.4$ $= 0.054 \text{ rad}$	B1 ft M1 A1 **3**	
3	Before:  Aft  $y = 4\sin 35 \quad (= 2.294)$ $0.6x + 0.3w = 0.6(4\cos 35)$ $w - x = 4\cos 35$ $x = \frac{4}{3}\cos 35 = 1.092$ Speed is $\sqrt{x^2 + y^2} = 2.54 \text{ m s}^{-1}$ Angle $\tan^{-1}\frac{y}{x} = 64.5°$ to line of centres	B1 M1 A1 B1 M1 M1 A1 A1 **8**	 Momentum equation or correct energy equation Obtaining a value of x Using $\sqrt{x^2 + y^2}$ or $\tan^{-1}\frac{y}{x}$

4	 Moments about *A* for *AB* $9V+12H-126\times4.5=0$ Moments about *C* for *BC* $7V-10.5H+78\times3.5=0$ $H=36,\quad V=15$ Magnitude is $\sqrt{H^2+V^2}=39$ N	M1 A1 M1 A1 M1 A1 M1A1 **8**	Moments equation Moments equation Obtaining *H* or *V* Both magnitudes correct *Solutions by other methods can earn full marks*
5 (i)	Driving force is $\frac{26000}{v}$ $\frac{26000}{v}-0.4v^2=840v\frac{dv}{dx}$ $65000-v^3=2100v^2\frac{dv}{dx}$ $\frac{2100v^2}{65000-v^3}\frac{dv}{dx}=1$	M1 M1 A1 A1 (ag) **4**	 Using N2L to obtain a diff eqn
(ii)	$x=\int\frac{2100v^2}{65000-v^3}\,dv$ $=-700\ln(65000-v^3)+C$ $v=10$ when $x=0\Rightarrow C=700\ln64000$ $x=-700\ln(65000-v^3)+700\ln64000$ When $x=350,\quad \ln(65000-v^3)=10.567$ $65000-v^3=38820$ $v=29.7\text{ m s}^{-1}$	M1 M1 A1 M1 A1 **5**	 or correct use of limits exponentiation
6 (i)	$T=0,\quad 0.3\times9.8\cos60=0.3\times\frac{u^2}{1.6}$ $u=2.8\text{ m s}^{-1}$	M1 A1 A1 **3**	N2L in radial direction

(ii)	By conservation of energy, $\frac{1}{2}(0.3)(v^2 - u^2) = 0.3 \times 9.8 \times 1.6$ $v^2 = 39.2$ Radial component of acceleration is $\frac{v^2}{1.6}$ $= 24.5 \text{ m s}^{-2}$	M1 A1 M1 A1 **4**	 *Dependent on previous M1* M0 if mass included
(iii)	$T - 0.3 \times 9.8 \cos 60 = 0.3 \times 24.5$ $T = 8.82 \text{ N}$	M1A1 ft A1 **3**	Must use a result from (ii) A1ft requires numerical substitution
7 (i)	$T = \frac{1960}{20}x \quad (= 98x)$ $90 \times 9.8 - T = 90a$ $a = 9.8 - \frac{98}{90}x$ When $x = 9$, $a = 9.8 - 9.8 = 0$	M1 A1 A1 (ag) **3**	
(ii)	Gain in EE is $\frac{1960x^2}{2 \times 20} = 49x^2$ Loss of PE is $90 \times 9.8(x + 20) = 882x + 17640$ By conservation of energy, $\frac{1}{2}(90)v^2 + 49x^2 = 882x + 17640$ $45v^2 + 49x^2 = 882x + 17640$	B1 B1 M1 A1 (ag) **4**	 Equation involving KE, EE and PE
(iii)	Maximum speed when $a = 0$, i.e. $x = 9$ $v = 21.9 \text{ m s}^{-1}$	M1 A1 **2**	
(iv)	Maximum extension when $v = 0$ $49x^2 = 882x + 17640$ $x^2 - 18x - 360 = 0$ $(x - 30)(x + 12) = 0$ $x = 30 \text{ m}$	 M1 M1 A1 **3**	 Solving to obtain a value of x
(v)	Maximum $\|a\|$ when $x = 30$ $\|a\| = 22.9 \text{ m s}^{-2}$	M1 A1 **2**	 Condone -22.9

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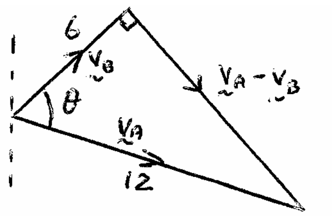
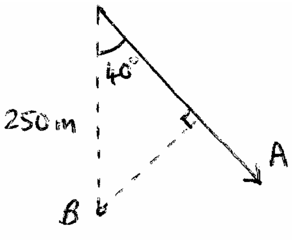
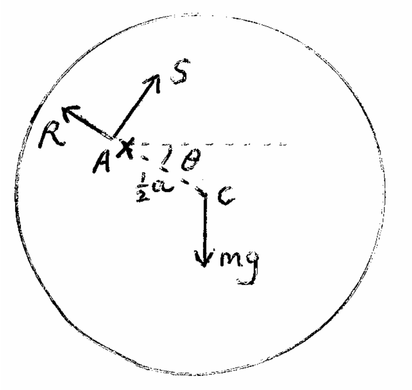
1 (i)	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $180 = 25 \times 5 + \frac{1}{2}\alpha \times 25$ $\alpha = 4.4 \text{ rad s}^{-2}$	M1 A1 **2**	
(ii)	Moment $= I\alpha = 0.65 \times 4.4$ $= 2.86 \text{ N m}$	M1 A1 ft **2**	
2	Area $= \int_0^9 \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^9 = 18$ $\int xy\,dx = \int_0^9 x^{\frac{3}{2}}\,dx = \left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^9 = 97.2$ $\bar{x} = \frac{97.2}{18}$ $= \frac{27}{5} = 5.4$ $\int \frac{1}{2}y^2\,dx = \int_0^9 \frac{1}{2}x\,dx = \left[\frac{1}{4}x^2\right]_0^9 = 20.25$ $\bar{y} = \frac{20.25}{18}$ $= \frac{9}{8} = 1.125$	B1 B1 M1 A1 B1 M1 A1 **7**	For $\frac{2}{5}x^{\frac{5}{2}}$ For $\frac{1}{4}x^2$ (*or* $\frac{9}{2}y^2 - \frac{1}{4}y^4$)
3 (i)	$I = 0.02 + 0.12 = 0.14$ Period is $2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{0.14}{1.5 \times 9.8 \times 0.2}}$ $= 1.37 \text{ s}$	B1 M1 A1 **3**	
(ii)	WD by couple is $3.2 \times \frac{1}{2}\pi$ $3.2 \times \frac{1}{2}\pi = 1.5 \times 9.8 \times 0.2 + \frac{1}{2}(0.14)\omega^2$ $\omega = 5.46 \text{ rad s}^{-1}$	B1 M1 A1 ft A1 **4**	For WD = PE + KE
4 (i)	 $\cos\theta = \frac{6}{12}$ $\theta = 60°$ Bearing of *B*'s velocity is $110 - 60 = 050°$	M1 A1 M1 A1 **4**	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity triangle

(ii)	As viewed from B: 	M1 A1	Considering relative displacement Relative velocity on bearing 140°
	Shortest distance is $250\sin 40$ $= 161\text{ m}$	M1 A1 **4**	

5	$m = \rho\int \pi y^2\,\mathrm{d}x = \rho\pi\int_0^{ka} a^2 \mathrm{e}^{-\frac{2x}{a}}\,\mathrm{d}x$	M1	Integral of $\left(\mathrm{e}^{-\frac{x}{a}}\right)^2$ (when finding mass or volume)
	$= \rho\pi\left[-\tfrac{1}{2}a^3\mathrm{e}^{-\frac{2x}{a}}\right]_0^{ka}$	A1	For $\int \mathrm{e}^{-\frac{2x}{a}}\,\mathrm{d}x = -\tfrac{1}{2}a\,\mathrm{e}^{-\frac{2x}{a}}$
	$= \tfrac{1}{2}\rho\pi a^3(1-\mathrm{e}^{-2k})$	A1	For mass or volume
	$I = \int \tfrac{1}{2}\rho\pi y^4\,\mathrm{d}x$ $= \tfrac{1}{2}\rho\pi\int_0^{ka} a^4\mathrm{e}^{-\frac{4x}{a}}\,\mathrm{d}x$	M1 A1 ft	Integral of y^4 Correct integral expression (in terms of x)
	$= \tfrac{1}{2}\rho\pi\left[-\tfrac{1}{4}a^5\mathrm{e}^{-\frac{4x}{a}}\right]_0^{ka} = \tfrac{1}{8}\rho\pi a^5(1-\mathrm{e}^{-4k})$	A1	
	$= \dfrac{\tfrac{1}{4}ma^2(1-\mathrm{e}^{-4k})}{1-\mathrm{e}^{-2k}}$	M1	*Dependent on previous M1M1*
	$= \dfrac{\tfrac{1}{4}ma^2(1-\mathrm{e}^{-2k})(1+\mathrm{e}^{-2k})}{1-\mathrm{e}^{-2k}} = \tfrac{1}{4}ma^2(1+\mathrm{e}^{-2k})$	A1 (ag) **8**	*Intermediate step not required, provided no wrong working seen*

6 (i)			
	$I = \frac{1}{2}ma^2 + m(\frac{1}{2}a)^2$	M1	Using parallel axes rule
	$= \frac{3}{4}ma^2$	A1	
	$mg(\frac{1}{2}a\cos\theta) = I\alpha = (\frac{3}{4}ma^2)\alpha$	M1	Or differentiating the energy equation
	$\alpha = \dfrac{2g\cos\theta}{3a}$	A1 (ag) **4**	
(ii)	$\frac{1}{2}I\omega^2 = mg(\frac{1}{2}a\sin\theta)$	M1 A1	Using $\frac{1}{2}I\omega^2$
	$\omega = \sqrt{\dfrac{4g\sin\theta}{3a}}$	A1 **3**	
(iii)		B1	For radial acc'n of C is $(\frac{1}{2}a)\omega^2$
	$R - mg\sin\theta = m(\frac{1}{2}a)\omega^2$	M1	$\pm R \pm mg\sin\theta = mr\omega^2$ *or* $kma\omega^2$ (with numerical k)
	$R = \frac{5}{3}mg\sin\theta$	A1	
		B1	For transverse acc'n of C is $(\frac{1}{2}a)\alpha$
	$mg\cos\theta - S = m(\frac{1}{2}a)\alpha$	M1	*as above*
	$S = \frac{2}{3}mg\cos\theta$	A1 **6**	Direction must be clear *Equations involving horizontal and vertical components can earn B1M1B1M1*
	OR $S(\frac{1}{2}a) = I_G\alpha$ M1 $S(\frac{1}{2}a) = (\frac{1}{2}ma^2)\alpha$ A1 $S = \frac{2}{3}mg\cos\theta$ A1		Must use I_G

7 (i)	$RB^2 = a^2 + (2a)^2 - 2(a)(2a)\cos(\theta + \frac{1}{4}\pi)$ $= 5a^2 - 4a^2(\cos\theta\cos\frac{1}{4}\pi - \sin\theta\sin\frac{1}{4}\pi)$ $= a^2(5 - 2\sqrt{2}\cos\theta + 2\sqrt{2}\sin\theta)$	M1 A1 (ag) **2**	
(ii)	$V = -mg(2a\sin\theta) + \dfrac{mg\sqrt{2}}{2a} \times RB^2$ $= \frac{5}{2}\sqrt{2}mga - 2mga\cos\theta$ $\dfrac{dV}{d\theta} = 2mga\sin\theta$ When $\theta = 0$, $\dfrac{dV}{d\theta} = 0$, hence equilibrium $\dfrac{d^2V}{d\theta^2} = 2mga\cos\theta$ When $\theta = 0$, $\dfrac{d^2V}{d\theta^2} = 2mga > 0$, hence stable	M1 A1 M1 A1 M1 A1 **6**	Considering PE and EE Correctly shown or other method for max / min Correctly shown
(iii)	KE is $\frac{1}{2}m(2a\dot{\theta})^2$ $\frac{5}{2}\sqrt{2}mga - 2mga\cos\theta + 2ma^2\dot{\theta}^2 = E$ Differentiating w.r.t. *t*, $2mga\sin\theta\,\dot{\theta} + 4ma^2\dot{\theta}\ddot{\theta} = 0$ $\ddot{\theta} = -\dfrac{g}{2a}\sin\theta$	B1 M1 M1 A1	 Requires fully correct working
	OR $(mg\cos\theta - T\sin\phi)(2a) = I\ddot{\theta}$, where $T = \dfrac{mg\sqrt{2}(RB)}{a}$ and $\dfrac{\sin\phi}{a} = \dfrac{\sin(\theta + \frac{1}{4}\pi)}{RB}$ M2 $\ddot{\theta} = -\dfrac{g}{2a}\sin\theta$ A2		or $mg\cos\theta - T\sin\phi = m(2a\ddot{\theta})$ Give A1 if just one minor error
	Period is $2\pi\sqrt{\dfrac{2a}{g}}$	B1 ft **5**	ft provided that *k* is in terms of *a* and *g* only

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1	(i)	R is smooth	B1 1	
	(ii)		M1	For resolving forces horizontally to obtain an equation in T (requires 3 relevant terms and at least one force resolved)
		$T + T\cos60^{o} = 1.6\cos45^{o}$ Tension is 0.754 N **AG**	A1 A1 3	
	(iii)		M1	For resolving forces vertically to obtain an equation for m (requires 3 relevant terms with both T and the 1.6 N force resolved)
		$mg = T\sin60^{o} + 1.6\sin 45^{o}$	A1 ft	ft sin/cos mix from (ii)
		$m = 0.182$	A1 3	
				SR $m = T\sin60^{o} + 1.6\sin 45^{o}$ M1 $m = 1.78$ B1

2	(i)		M1	For applying $F = ma$ (requires at least ma, T and air resistance in linear combination in at least one equation).
			A1	At least one equation with not more than one error.
		$0.2g + T – 0.4 = 0.2a$ $0.3g – T – 0.25 = 0.3a$	A1 A1 4	SR $0.2g - T – 0.4 = 0.2a$ **and** $0.3g + T – 0.25 = 0.3a$ B1
	(ii)		M1	For obtaining an equation in T or a only, either by eliminating a or T from the equations in (i) or by applying $F = ma$ to the complete system
		$0.5g – 0.65 = 0.5a$ or $5T – 0.7 = 0$	A1 ft	For a correct equation in a only or T only ft opposite direction of T only
		$a = 8.5$ **and** $T = 0.14$ (positive only)	A1 3	

3	(i)	Momentum before=$0.1\times4 - 0.2\times3$ Momentum after = $-0.1u + 0.2(3.5 - u)$ $0.1\times4 - 0.2\times3 = -0.1u + 0.2(3.5 - u)$ $u = 3$ (positive value only)	B1 B1 M1 A1 4	or Loss by $P = 0.1\times4 + 0.1u$ or Gain by $Q = 0.2(3.5 - u) + 0.2\times3$ For using the principle of conservation of momentum
				SR If mgv used for momentum instead of mv, then $u = 3$ B1
	(ii)	 $0 = 3^2 - 10s_1$ and $0 = 0.5^2 - 10s_2$ $0.9 + 0.025$ Distance is 0.925 m cao	M1 A1 ft M1 A1 4	For using $v^2 = u^2 + 2as$ with $v = 0$ (either case) or equivalent equations ft value of u from (i) For using $PQ = s_1 + s_2$

4	(i) α	 $2 = 0.8u + ½\, a(0.8)^2$ $8 = 2u + ½\, a2^2$ or $6 = 1.2(u + 0.8a) + ½\, a(1.2)^2$ or $6 = 1.2(2\times2\div0.8 - u) + ½\, a(1.2)^2$ $u = 1.5$ Acceleration is 2.5 ms^{-2}	M1 A1 M1 A1 M1 A1 A1 7	For using $s = ut + ½\, at^2$ for the first stage For obtaining another equation in u and a with relevant values of velocity, displacement and time For eliminating a or u
	(i) β	 $2 = 0.8v - ½\, a(0.8)^2$ $6 = 1.2v + ½\, a(1.2)^2$ Acceleration is 2.5 ms^{-2} ($v = 3.5$) $u = 1.5$	M1 A1 M1 A1 M1 A1 A1 7	For using $s = vt - ½\, at^2$ for the first stage For using $s = ut + ½\, at^2$ for the second stage For obtaining values of a and v and using $v = u + at$ for first stage to find u
	(i) γ	$2\div0.8$ ms^{-1} and $6\div1.2$ ms^{-1} = 2.5 ms^{-1} and 5ms^{-1} $t_1 = 0.4$ and $t_2 = (0.8 +)\ 0.6$ $5 = 2.5 + a\,(1.4 - 0.4)$ Acceleration is 2.5 ms^{-2}	M1 A1 B1 M1 A1	For finding average speeds in both intervals For finding mid-interval times For using $v = u + at$ between the mid-interval times

		$2.5 = u + 2.5 \times 0.4$ or $5 = u + 2.5 \times 1.4$	M1	
		$u = 1.5$	A1 7	For using $v = u + at$ between t = 0 and one of the mid-interval times
	(ii)	$2.5 = 9.8\sin\alpha$ $\alpha = 14.8^o$	M1 A1ft 2	For using $(m)a = (m)g\sin\alpha$ ft value of acceleration

5	(i)		M1	For resolving forces on A vertically (3 terms)
		$F = 2 + 7\cos\alpha$ $F = 3.96$ (may be implied)	A1 A1	
		$N = 7\sin\alpha$	M1	For resolving forces on A horizontally (2 terms)
		$N = 6.72$ (may be implied)	A1	
		$3.96 = \mu\, 6.72$ Coefficient is 0.589 or 33/56 cao	M1 A1 7	For using $F = \mu N$
	(ii)	$T\cos\beta = 7\cos\alpha$	M1	For resolving forces at P vertically (2 terms)
		$T\cos\beta = 7 \times 0.28$ ($= 1.96$ **AG)**	A1 2	
	(iii)		M1	For resolving forces on B vertically (2 terms)
		$T\cos\beta - mg = 0$ Mass is 0.2 kg	A1 A1 3	

6	(i)(a)	$V = P\cos 20^o - 0.04g$	B1	
		$P = 0.417$	M1 A1 3	For setting $V = 0$
	(i)(b)	$R = P\sin 20^o$	M1	For using R = horizontal component of P
		Magnitude is 0.143 N	A1ft 2	ft value of P
	(i)(c)	$0.143 = 0.04a$ Acceleration is 3.57 ms^{-2}	M1 A1ft 2	For using Newton's second law ft magnitude of the resultant
	(ii)	$R^2 = 0.08^2 + (0.04g)^2$	M1	For using $R^2 = P^2 + W^2$
		Magnitude is 0.400 N (or 0.40 or 0.4)	A1	
		$\tan\theta = +/-0.04g/0.08$ or $\tan(90^o - \theta) = +/-0.08/0.04g$	M1	For using $\tan\theta = Y/X$ or $\tan(90^o - \theta) = X/Y$
		Angle made with horizontal is 78.5^o or 1.37 radians, or angle made with vertical is 11.5^o or 0.201 radians	A1	
		Downwards or below horizontal	B1 5	Direction may alternatively be shown clearly on a diagram or given as a bearing

7	(i)	½ 200×16 + 300× ½ (16 + 25) + ½ 100×25 (=1600 + 6150 + 1250) Distance is 9000m	M1 A1 A1	3	For using the idea that the area of the quadrilateral represents distance
	(ii)	$a = (0 - 25)/(600 - 500)$ Deceleration is 0.25 ms^{-2}	M1 A1	2	For using the idea that gradient (= vel ÷ time) represents acceleration Or for using v = u + at Allow acceleration = - 0.25 ms^{-2}
	(iii)	Acceleration is $(1200t - 3t^2)\times 10^{-6}$	M1 A1	2	For using $a(t) = \dot{v}(t)$
	(iv)	0.25 – 0.2475 Amount is +/- 0.0025 ms^{-2}	M1 A1ft	2	For using 'ans(ii) – $\lvert a_Q(550)\rvert$' ft ans(ii) only
	(v)	$1200t - 3t^2 = 0$ t = (0 or) 400 **AG**	M1 A1	2	For solving $a_Q(t) = 0$ or for finding $a_Q(400)$ Or for obtaining $a_Q(400) = 0$
	(vi)	½ 200×16 + 200× ½ (16 + 22) $s_Q(t) = (200t^3 - t^4/4) \times 10^{-6}$ (+C) 6400 – 5400 Distance is 1000 m	M1 A1 M1 A1 M1 A1	6	For correct method for $s_P(400)$ For using $s_Q(t) = \int v_Q\,dt$ For using correct limits and finding $\lvert s_Q(400) - s_P(400)\rvert$

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1	(i)	use of h/4	B1			
		com vert above lowest pt of contact	B1		can be implied	
		r = 5 x tan24°	M1			
		r = 2.2	A1	4	2.226	
	(ii)	No & valid reason (eg 24°👍26.6°)	B1√	1	√Yes if their r ☜ 2.5	**5**

2		v^2 = 2x9.8x10	M1		energy:½mv^2=½mu^2 + mgh	
		v = 14	A1		½v^2 = ½.36 + 9.8x10	
		speed = √(14^2 + 6^2)	M1		(must be 6^2) v^2 = 36+196=232	
		speed = 15.2 ms^{-1}	A1			
		tanθ = 14/6	M1		cos^{-1}(6/15.2) etc	
		θ=66.8°(below) horiz.	A1	6	or 23.2° to the vertical	**6**

3	(i)	Tcosθ = 0.01 x 9.8	M1		resolving vertically	
		8/10T = 0.01 x 9.8	A1		with cosθ = 8/10	
		T = 0.1225 N	A1	3	AG	
	(ii)	T + Tsinθ = ma	M1		resolving horizontally	
		use of mr$ω^2$	M1			
		ω = 5.72 $rads^{-1}$	A1	3		
	(iii)	K.E.=½x0.01x$(rω)^2$	M1		½mv^2 with v=rw	
		K.E.= 0.0588	A1√	2	√0.0018 x their $ω^2$	**8**

4	(i)	5m = mu + 4m	M1		cons. of mom.	
		u = 1	A1			
		e = (2-1)/5	M1			
		e = 🗐	A1	4		
	(ii)	I = 4m	B1			
		→	B1	2	to the right	
	(iii)	4m = 5mv	M1			
		v = 😐	A1			
		😐 < 1	B1	3		**9**

5	(i)	60T = 15x30cosθ	M1		moments about A	
		"	A1			
		60T = 15x30 x 0.6	A1		cosθ = 0.6	
		T = 4.5 N	A1	4	**AG**	
	(ii)	X = Tsinθ	M1		res. horiz. (or moments)	
		X = 3.6 N	A1			
		Y + Tcosθ = 15	M1		res. vert.(3 terms) (or moments)	
		Y = 12.3 N	A1			
		R = 12.8 N	A1√		√(their X^2 + Y^2)	
		73.7° to horizontal	A1√	6	or 16.3° to vert.√tan^{-1}their(Y/X)	**10**
		or triangle of forces: Triangle (M1) R^2 =15^2+4.5^2-2x4.5x15x0.6(M1A1)				
		R= 12.8 (A1) sinθ/4.5=sinα/12.8 (M1) θ = 16.3° to vert. (A1)				

6	(i)	½.700.20^2 or ½.700.15^2	B1		either K.E.	
		700x9.8x400sin5°	B1		correct P.E.	
		½.700.15^2+700.9.8.400sin5°= ½.700.20^2 + W.D.	M1		for 4 terms with W.D.	
		W.D. = 178,000 J	A1	4	or 178 kJ	
	(ii)	D=200 + 700.9.8sin5°	M1			
		D = 798 N	A1		may be implied	
		P = Dx15 = 12,000 = 12kW	A1	3	AG **(11,968W)**	
	(iii)	D' = 11,968÷20 = 598	M1			
		D'-700.9.8sin5°-200 = 700a	M1			
		a = 0.285 ms^{-2} (±)	A1	3	allow 0.283 (from 12kW)	**10**
		Alternative for false assumption			**of constant acceleration**	
	(i)	D-700 x 9.8sin5° = 700a and 15^2 = 20^2 + 2a. 400	M1		(D = 445, a = -0.21875)	
		W.D. = 400xD = 178,000	A1		2 marks (out of 4) maximum	

7	(i)	50x9.8x2 = Rx3.75 +80x9.8x0.25	M1		moments about D.	
		"	A1		SR/no g/ R = 21.3 (M1A1A0)	
		R = 209 N	A1	3		
	(ii)	130$\bar{x}$ = 50 x 2 + 80 x 4.25	M1 A1		moments about BC or FE….. 130$\bar{x}$ =80x0.25 + 50x2.5	
		$\bar{x}$ = 3.385	A1		$\bar{x}$ = 1.115	
		130$\bar{y}$ = 50 x 0.125 + 80 x 0.25	M1 A1		moments about EC	
		$\bar{y}$ = 0.202	A1			
		tanθ = 0.615/0.202	M1			
		θ = 71.8° to the horizontal	A1	8	71.6° to 72.0°	**11**

8	(i)	x = 49cosθ . t	B1			
		y=49sinθ.t - ½.9.8.t^2	B1			
		y=xtanθ - 4.9x^2/49^2.cos^{2}θ	M 1		aef (eliminating t)	
		y=xtanθ – x^2(1+tan^{2}θ)/490	A1	4	AG	
	(ii)	30 = 70tanθ – 10(1+tan^{2}θ)	M 1			
		tanθ=(70±√3300)÷20	M 1		(6.37/0.628)	
		81.1°	A1		θ_1 or θ_2	
		32.1°	A1	4	"	
	(iii)	x^2 (1+tan^2 θ)/490 = xtanθ	M 1		set y = 0	
		x = 490tanθ/(1+tan^2 θ)	A1			
		x = 75.0	A1			

		x = 221 (220.6)	A1			
		d = 146 m	A1 √	5	√	**13**
	(iii)	Alternatively (1st 2 marks)				
		t=49sinθ/4.9 and (9.88/5.31) x=49cosθ.t	M 1		<u>$s=ut+½at^2$ and</u> <u>x=49cosθ.t</u> or $R = u^2\sin2θ/g$ (precise)	
		x = 490sinθcosθ	A1		245sin2θ	

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1.	(i)	Mean=35.2/80=0.44 Variance=175.08/80-0.44^2 1.99(49)	B1 M1 A1	3	
	(ii)	mean of x=11.44 Variance =1.99(49)	B1 ft B1 ft	2	From(i) From(i)
2.	(i)	9!/(4!3!2!) 1260	M1 A1	2	Use of formula
	(ii)	Perm remaining 5 5!/(3!2!)=10	M1 A1	2	Stated or implied
	(iii)	Ans(ii)/Ans(i) =1/126	B1ft	1	Allow 10/1260,0.00794
3.	(i)	L.Q.=2.75	B1		£ not required
		Median=3.50	B1		
		U.Q.=4.55 Allow slight variations for L.Q.,U.Q.(+/- 5p) SR Key misinterpreted. Acceptable answers x or / by 10 or 100	B1 B1	3	
	(ii)	Box-plot. Show 1.00,5.30, quartiles and Median. Scale indicated or implied.	M1 A1ft A1ft	4 3	Recognisable box-plot At least correct (ft) All correct (ft)
	(iii)(a)	Store a has greater variability.	B1		
	(b)	Sensible comment about skewness or symmetry.	B1	2	
4.	(i)	(5/6)^2x(5/6)	M1		
		a=125/216 b=1-125/216-1/36=85/216	A1 B1ft	3	aef Or independently
	(ii)	85/216+2x1/36 97/216	M1 A1	2	use of sum of xp (0.449)
	(iii)	Use B(5,125/216)	M1		Binomial recognized. 5C3 essential
		5C3(125/216)^3x(91/216)^2 0.344	A1ft A1	3	
5.	(i)	Scatter diagram	B1		Uniform scale, axes and points labelled.
			B1 B1		At least 6 pts. correct. All 9 correct.
	(ii)	e.g. C lower than B	B1	1	
	(iii)	9 8 7 6 5 4 3 2 1 9 7 8 6 5 3 2 4 1	B1		Correct ranks(or reversed)

		Sum of d^2=8	M1		attempt at d or d^2
		1-(6x8)/(9(9^2-1)) 14/15	M1 A1	4	Correct use of formula (0.933)
	(iv)	Strong association between heights	B1	1	Or equivalent
	(v)	None	B1	1	
6.	(i)	Sxy=21020-360x367/8=4505			
		Sxx=20400-360x360/8=4200			
		Syy=21673-367x367/8	M1		Any 1 of Sxy,Sxx,Syy
		=4836.875			Correct.
		r=4505/(4200x4836.875)^0.5 =0.9995	A1 A1	3	
	(ii)	Since x values are exactly is the dep. variable.	B1	1	or equivalent.
	(iii)	b=4505/4200	M1		x on y used
		=1.07(3)	A1		allow M1s for
		a=367/8-1.07..x45 y=-2.39+1.07x	M1 A1	4	b',a' a=[-2.41,-2.39]
	(iv)	(a) 54.4 (g)	B1		[54.3,54.6]
		(b) 95.4 (g)	B1	2	[95.35,95.75]
	(v)	High value of r means that (a) is reliable but (b) is out of data range, so unreliable	B1ft B1		
7.	(i)	Imperfections occur independently with constant prob. Or reference to random sample	B1 B1	2	or at constant rate.
	(ii)	B(20,0.03)orB(20,0.97) stated or implied	M1		
		$0.97^{20}+20\text{x}0.97^{19}\text{x}0.03$ 1-[" " "] 0.1198	M1 M1 A1	4	 allow 1,2or 3 terms in[] allow 0.12
	(iii)	1/0.1198 8.35	M1 A1	2	1/their(ii)prov.not 0.03,0.97 [8.33,8.35]
	(iv)	P(U>10.35)=P(U>10)	M1		correct rounding of value to integer
		(1-0.1198)^10 0.279	M1A1ft A1	4	M1 for (1-(ii))^integral part of (iii)+2,3or 4 A1 ft for index [(iii)]+2

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1	Po(1.2) Tables or correct formula used 0.8795	B1 M1 A1 **3**	Po(1.2) stated or implied Correct method for probability Answer, 0.8795 or 0.879 or 0.88(0)
2 (i)	$\mu = 17 + 5 \times 2$; $\Phi\left(\frac{25-27}{3}\right)$ $1 - 0.747 = 0.253$	M1 M1 A1 **3**	$\mu = 17 + 5 \times 2$ Standardise, their μ; 3, 3^2 or $\sqrt{3}$ Answer in range [0.252, 0.253]
(ii)	$(40 - \mu)/3 = \Phi^{-1}(0.975)$ $= 1.96$ $\mu = 34.12$ $d = 8.56$	M1 B1 A1 A1 **4**	Standardise and equate to Φ^{-1}; 3, 3^2 or $\sqrt{3}$ 1.96 seen Correct value of μ, can be implied Correct value of d, allow 8.6
3 (i)	$\int_1^4 kt^{-2}dt = \left[-\frac{k}{t}\right]_1^4 = \frac{3k}{4}$ so $k = \frac{4}{3}$	M1 A1 **2**	Use $\int f(t)dt = 1$ Correctly obtain $k = 4/3$ [A.G.]
(ii)	$\mu = \frac{4}{3}\int_1^4 \frac{1}{t}dt = \frac{4}{3}[\ln t]_1^4$ $= \frac{4}{3}\ln 4$.	M1 A1 A1 **3**	Use $\int_1^4 tf(t)dt$; correct indefinite integral Exact answer only, allow equivalents but not ln1
(iii)	$\frac{4}{3}\int_{t_0}^4 \frac{1}{t^2}dt = 0.1$; $\frac{4}{3}\left[\frac{1}{t_0} - \frac{1}{4}\right] = 0.1$ $1/t_0 = 0.325$ so $t_0 = 3.077$.	M1 M1 A1 **3**	Attempt correct integral & limits (can be complement) Substitute and solve for t_0 answer, art 3.08 or $\frac{40}{13}$
4 (i)	$\hat{\sigma}^2 = \frac{50}{49} \times 0.0967^2 = 0.0987$	M1 A1 **2**	Use $\frac{n}{n-1} \times s^2$, allow √ Answer, a.r.t. 0.0987
(ii) α: β:	$H_0: \mu = 1.8$, $H_1: \mu \neq 1.8$ where μ is population mean $z = \frac{(1.72 - 1.8)}{\hat{\sigma}/\sqrt{50}} = -1.8(01)$ CV $1.8 - k.\sigma/\sqrt{50}$ Compare z with 1.645 *or* 1.72 with CV Reject H_0 Significant evidence that mean height is not 1.8	B1 B1 M1 A1 B1 M1 A1 **7**	Hypotheses correctly stated in terms of μ SR: μ wrong/omitted: B1 only, but $\overline{X}$: B0 Standardise with $\sqrt{n}$, allow + or – $z = -1.80 \pm 0.01$, *or* correct expression for CV, any reasonable k, don't allow + 1.645 and comparison Correct method and comparison, needs $\sqrt{50}$, μ = 1.8; allow cc, √ errors or biased σ estimate Conclusion stated in context
5 (i)	${}^{30}C_{10}(0.4)^{10}(0.6)^{20}$ $= 0.1152$	M1 A1 **2**	Correct formula or use of tables Answer, a.r.t. 0.115
(ii)	$30p > 5$ so $p > 1/6$ $30q > 5$ so $q > 1/6$ $1/6 < p < 5/6$	M1 M1 A1 **3**	$30p$ and 5 used $30q$ and 5 used Answer completely correct, except allow ≤
(iii)	N(12, 7.2) $\frac{10.5 - np}{\sqrt{npq}}$ [= –0.559] $\Phi(-0.559) = 0.2881$	B1 B1 M1 A1 A1 **5**	12 seen 7.2 or 2.683 seen Standardise, allow wrong/no cc, their npq $\sqrt{npq}$, and 10.5 Answer, a.r.t. 0.288

6	(i)	$R \sim B(25, 0.8)$ $P(R \le 16) = 0.0468$, $P(R \le 17) = 0.1091$ $k = 16$	B1 M1 A1	 **3**	B(25, 0.8) stated or implied One relevant probability seen Answer $k = 16$ only
	(ii)	$20p$ $= 0.936$	M1 A1	 **2**	$20 \times$ their p *OR* 20×0.05 Answer, a.r.t. 0.936
	(iii)	$P(R \le 16 \mid p = 0.6)$ $= 0.7265$	M1 A1	 **2**	Find $P(r \le k \mid p = 0.6)$ Answer 0.7265 or 0.727
	(iv)	$p' = 0.5 \times 0.0468 + 0.5 \times 0.7265$ $= 0.38665$ $2 \times p' \times (1 - p')$ $= 0.474$	M1 A1 M1 A1	 **4**	"Tree diagram" probability; 0.5 x (i)prob + 0.5 x (iii)prob Value in range [0.38, 0.39] Correct formula, including 2 Answer in range [0.47, 0.48]
		or 0.8 A 0.8 R $.5^2 \times .9532 \times .0468 = .0112$ 0.8 R 0.8 A $.5^2 \times .0468 \times .9532 = .0112$ 0.6 A 0.8 R $.5^2 \times .2735 \times .0468 = .0032$ 0.6 R 0.8 A $.5^2 \times .7265 \times .9532 = .1731$ 0.8 A 0.6 R $.5^2 \times .9532 \times .7265 = .1731$ 0.8 R 0.6 A $.5^2 \times .0468 \times .2735 = .0032$ 0.6 A 0.6 R $.5^2 \times .2735 \times .7265 = .0497$ 0.6 R 0.6 A $.5^2 \times .7265 \times .2735 = .0497$	M1 A1 M1 A1		2 products added; can omit 0.5^2 4 products; can omit 0.5^2 Completely correct list of cases and probabilities Answer in range [0.47, 0.48]
7	(i)	Coins occur at constant average rate and independently of one another	B1 B1	 **2**	Constant average rate, contextualised Independence, contextualised
	(ii)	$R \sim Po(5.4)$ $e^{-5.4}\dfrac{5.4^3}{3!} = 0.1185$	B1 M1 A1	 **3**	Poisson (5.4) stated or implied Correct formula, any λ Answer, in range [0.118, 0.119]
	(iii)	$R \sim Po(3)$ Tables, looking for 0.05 or 0.95 $P(R \ge 7) = 0.0335$ Therefore smallest number is 7	B1 M1 A1 A1	 **4**	Poisson (3) stated or implied Evidence of correct use of tables One relevant correct probability seen $r = 7$ only, ignore inequalities
	(iv)	$R \sim Po(4.8)$ Type II error is $R < 7$ when $\mu = 4.8$ $P(< 7) = 0.7908$	B1 M1 A1	 **3**	Poisson (4.8) used Correct context for Type II error $P(< 7)$, a.r.t. 0.791

Clarification Question 4 (ii)

$\alpha: \quad z = \frac{(1.72 - 1.8)}{\hat{\sigma}\sqrt{50}}$	M1	Standardise with $\sqrt{n}$
$= -1.8(01)$	A1	$z = -1.801 \pm 0.01$
$-1.801 < -1.645$	B1	1.645 seen
	M1	Compare z with 1.645
Reject H_0		
Significant evidence that mean height is not 1.8	A1	Conclusion in context
$\beta: \quad z =$	M1	
$= -1.8(01)$	A1	
$\Phi(+z) = 0.9642$		
P('2 tails') = 2 x 0.0358		
= 0.0716	B1	Answer in range 0.071 to 0.072
7.16%<10%	M1	Compare p with 10%
Reject H_0		
Conclusion in context	A1	
$\gamma: \quad z_{\text{crit}} = \pm 1.645$	B1	1.645 seen
$\text{CV} = 1.8 \pm k.\hat{\sigma} / \sqrt{50}$	M1	Correct expression for CV
=(1.727, 1.873)	A1	1.727
1.2 < 1.727	M1	Compare 1.72 with CV
Reject H_0		
Conclusion in context	A1	

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1	(i)	Use Po(2.35) or equivalent $e^{-2.35} = 0.0954$	M1 A1	 2	Use formula, summing means Or from table. (Require 3sf but only penalise once in paper)
	(ii)	EITHER: S=0, T=0, F=2	M1		
		OR:S+T=0,F=2 $e^{-1.9} \times e^{-0.45} \times 0.45^2/2$ 0.00966	 A1 A1	 3	Not S+T+F=2 and F=2
2	(i)	There is a 95% chance that this interval includes the population mean	B1G B1H	2	OR: Of 100 such intervals, 95 would contain the population mean
	(ii)	Width of interval = 2.10	B1		Allow 1.05
		Use $(x) + 1.96\ \sigma/\sqrt{100}$ $(2)\times 1.96\sigma/\sqrt{100} = (2)\times 1.05$ $\sigma = 5.36$	M1 M1 A1	 4	
3		H_0: Type and area are independent			May be implied later
		E values 45.91 54.09 55.09 64.91	M1 A1		
		Use of $\sum(O-E)^2/E$	M1*		With or without Yates - at least one correct
		0.332.. 0.282.. 0.277.. 0.277..			With Yates – use of modulus correct
		0.253.. 0.213.. 0.211.. 0.179..	M1		
		$\sum(\lvert O-E\rvert-0.5)^2/E = 0.85(8..)$ Compare with 2.706 Accept type of flower is independent of area in which plant grows	A1 M1 A1	 7	Accept 0.85 or 0.86 dep* Conclusion in context
4	(i)	$H_0: \mu_1 - \mu_2 = 0,\ H_1: \mu_1 - \mu_2 > 0$	B1		Both – allow μ_d

		Differences			
		-0.3 0.3 0.4 0.8 0.4 0.2 0.2 0.5	M1		
		mean = 0.31(25); variance = 0.098(3.)	A1 A1		
		$t = 0.3125/\sqrt{(0.098../8)}$	M1*		
		= 2.8(1..) Compare with 1.895 Accept mean biomass has reduced	A1 M1 A1	8	dep*
	(ii)	SR: Two sample test			
		Hypotheses	B1		
		$t = (2.75 - 2.4375)/\sqrt{(\sigma^2(1/8 + 1/8))}$	M1*		Using pooled sample estimate
		= 3.0(22..)	A1		
		Compare with 1.761 Accept mean biomass has reduced	M1 A1		dep* Max 5 out of 8
	(iii)	Test indicates reduced mean, not the cause- may be other reasons for reduction.	A1	1	
5	(i)	$\int_1^3 a\,dx + \int_3^6 9a/x^2\,dx = 1$	M1		Correct limits
		2a + 3a/2 = 1 a = 2/7	A1 AG A1	3	
	(ii)	$\int_5^6 18/(7x^2)\,dx$ = 3/35	M1 A1	2	or 0.0857 ; 32/35 or 0.914 => M1A0
	(iii)	$\int_1^m (2/7)\,dx = ½$ m = 275 litres	M1 A1	2	2.75 => M1A0
	(iv)	Use $P = 0.4x^2 \times 100$	B1		
		$\int_1^3 (2P/7)\,dx + \int_3^6 (18P/7x^2)\,dx$ £ 408	M1 A1	3	
6	(i)	$R \sim N(6.01, 0.24^2)$, $H \sim N(0.85, 0.17^2)$ Use distribution of C = R – H E(C) = 5.16; Var(C) = 0.0865	 M1 A1 A1		

		Find P(4.5 < C < 5.5) $\Phi(1.156) - \Phi(-2.244)$ $0.8637 \approx 86\%$	M1 A1 AG A1	6	For both z values
	(ii)	Use N $(5.16n, 0.0865n)$	M1		
		$(125-5.16n)/\sqrt{(0.0865n)} > 1.645$ $5.16n + 0.4838\sqrt{n} - 125 < 0$	A1 AG A1	3	
	(iii)	$f(n) = 5.16n + 0.4838\sqrt{n} - 125$ f(23) = -3.99.. < 0; f(24) = 1.21..> 0	M1 A1	2	
7	(i)	z = (6/50-12/60)/s $p = (6+12)/(50+60) = 9/55$ $s = \sqrt{(\frac{9}{55}\cdot\frac{46}{55}\cdot(\frac{1}{50}+\frac{1}{60}))}$ -1.1(29..) Compare with 1.96 Accept NH – there is insufficient evidence of a difference in the proportion of misprints in the books. SR $z = (6/50-12/60)/[(\frac{6}{50}\cdot\frac{44}{50})/50 + (\frac{12}{60}\cdot\frac{48}{60})/60]$ = - 1.157 Compare with 1.96 Accept NH – there is insufficient evidence of a difference in the proportion of misprints in the books.	M1* B1 M1* A1 A1 M1* A1 M1 A1	6	Or 1.129 M1dep* dep* Max 4 out of 6
	(ii)	Either use Calculus to establish a maximum or Sketch the parabola	M1 A1	2	
	(iii)	Use 1.645 (2) z. $\sqrt{p(1-p)}/\sqrt{100}$ 2 x 1.645 x (1/20) 0.1645	B1 M1 A1 A1	4	

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1	(i)	M(0) = 1; $a = 0.5$	**B1**	1	
	(ii)	$M'(t) = a(2be^{2bt} - be^{-bt})$	**M1**		aef
		$t = 0$, $ab = 0.125$	**M1**		
		$b = 0.25$	**A1**	3	Only if a found
2	(i)	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$ $- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$	**B1**	1	
	(ii)	Substitute ¼ for $P(A \cap B)$ etc and other probs	**M1A1**		
		Answer $^3/_{16}$	**A1**	3	
	(iii)	Show EITHER $P(A \cap B \mid C) \neq P(A \cap B)$			
		OR: $P(A \cap B \cap C) \neq P(A \cap B)P(C)$			
		OR: $P(A)P(B)P(C) \neq P(A \cap B)P(C)$	**M1A1ft**	2	ft $^3/_{16}$
3	(i)	Distribution of % titanium symmetric	**B1**	1	
	(ii)	H_0: $m = 8.5$, H_1: $m \neq 8.5$	**B1**		Not mean or average
		Subtract 8.5 and rank	**M1**		Or subtract from 8.5
		-0.10 0.13 -0.33 0.22 -0.08 0.11 0.15 0.20 -0.12 0.09 0.03 0.18 0.25 4 7 13 11 2 5 8 10 6 3 1 9 12			A1
		$Q = 25$ ($P = 66$)	**B1**		
		Use 25	**B1ft**		ft P, Q
		5% critical region: $T \leq 17$	**B1**		
		Insufficient evidence that average is not .5%	**B1ft**	7	ft 25
		SR: 1-tail test: B0M1A1B1B1B1B1			critical value 21
4	(i)	$E(S^2) = \sigma^2$	**B1**	1	
	(ii)	$Var(S) = E(S^2) - [E(S)]^2$	**B1**	1	Or $E(S^2) - \mu^2$
	(iii)	EITHER $Var(S) > 0$; $[E(S)]^2 < \sigma^2$	**M1M1**		
		$\Rightarrow E(S) < \sigma$	**A1**		
		S is not an unbiased estimator of σ	**A1**		
		Underestimates σ on average	**A1**	5	
		OR: S unbiased estimator $\Rightarrow E(S) = \sigma$	**M1**		
		$\Rightarrow Var(S) = 0$ which is untrue			
		$\Rightarrow$ S is not an unbiased estimate of σ	**A1**		
		$Var(S) > 0 \Rightarrow [E(S)]^2 < \sigma^2$	**M1**		
		$\Rightarrow E(S) < \sigma$	**A1**		
		$\Rightarrow$ S underestimates σ	**A1**	5	B1/3 for unsupported statement
5	(i)	Consider 2,2,3 and 2,3,3 with permutations	**M1**		
		3(⅓)(⅓)(½) + 3(½)(½)(⅓)	**A1**	2	
	(ii)	Correct method for finding $E(L)$ or $E(S)$	**M1**		
		$E(L) = {}^{155}/_{54}$; $E(S) = {}^{46}/_{27}$	**A1A1**	3	

	(iii)	$1(^1/_{216})+2(^1/_{12})+3(^1/_3)+4(^1/_{27})+6(^5/_{12})+9(^1/_8)$ $= {}^{89}/_{18}$	**M1** **A1**	2	Allow one error
	(iv)	${}^{89}/_{18} - ({}^{46}/_{27})({}^{155}/_{54})$	**M1**		Not $E(LS)=E(L)E(S)$
		$= {}^{79}/_{1458}$	**A1**	2	art 0.0542
6	(i)	$\sum e^{-\lambda}\lambda^x t^x/x!$ for $x=0$ to ∞	**M1**		Allow finite, or no $x=0$
		$=e^{-\lambda}\sum(\lambda t)^x/x!$	**A1**		
		$=e^{-\lambda}e^{\lambda t} = e^{-\lambda(1-t)}$	**AG** **A1**	3	
	(ii)	$e^{-\lambda(1-t)}\ e^{-\mu(1-t)}$	**M1**		
		$=e^{-(\lambda+\mu)(1-t)}$ => $Po(\lambda+\mu)$	**A1**	2	Correct form
	(iii)	EITHER: $X - Y$ takes negative values			
		OR: $E(X-Y) = \lambda - \mu$, $Var(X-Y) = \lambda + \mu$, unequal	**B1**	1	
		OR: $E(X-Y)$ could be negative			
	(iv)	Use Po(6) to find $P(X+Y=5)$	**M1**		$\lambda=6$
		EITHER $e^{-6}6^5/5!$ OR 0.4457 – 0.2851	**M1**		Attempt at 6
		0.1606	**A1**	3	art 0.161
	(v)	$P(X=3,Y=2$ or $X=4,Y=1$ or $X=5,Y=0)$	**M1**		Allow one pair missing
		$e^{-6}[(2.8^3/3!)(3.2^2/2!)+(2.8^4/4!)3.2+2.8^5/5!]$	**A1**		
		$\div e^{-6}6^5/5!$	**M1**		
		$0.4377 \approx 0.438$	**A1**	4	Cao
7	(i)				
		A and *B* have identical (continuous) distns (apart from location).	**B1**	1	
	(ii)	$H_0: m_A = m_B$, $H_1: m_A > m_B$	**B1**		Allow medians
		Rank samples	**M1**		
		22 26 34 36 42 43 47 48 50 51 58 59 60 61 63 74 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 *B B B A B A A A B A B B A B A A*			A1
		$R_A = 79, R_B = 57$	**B1**		
		$W = 57$	**B1ft**		ft R_A, R_B
		5% critical region, $W \le 51$	**M1**		
		Accept new variety not less noisy than old	**A1ft**	7	ft W
	(iii)	$W \sim N(68, 90⅔)$	**B1B1**		Mean and variance
		$\pm[57.5 - 68]/\sqrt{(90⅔)}$	**M1**		With or without cc
		–1.103	**A1ft**		ft μ, σ with cc
		0.135	**A1**	5	

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding

1 (i) Σd^2 = 14 $1-\frac{6\times their\,14}{5\times(25-1)}$ = 0.3	M1 A1 M1 A1 4	Subtr & squ 5 pairs & add dep 1st M1 $S_{xy} = 48 - \frac{15\times15}{5}$ {= 3}; $S_{xx} = 55 - \frac{15^2}{5}$ {= 10}; $S_{yy} = 55 - \frac{15^2}{5}$ {= 10} M1 A1 their $\frac{S_{xy}}{\sqrt{(S_{xx}S_{yy})}}$ M1dep = 0.3 A1
(ii) Reverse rankings attempted 2 5 3 4 1	M1 A1 2	3 correct T & I to make $\Sigma d^2 = 40$: 2 mks or 0 mks
	6	
2 (i) (a) Geo(0.14) stated in (a) or (b) $(0.86)^4$ x 0.14 = 0.0766 (3 sfs)	B1 M1 A1 3	or 0.86^nx0.14 or 0.14^nx0.86 in (a) or ≥ M1 in (b) or Geo(0.86) stated in (a) or (b) No wking: 0.077: B1M1A0
(b) $1 - 0.86^7$ or 0.14 + 0.86x0.14…+ 0.86^6x0.14 = 0.652 (3 sfs)	M2 A1 3	$1 - 0.86^8$: M1 +8th term (r = 7 or 0) or 1 missing term: M1
(ii) 1/0.14 = $^{50}/_7$ or 7.14 (3 sfs)	M1 A1 2	
	8	
3 (i) (a) B(16, 0.35) stated 1 – 0.8406 = 0.159 (3 sfs)	B1 M1 A1 3	Or implied by use of tables or 0.35^ax0.65^b ($a+b$ = 16) in (a) or (b) Allow 1 – 0.9329 or 0.0671 Or complete method using formula, P(r = 8-16 or 9-16) or 1-P(r = 0-7 or 0-8)
(b) 0.9771 – 0.1339 = 0.843 (3 sfs)	M1 A1 2	Allow 0.9771 – 0.2892 Or complete method using formula (r = 4-9)
(ii) $^{16}C_6(0.38)^6(0.62)^{10}$ = 0.202 (3 sfs)	M2 A1 3	Absent or incorr coeff : M1 or $^{16}C_6(0.38)^{10}(0.62)^6$: M1
	8	
4 (i) Correct subst in ≥ two S formulae $\frac{14464.1-\frac{265\times274.6}{5}}{\sqrt{\left(14176.54-\frac{265^2}{5}\right)\left(15162.22-\frac{274.6^2}{5}\right)}}$ = –0.868 (3 sfs)	M1 M1 A1 3	Any correct version or $\frac{14464.1-5\times53\times54.92}{\sqrt{(14176.54-5\times53^2)(15162.22-5\times54.92^2)}}$ or fully correct method with $(x-\bar{x})^2$ etc
(ii) No difference oe	B1 1	Or slightly diff or more acc because of rounding errors when mult by 2.54 oe Not just "more accurate"
(iii)Choose y on x stated	B1ind	or implied, eg by S_{xy}/S_{xx} or $y = ax + b$

$\dfrac{14464.1-\frac{265\times274.6}{5}}{14176.54-\frac{265^2}{5}}$ or – 0.682	M1	If state x on y, but wking is y on x: B1 or their $\dfrac{-89.7}{131.54}$ seen or $\dfrac{14464.1-5\times53\times54.92}{14176.54-5\times53^2}$ or correct subst into a correct formula $\dfrac{S_{xy}}{S_{xx}}$
$y - {}^{274.6}/_5$ = (their – 0.682)(x – ${}^{265}/_5$) y = 91(.1) – 0.68(2) x	M1ind A1	or $a = {}^{274.6}/_5$ - (their – 0.682) x ${}^{265}/_5$ Simplif to 3 terms. Coeffs to $\geq$ 2 sfs
49.9 (3sfs) or 50	A1 5	cao Use of x on y: equiv M mks as above
	9	
5 (i) Read at 300 or 300.25 and 900 or 900.75 44.5 to 45.5 and 69 to 69.9 IQR 23.5 to 25.4	M1 A1 A1 3	or 44-46 and 68-70 incl. dep A1 Must look back, see method. No wking, ans in range: M1A1A1
(ii) 0.6 or 60% CF 720 63 to 64	M1 M1 A1 3	Seen or implied Seen or implied 55.5 to 56: SC B1
(iii) 1200 – 860 = 340	M1 A1 2	Allow 1200 – (850 to 890) 310 to 350
(iv) 340/1200 0.283^5 = 0.00183	M1 M1dep A1 3	their (iii)/1200 [their (iii)/1200]^5 exactly Allow 0.00114 to 0.00212 $\geq$ 2 sfs $^{340}C_5/^{1200}C_5$ M1
(v) Incorrect reason or ambiguity: B0B0. Otherwise: Too low, or should be 26 or 27 or 2 or 3 higher	B2 2	eg IQR = 55–35 = 20 or IQR = value >27 or new info' implies straight line: B1 or originally, majority in range 35 – 55 are at top of this range: B1
	13	
6 (i) $a = {}^4/_5$, $b = {}^1/_5$ c = ¼ , d = ¾ e = ¾ , f = ¼	B1 B1B1 B1 4	Or : B1 { ie: a, b : B1 B1 { another pair : B1B1 B1B1 { third pair : B1
(ii) ${}^1/_2 \times {}^4/_5 \times {}^1/_2 + {}^1/_2 \times {}^1/_5 \times {}^1/_4 + {}^1/_2 \times {}^3/_5 \times {}^3/_4$ $= \frac{9}{20}$ (**AG**) with no errors seen	M2 A1 3	M1: one correct product (M2 needs +) ft their values for M mks only
(iii) 1/10 + 9/20 + k + 1/5 = 1 oe or ${}^1/_2 \times {}^1/_5 \times {}^3/_4 + {}^1/_2 \times {}^3/_5 \times {}^1/_4 + {}^1/_2 \times {}^2/_5 \times {}^1/_2$ k = ¼ oe	M1 A1 2	 ft their values for M mk only
(iv) $\Sigma x\mathrm{p}(x)$ = $1\frac{3}{4}$ oe $\Sigma x^2\mathrm{p}(x)$ [= $3\frac{17}{20}$] $\Sigma x^2\mathrm{p}(x)$ – (their μ)^2 63/80 or 0.788 (3 sfs)	M1 A1 M1 M1ind A1 5	Allow omit 1st term only. Not ISW, eg ÷ 4 cao Allow omit 1st term only. Not ISW, eg ÷ 4 Subtract (their μ)^2, if result +ve Follow their k for M mks only $\Sigma(x - \mu)^2\mathrm{p}(x)$: Single consistent pair: M1 Rest correct : M1
	14	

7 (i) ${}^{18}C_7$ or ${}^{18!}/_{(11!\times 7!)}$ = 31824	M1 A1	 2	 cao
(ii) ${}^5C_2 \times {}^6C_2 \times {}^7C_3$ or 5250 ÷ 31824 = 875/5304 or 5250/31824 oe or 0.165 (3 sfs)	M2 M1 A1	 4	M1: 1 correct ${}^nC_r$ or mult any three ${}^nC_r$s Divide by their (i). Indep If cancelled, must be clear have ÷ 31824 $\frac{5 \times 4 \times 6 \times 5 \times 7 \times 6 \times 5 \times 7!}{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 2!^2 \times 3!}$ Correct 7 fractions mult: M1 x7!: M1} ÷ (2!²x3!): M1}both dep any 7 fracts mult
(iii) 5 from W & 2 from (G + H) ${}^7C_5 \times {}^{11}C_2$ or 1155 ÷ 31824 = 385/10608 or 1155/31824 oe or 0.0363 (3 sfs)	M1 M1 M1 A1	 4	Seen or implied, eg by combs or list Divide by their (i). Indep $\frac{7 \times 6 \times 5 \times 4 \times 3 \times 11 \times 10 \times 7!}{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 5! \times 2!}$ Correct 7 fractions mult: M1 x 7! : M1} ÷ (5! x 2!): M1} both dep any 7 fracts mult
(iv) (2, 2, 3) or (2, 3, 2) or (3, 2, 2) ${}^5C_2 \times {}^6C_2 \times {}^7C_3 + {}^5C_2 \times {}^6C_3 \times {}^7C_2$ $+ {}^5C_3 \times {}^6C_2 \times {}^7C_2$ (÷ 31824) = 175/442 or 12600/31824 oe or 0.396 (3 sfs)	M1 M2 A1	 4	Any one. Seen or implied eg by combs M1: one correct product. NOT ${}^5C_2 \times {}^6C_2 \times {}^7C_2$ (No mk for ÷ 31824) Equiv method; ((ii) + etc) can imply M mks $\frac{5 \times 4 \times 6 \times 5 \times 7 \times 6 \times 7!}{18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 2!^2 \times 3!}$ Correct 6 fractions mult: M1 x 7! : M1} ÷ (2!² x 3!) : M1}both dep any 6 fracts mult Complement method: Triple with total 7, incl at least one 0 or 1 or (0, 7) or (1, 6) seen or implied: M1 One correct prod seen, eg ${}^5C_0 \times {}^6C_2 \times {}^7C_5$ M1 Full correct method, incl "1 – " M1
	14		

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1	(i)	Method is biased because many pupils cannot be chosen	B1 B1 **2**	"Biased" or equivalent stated, allow "not random" Valid relevant reason
	(ii)	Allocate a number to each pupil Select using random numbers	B1 B1 **2**	State "list numbered" Use random numbers [*not* "hat"]
2		$\frac{20-25}{\sigma} = \Phi^{-1}(0.25) = -0.674$ $\sigma = 5 \div 0.674$ $= 7.42$	M1 B1 M1 A1 **4**	Standardise and equate to Φ^{-1} [*not* .7754 or .5987] z in range [–0.675, –0.674], allow + (±) 5 ÷ z-value [*not* $\Phi(z)$ or 0.75] Answer in range [7.41, 7.42], no sign fudges [SR: σ^2: M1B1M0A0 cc: M1B1M1A0]
3	(a)	Po(1.2) Tables or correct formula used 0.8795	B1 M1 A1 **3**	Po(1.2) stated or implied Correct method for Poisson probability, allow "1 –" Answer, 0.8795 or 0.879 or 0.88(0)
	(b)	N(30, 30) $\frac{38.5-30}{\sqrt{30}}$ [= 1.55] [$\Phi(1.55)$ =] 0.9396	B1 B1 M1 A1 A1 **5**	Normal, mean 30 stated or implied Variance 30 stated or implied, allow √30 or 30^2 Standardise using $\sigma^2 = \mu$, allow √ or cc errors $\sqrt{\mu}$ and 38.5 both correct Answer in range [0.939, 0.94(0)]
4	(i)	$\hat{\sigma}^2 = \frac{50}{49} \times 0.0967 = 0.0987$	M1 A1 **2**	Use $\frac{n}{n-1} \times s$ or s^2, allow √ Answer, a.r.t. 0.0987
	(ii)	$H_0: \mu = 1.8$, $H_1: \mu \neq 1.8$ where μ is the population mean	B1B1	Hypotheses correctly stated in terms of μ SR: μ wrong/omitted: B1 both, but $\overline{X}$: B0
	α, β:	$z = \frac{(1.72-1.8)}{\hat{\sigma}/\sqrt{50}} = -1.8(006)$	M1 A1	Standardise with $\sqrt{n}$, allow +, biased σ, √ errors $z = -1.80 \pm 0.01$, don't allow +
	α:	$-1.8 < -1.645$	B1√	Compare ± z with ±1.645, signs consistent
	β:	$\Phi(-1.8) = 1 - 0.9641 < 0.05$	B1	Explicitly compare $\Phi(z)$ with 0.05, correct tail
	γ:	CV $1.8 - k.\sigma/\sqrt{50}$ $k = 1.645$, CV = 1.727 $1.72 < 1.727$	M1 A1√ B1√	Correct expression for CV, – or ±, k from Φ^{-1} CV = 1.727, √ on their k, ignore upper limit k = 1.645 and compare CV with 1.72
		Reject H_0 Significant evidence that mean height is not 1.8	M1 A1√ **7**	Reject H_0√, correct method, needs √50, μ = 1.8; allow cc,√ or k error or biased σ estimate Conclusion stated in context [SR: 1.8, 1.72 interchanged: B0B0M1A0B1M0]
5	(i)	${}^{30}C_{10}(0.4)^{10}(0.6)^{20}$ or 0.2915 – 0.1763 = 0.1152	M1 A1 **2**	Correct formula or use of tables Answer, a.r.t. 0.115
	(ii)	$30p > 5$ so $p > \frac{1}{6}$ $30q > 5$ so $q > \frac{1}{6}$ $\frac{1}{6} < p < \frac{5}{6}$	M1 M1 A1 **3**	$30p$ or $30pq$ used $30q$ or both solutions from $30pq$ used *Either* $\frac{1}{6} < p < \frac{5}{6}$ *or* $\left[\frac{1}{2} - \frac{\sqrt{3}}{6} < p < \frac{1}{2} + \frac{\sqrt{3}}{6}\right]$ [$0.211 < p < 0.789$], allow ≤
	(iii)	N(12, 7.2) $\frac{10.5-np}{\sqrt{npq}}$ and $\frac{9.5-np}{\sqrt{npq}}$ $\Phi(-0.559) - \Phi(-0.9317)$ = 0.8243 – 0.7119 = 0.1124	B1 B1 M1 A1√ M1 A1 **6**	12 seen 7.2 or 2.683 seen, allow 7.2^2 Both standardised, allow wrong/no cc, npq $\sqrt{npq}$, 10.5 and 9.5 correct, √ on their np, npq Correct use of tails Answer, in range [0.112, 0.113] [SR: $\frac{1}{\sqrt{2\pi \times 7.2}} e^{-\frac{1}{2}\frac{(10-12)^2}{7.2}}$ M1A1, answer A2]

6	(i)	$R \sim B(25, 0.8)$ $P(R \le 16) = 0.0468$, $P(R \le 17) = 0.1091$ $k = 16$	B1 M1 A1 **3**	B(25, 0.8) stated or implied, e.g. from N(20, 4) One relevant probability seen [Normal: M0A0] Answer $k = 16$ only [SR: unsupported 16, B1M0B1]
	(ii)	$20p$ $= 0.936$	M1 A1 **2**	$20 \times$ their p *or* 20×0.05 Answer, a.r.t. 0.936, i.s.w.
	(iii)	$P(R \le 16 \mid p = 0.6)$ $= 0.7265$	M1 A1 **2**	Find $P(R \le k \mid p = 0.6)$ Answer 0.7265 or 0.727
	(iv) α:	$p' = 0.5 \times 0.0468 + 0.5 \times 0.7265$ $= 0.38665$ $2 \times p' \times (1 - p')$ $= 0.474$	M1 A1 M1 A1 **4**	"Tree diagram" probability, any sensible p Value in range [0.38, 0.39] Correct formula, including 2, any p' Answer in range [0.47, 0.48]
or β:		0.8 A 0.8 R $.5^2 \times .9532 \times .0468 = .0112$ 0.8 R 0.8 A $.5^2 \times .0468 \times .9532 = .0112$ 0.6 A 0.8 R $.5^2 \times .2735 \times .0468 = .0032$ 0.6 R 0.8 A $.5^2 \times .7265 \times .9532 = .1731$ 0.8 A 0.6 R $.5^2 \times .9532 \times .7265 = .1731$ 0.8 R 0.6 A $.5^2 \times .0468 \times .2735 = .0032$ 0.6 A 0.6 R $.5^2 \times .2735 \times .7265 = .0497$ 0.6 R 0.6 A $.5^2 \times .7265 \times .2735 = .0497$	M1 A1 M1 A1	$p_1q_2 + p_2q_1$ etc (0.5 not needed) 4 cases, √ on their ps and qs, 0.5 not needed e.g. $2(p_1q_2 + p_2q_1)$ Completely correct list of cases and probabilities, including 0.5 Answer in range [0.47, 0.48]
7	(i)	$(11 - 3)k = 1$ $k = 1/8$	M1 A1 **2**	Use area = 1 [e.g. $\int kx\,dx = 1$ with limits 3, 11] Answer 1/8 or 0.125 only
	(ii)	$\mu = ½(3 + 11) = 7$ $\int_3^{11} \frac{1}{8}x^2\,dx = \left[\frac{x^3}{24}\right]_3^{11}$ [= $54\frac{1}{3}$] $\sigma^2 = 54\frac{1}{3} - 7^2$ $= 5\frac{1}{3}$	B1 M1 A1 M1 A1 **5**	Mean 7, cwd Attempt $\int x^2 f(x)\,dx$, correct limits Indefinite integral $\frac{x^3}{3k}$, their k Subtract their μ^2 Correct answer, $5\frac{1}{3}$ or a.r.t. 5.33
	(iii)	$P(X < 9) = 6k$ [= ¾] $(¾)^3$ $= \frac{27}{64}$ or 0.421875	B1√ M1 A1 **3**	Correct p for their k Work out their p^3, $0 < p < 1$ Answer $\frac{27}{64}$ or a.r.t. 0.422
	(iv)	Normal Mean is 7 Variance is $5\frac{1}{3} \div 32$ (= $\frac{1}{6}$)	B1 B1√ B1√ **3**	"Normal" distribution stated Mean same as in (ii) √ Variance is [(iii) ÷ 32] √ [*not* √ errors]
8	(i)	Coins occur at constant average rate and independently of one another	B1 B1 **2**	One contextualised condition, e.g. independent A different one, e.g. constant average rate, or "not in hoards" ["singly" not enough]. Treat "random" as equivalent to "independent". Allow "They…"
	(ii)	$R \sim Po(5.4)$ $e^{-5.4}\frac{5.4^3}{3!} = 0.1185$	B1 M1 A1 **3**	Poisson (5.4) stated or implied Correct formula, any λ Answer, in range [0.118, 0.119]
	(iii)	$R \sim Po(3)$ Tables, looking for 0.05 or 0.95 $P(R \ge 7) = 0.0335$ Therefore smallest number is 7	B1 M1 A1√ A1 **4**	Poisson (3) stated or implied Evidence of correct use of tables One relevant correct probability seen $r = 7$ only, ignore inequalities
	(iv)	$R \sim Po(4.8)$ Type II error is $R < 7$ when $\mu = 4.8$ $P(< 7) = 0.7908$	B1 M1 A1 **3**	Poisson (4.8) used Correct context for Type II error, √ on their r $P(< 7)$, a.r.t. 0.791, c.w.o. [$P(\ge 7)$: M0]

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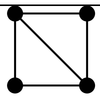
1 (i)	Original list 8 9 5 4 3 comps swaps After 1st pass 8 5 4 3 9 4 3 After 2nd pass 5 4 3 8 9 3 3 After 3rd pass 4 3 5 8 9 2 2 After 4th pass 3 4 5 8 9 1 1	M1 A1 A1 **3**	First pass correct Results of passes all correct Counting comparisons and swaps correctly
(ii)	$(0.1) \times (500 \div 100)^2$ = 2.5 seconds	M1 A1 **2**	$0.1 = (1\times10^{-5})\times100^2$ and $(1\times10^{-5})\times500^2$ Cao Working can be implied by correct answer **Total = 5**
2 (i)		B1 **1**	Or anything topologically equivalent to this
(ii)	The sum of the orders of the vertices is twice the number of arcs, and hence is even. Hence the sum of the odd orders must be even and so there must be an even number of odd vertices.	M1 A1 **2**	Start from a null graph and successively add in arcs. Each time an arc is added the number of odd vertices is either unchanged or it increases or decreases by 2. So the number of odd nodes is always even
(iii)	5 arcs ⇒ sum of orders of vertices = 10 Simple graph so each vertex has order 1, 2 or 3 1 + 3 + 3 + 3 = 10 or 2 + 2 + 3 + 3 = 10 But 1 + 3 + 3 + 3 is not possible since if three vertices have order 3 they are all connected to the fourth vertex so it also has order 3. With 2 + 2 + 3 + 3 the two vertices of order 2 cannot be adjacent. Hence only one possible graph.	M1 A1 A1 **3**	Explaining why 1 + 3 + 3 + 3 is not possible. Expaining why there is only one graph with nodes of orders 2, 2, 3, 3. **Total = 6**
3 (i)	*DF* , *CD*, *BD* and *EF*, *FH*, *AC*, *EG* 40	M1 A1 B1 **3**	Correct arcs listed or drawn A valid order (Prim or Kruskal)
(ii)	*A C D F E G H B A*	M1 A1 **2**	At least as far as *A C D F E G* or diagram with no arrows. Fully directed diagram
(iii)	(a) *ACEG* and *ABGH* (b) 5 (c) *ABCD*	B1 B1 B1 **3**	 Both **Total = 8**

4 (i)

P	x	y	z	s	t	u	-
1	-8	1	4	0	0	0	0
0	10	1	8	1	0	0	40
0	12	4	9	0	1	0	72
0	-6	4	3	0	0	1	48

B1 $\pm$(-8 1 4 0 0 0)

B1 Constraint rows correct

2

(ii) Pivot on 10 in x column — B1 Correct choice of pivot

P	x	y	z	S	t	u	-
1	0	1.8	10.4	0.8	0	0	32
0	1	0.1	0.8	0.1	0	0	4
0	0	2.8	-0.6	-1.2	1	0	24

$x = 4$, $y = 0$, $z = 0$

$P = 32$

M1 P has increased

M1 Three basis columns (apart from P)

A1 Correct final tableau

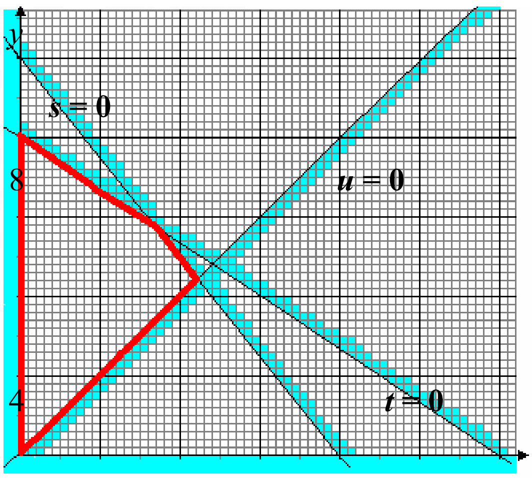
B1 follow through their final tableau

B1 follow through their final tableau,

6

Total = 8

5 (i)



M2 Plotting lines accurately
-1 each error or omission

A1 Shading

3

(ii) $5x + 2y + s = 20$ or equivalent

$4x + 3y + t = 24$

$2x - y + u = 0$

Slack (s, t and u) must be <u>added</u>

B1 Both $5x + 2y + s = 20$ and $4x + 3y + t = 24$

B1 <u>Not</u> $-2x + y + u = 0$

2

(iii) Labelling $s = 0$, $t = 0$, $u = 0$ (see diagram)

B2 follow through equations where possible (if labelled)
-1 each error or omission condone s, t, u

2

(iv) Checking vertices of feasible region

($x = 0, y = 0 \Rightarrow P = 0$)

$x = 2.2, y = 4.4 \Rightarrow P = 8.9$ (8.8)

$\boldsymbol{x = 1.7, y = 5.7 \Rightarrow P = 9.1}$ *

$x = 0, y = 8 \Rightarrow P = 8$

M1 Or using a line of constant profit

A1 $x = 1.7$ (12/7), $y = 5.7$ (40/7) at optimum
$P = 9.1$ (64/7) at optimum
Allow $\pm$0.1 for x, y, P

3

Total = 10

6 (i)	(Minimise) $C = 15x + 14y + 10z$	B1 **1**	$15x + 14y + 10z$ or scaled
(ii)	Number of large tiles = $20x + 25y + 10z$ Number of small tiles = $40x + 10y + 50z$	B1	
	large + 0.25× small ≥ 250	B1	4 small are equivalent to 1 large
	$20x + 25y + 10z + 10x + 2.5y + 12.5z \geq 250$ $\Rightarrow\ 30x + 27.5y + 22.5z \geq 250$ $\Rightarrow\ 12x + 11y + 9z \geq 100$	M1 A1 **4**	Simplifying the constraint given (remove brackets) cao
(iii)	small ≥ 0.5 × large $\Rightarrow\ 40x + 10y + 50z \geq 10x + 12.5y + 5z$ $\Rightarrow\ 12x + 18z \geq y$	M1 M1 A1	 A contraint equivalent to this cao
	Number of edging tiles = $10x + 5y + 20z$ $\Rightarrow\ 10x + 5y + 20z \geq 25$ $\Rightarrow\ 2x + y + 4z \geq 5$	M1 A1	A constraint equivalent to this cao
	$x \geq 0$, $y \geq 0$ and $z \geq 0$	B1 **6**	All three non-negativity constraints (ignore integer requirement) **Total = 11**
7 (i)	*A*: 1 | 0 *B*: 3 | 3; 3 *C*: 2 | 2; 2 *D*: 5 | 18; 18 *E*: 4 | 17; 17 *F*: 7 | 25; 26 25 *G*: 6 | 20; 20 *H*: 8 | 26; 29 26	M1 A1	Attempt at updating labels (Check *F* and *H*, ignore any extras) All permanent labels correct
	Order of labelling: *A C B E D G F H*	B1	A valid order of labelling (May leave out *A* and have *C* as 1st, *B* as 2nd, etc)
(ii)	Shortest path: *A - G - H*	B1 **4**	
	AB = 3, *CH* = 27, total 30 *AC* = 2, *BH* = 29, total 31 *AH* = 26, *BC* = 5, total 31	B1 M1 M1 A1	Odd nodes *A*, *B*, *C* and *H* All three pairings with lengths attempted Any one pairing with correct total (or implied) Identifying 30 (or arcs *AB* and *CH*)
(iii)	Length = 155 km	B1 **5**	cao
	AC = 2, *DG* = 8 and *GH* = 6 141 km	M1 M1 A1 **3**	Identify *AC DH* as odd nodes to pair 2 + 14 Cao Method marks can be implied from correct answer **Total = 12**

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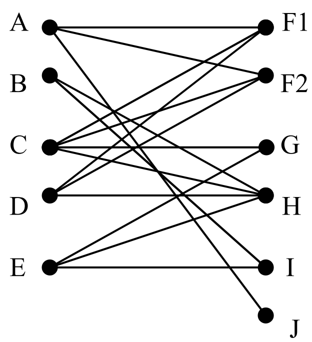
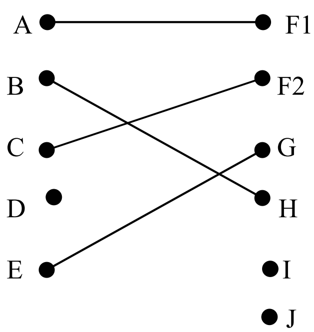
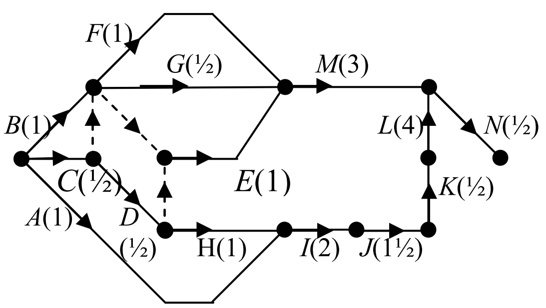
1 (i)		M1 A1 **2**	All correct apart from football cao
(ii)		B1 **1**	Allow any reasonable way of dealing with football
(iii)	I-B-H-D or I-E-G-C-F2-D or equivalent listed or clearly shown on a diagram	B1	Must start from I F2 may be replaced by F1 or F
	A - F1, B – I, C – F2, D – H, E - G or A - F1, B – H, C – G, D – F2, E – I	B1 **2**	Need not follow from their path F2 and F1 may be interchanged, or both just F
(iv)	Alan gets judo Belinda gets hockey Charlie gets football Doug gets football Emma gets golf	B1 **1**	Or on a diagram or listed using letters A – J, B – H, C – F, D – F, E – G Need <u>all five</u> pairings shown, or state A-J, D-F <u>and</u> say that the other pupils are as before **Total = 6**

2	Subtract all entries from a constant eg L M N O P 3 3 4 6 Q 5 3 4 0 R 0 3 0 5 S 4 3 4 1	B1	Accept any 'constant value – entry' or each row subtracted from its row maximum
	Reduce rows 0 0 1 3 5 3 4 0 0 3 0 5 3 2 3 0 Columns are reduced	M1	Reducing rows and columns Follow through their matrix (even if now minimising instead of maximising)
	Incomplete matching 0 0 1 3 5 3 4 0 0 3 0 5 3 2 3 0	A1	Covering with minimum number of lines ft their reduced matrix
	Augment by adding 2 0 0 1 5 3 1 2 0 0 3 0 7 1 0 1 0	M1	Correctly augmenting their matrix (do not award if no augmenting needed)
		A1	cao (not follow through)
	Complete matching Lucy = Parachute jump Mark = Steeple climb Nicola = Racing biker Oliver = Quarterback	B1 **6**	cao listed or shown on matrix **Total = 6**

3 (i)	*AD*, *CF* and *EB* 8 + 7 + 2 = 17	B1 M1 A1 **3**	cao (NOT *BE* and no extras) 8+7+0 or 8+7-2 seen ⇒ M1, A0
(ii)	*SA* = 0 *AD* = 3 *BD* = 4 *CE* = 5 *DT* = 2 *AS* = 5 *DA* = 5 *DB* = 0 *EC* = 0 *TD* = 5 *SC* = 2 *AE* = 6 *BE* = 0 *CF* = 1 *FT* = 0 *CS* = 6 *EA* = 0 *EB* = 2 *FC* = 6 *TF* = 6 *BF* = 5 *FB* = 0	M1 A1 **2**	condone all reversed and up to 3 errors cao (or all reversed)
(iii)	*S C E B D T* *SA* = 0 *AD* = 3 ***BD* = 2** ***CE* = 3** ***DT* = 0** *AS* = 5 *DA* = 5 ***DB* = 2** ***EC* = 2** ***TD* = 7** ***SC* = 0** *AE* = 6 ***BE* = 2** *CF* = 1 *FT* = 0 ***CS* = 8** *EA* = 0 ***EB* = 0** *FC* = 6 *TF* = 6 *BF* = 5 *FB* = 0	B1 B1 **2**	Correct flow augmenting route listed cao (or all reversed)
(iv)	Cut {*S*,*A*,*B*,*C*,*D*,*E*,*F*} {*T*} = 13 Flow in (iii) is 13 litres per second	B1 B1 **2**	For this cut or the cut {*S*} {*A*, *B*, *C*, *D*, *E*, *F*, *T*} in any form, or for 'no more can flow into *T*' or 'no more can flow out of *S*', or equivalent. For flow shown = 13 <u>or</u> max flow ≥ 13 ≥ min cut (but NOT just stating max flow = min cut) Value 13 given in question **Total = 9**

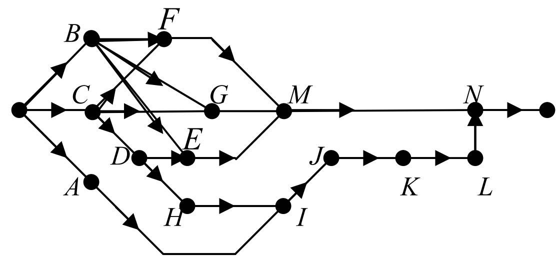
4 (i)

Either activity on arc (durations not necessary)

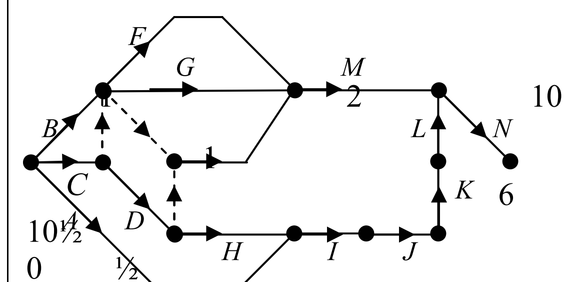


M1 A1

Or activity on node (may omit start and/or end)



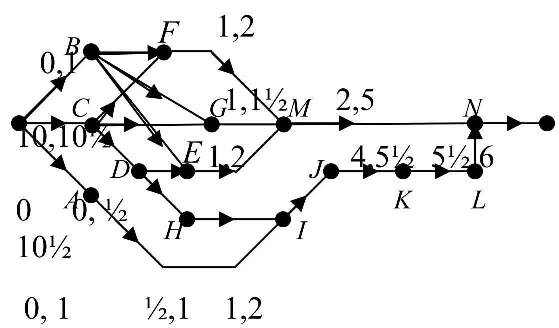
Forward pass



Minimum completion time = 10½ (hours)

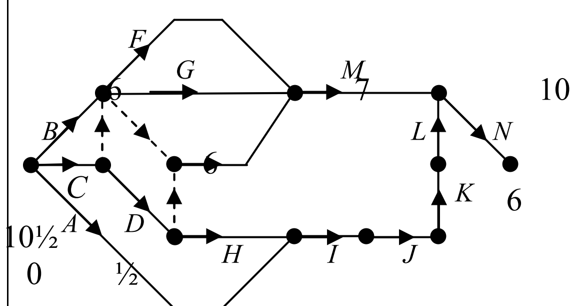
M1 A1 B1

Forward pass (either start or end, consistently)



Minimum completion time = 10½ (hours)

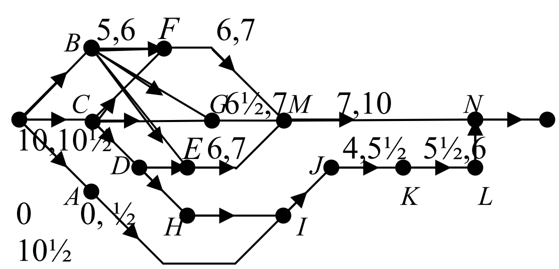
Backward pass



Critical activities: *C*, *D*, *H*, *I*, *J*, *K*, *L*, *N*

M1 A1 B1 **8**

Backward pass (either start or end, consistently)

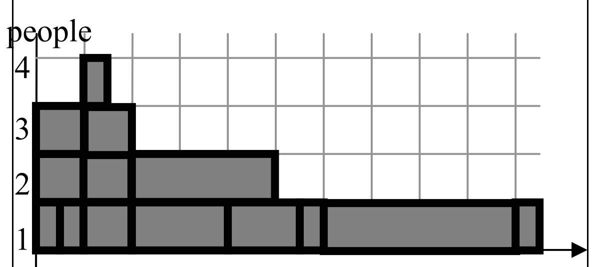


Critical activities: *C*, *D*, *H*, *I*, *J*, *K*, *L*, *N*

(ii) None, activity *M* can be finished before *L* starts — B1 **1**

None

(iii)



B1 — Axes labelled and scaled appropriately (scaling may be implied)

M1 — Substantially correct attempt. No hanging blocks or gaps

A1 — cao (no need to see individual activities)

Can be reduced to 3 people by, for example, delaying the start of *G* until after *F* and then following this with *M*

M1 — Substantially correct attempt to reduce to 3 people

A1 — A correct set of changes described.

5

Total = 14

5

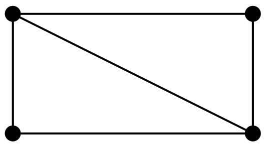
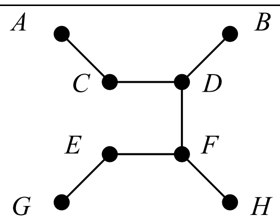
stage	state	action	working	maximum
1	0	0	4	*
	1	0	4	*
2	0	0	4+4= 8	
		1	5+4= 9	*
	1	0	6+4=10	*
	2	1	7+4=11	*
	3	0	5+4= 9	
		1	6+4=10	*
3	0	1	8+10=18	*
		3	6+10=16	
	1	0	7+ 9=16	
		2	6+11=17	*
	2	0	7+ 9=16	
		2	6+11=17	
		3	8+10=18	*
4	0	0	5+18=23	
		1	8+17=25	*
	1	0	7+18=25	*
		2	5+18=23	
5	0	0	6+25=31	
		1	8+25=33	*

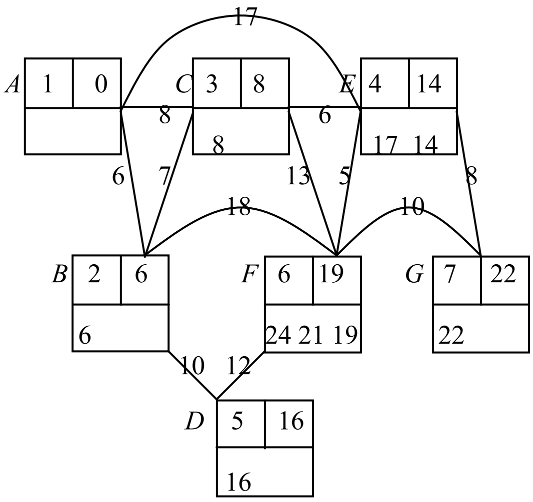
Route: (0; 0) – (1; 0) – (2; 1) – (3; 0) – (4; 1) – (5; 0)
33 plants

Mark	Guidance
M1	Structure of table essentially correct
A1	Stage and state columns correct
A1	Action values correct
M1	All calculations correct for stages 1 and 2 (may be seen as an addition or the result and may be shown in final column)
A1	Suboptimal maxima identified correctly (may be implied from next stage)
M1	Calculations correct for stage 3, ft from stage 2
A1	Suboptimal maxima correct, ft their totals (may be implied from next stage)
M1	Calculations correct for stages 4 and 5, (follow through from stage 3)
A1	Calculations correct for entire table
B1	Correct route or in reverse (cao)
B1	33(cao)
11	**Total = 11**

Question	Answer	Marks	Guidance
6 (i)	What one player wins the other loses	B1 **1**	Total is always zero
(ii)	*S* and *T*: $3>-2$ but $-2<1$ (or $-1<2$) *S* and *U*: $3>1$ (or $-1>-2$) but $-2<3$ *T* and *U*: $-2<1$ (or $1<3$) but $2>-2$	M1 A1	Or using differences or by considering rows where column maxima and/or minima occur or in words Valid explanation
	D and *E*: $3>-2$ but $-2<1$ (or $1<3$) *D* and *F*: $3>-1$ (or $1>-2$) but $-2<2$ *E* and *F*: $-2<-1$ (or $1<2$) but $3>-2$	M1 A1 **4**	Or using differences or by considering columns where row maxima and/or minima occur or in words Valid explanation
(iii)	Row minima are -2, -2, -2 $\Rightarrow$ row maximin = -2 Column maxima are 3, 3, 2 $\Rightarrow$ col minimax = 2 $2 \neq -2 \Rightarrow$ not stable	M1 A1 **2**	Or identifying <u>all</u> rows <u>and</u> column *F* Or equivalent, or described in words
(iv)	So that for $p_1, p_2, p_3 \geq 0$ we will have $m \geq 0$	B1 **1**	So that $m \geq 0$ (NOT sufficient to just say that we need to make all the entries non-negative)
(v)	If Colin plays *D*, with the augmented payoffs Rhoda will expect to win $5p_1+0p_2+3p_3$, and similarly for when Colin chooses *E* or *F* *m* is the minimum of the augmented expected winnings	B1 B1 **2**	Explaining any of the three RHS expressions Explaining why $m \leq$ each expression
(vi)	- 0.125	B1 **1**	-1/8
(vii)	eg toss the coin 3 times, to give 8 possible outcomes allocate 3 of these to 'play *S*' and 5 to 'play *T*' In the long run she expects to lose 1/8 per game	M1 A1 B1 **3**	Or give a specific example, or any other equivalent method lose (at least) 1/8 per game **Total = 14**

Mark Scheme 4736
June 2005

1	(a) (i)	8 7 5 4 3 3 3 3 2 2 First bag 8 2 Second bag 7 3 Third bag 5 4 Fourth bag 3 3 3 Fifth bag 2	M1 M1 A1	For sorting the list into decreasing order For trying to apply first-fit to their list For a completely correct solution
	(ii)	A packing that uses fewer bags could be First bag 8 2 Second bag 7 3 Third bag 5 3 2 Fourth bag 4 3 3	B1	For any valid packing into four bags (may be as an incorrect answer to using algorithm, need not be packed in this order)
	(b)	$\left(\frac{500}{100}\right)^3 \times 4$ or $125000000 \times 0.000004$ $= 500$	M1 A1 6	For scaling 4 seconds by 5^3 or for an equivalent valid and complete method. Condone minor errors with the number of zeros. For 500 or 500 seconds or 500 s. Accept 8 minutes 20 seconds or 8.3 minutes
2	(i)	eg 	B1	For any simple graph with 4 vertices and 5 arcs Vertices need not be labelled Need not be planar
	(ii)	The sum of the orders of the vertices is twice the number of arcs, and hence is even. Hence the sum of the odd orders must be even and so there must be an even number of odd vertices.	M1 A1	Or start from a null graph and successively add in arcs. Each time an arc is added the number of odd vertices is either unchanged or it increases or decreases by 2. So the number of odd nodes is always even
	(iii)	5 arcs ⇒ sum of orders of vertices = 10 Simple graph connecting vertices so each vertex has order 1, 2 or 3 1 + 3 + 3 + 3 = 10 or 2 + 2 + 3 + 3 = 10 But 1 + 3 + 3 + 3 is not possible since if three vertices have order 3 they are all connected to the fourth vertex so it also has order 3. With 2 + 2 + 3 + 3 the two vertices of order 2 cannot be adjacent, since otherwise two arcs connect the other two vertices so not simple. Hence only one possible graph.	M1 A1 A1 6	 Explaining why 1 + 3 + 3 + 3 is not possible. Explaining why there is only one graph with nodes of orders 2, 2, 3, 3.
3	(i)	 Kruskal: *DF*, *CD*, *BD* and *EF*, *FH*, *AC*, *EG* 40	M1 A1 B1	For a correct tree (labels not required) For a valid order (using Prim or Kruskal) For length = 40
	(ii)	*A C D F E G H B A*	M1	At getting at least as far as *A C D F E* (or shown on a diagram)

		Answer	Marks	Guidance
			A1	For a correct cycle, ending back at *A* (if shown on a diagram, needs direction shown)
	(iii) **(A)**	*ACEG* and *ABGH*	B1	For both, vertices in any order
	(B)	5	B1	For 5
	(C)	*ABCD*	B1 **8**	For *ABCD*, vertices in any order
4	**(i)**			Answer should be on insert sheet
			M1	For using Dijkstra's algorithm – updating at *E* and *F* (even if incomplete)
			A1	For all permanent labels correct
			B1	For valid order of assigning permanent labels
		Vertex *B* *C* *D* *E* *F* *G* Length 6 8 16 14 19 22	B1	For copying their permanent labels, or correct values
		A – *C* – *E* – *G*	B1	Correct answer only
	(ii)	The only odd nodes are *A* and *F* Shortest path from *A* to *F* has length 19 km	M1	For identifying *A* and *F* or value 19 or their 19
		120 + 19	M1	For 120 + their 19
		= 139 km	A1	For 139 (cao)
	(iii)	Need *A* and *G* odd and all other nodes even so need to connect *F* to *G* = 10 km	M1	For identifying *F* and *G* or value 10 as only extra
		120 + 10 = 130 km	A1 **10**	For 130 (cao)

5 (i)

X	N	T	S
	0	0	0
3	1	3	9
6	2	9	45
5	3	14	70
7	4	21	119
3	5	24	128

$M = 4.8$
$D = 1.6$

Mark	Guidance
M1	For initial pass through step 3 correct
M1	For updating each of *N*, *T* and *S* correctly
A1	For final values of *N*, *T* and *S* correct
B1	For 4.8 (ft their $T \div N$)
B1	For 1.6 (ft their $\sqrt{\{(S \div N) - (M^2)\}}$

(ii)

Answer	Mark	Guidance
15 additions and 5 multiplications	B1	
20 + 5 = 25	B1	For 'their 20' + 5

(iii)

Answer	Mark	Guidance
$3n + n + 5$	M1	For any function of *n* that gives their answer from (ii) when $n = 5$
$= 4n + 5$	A1	For any expression that simplifies to $4n + 5$

(iv)

Answer	Mark	Guidance
$(5000 \div 1000) \times 2 = 10$ seconds	B1	Or $2 \div 4005 \times 20005 = 9.99 \sim 10$ seconds

[10]

6 (i)

P	x	y	z	s	t	u	
1	-15	4	4	0	0	0	0
0	10	-4	8	1	0	0	40
0	10	6	9	0	1	0	72
0	-6	4	3	0	0	1	48

Mark	Guidance
M1	For overall structure correct, including three slack variables
A1	For a correct initial tableau, with no extra constraints added. Accept equivalent forms.

(ii) Pivot on 10 in *x* column 40 row

1	0	-2	16	1.5	0	0	60
0	1	-0.4	0.8	0.1	0	0	4
0	0	10	1	-1	1	0	32
0	0	1.6	7.8	0.6	0	1	72

$x = 4, y = 0, z = 0$
$P = 60$

Mark	Guidance
M1	For the correct pivot choice for their tableau
A1	For dealing with the pivot row correctly
M1	For dealing with the other rows correctly
A1	For a correct tableau
B1	For reading off *x*, *y* and *z* from their tableau
B1	For reading off *P* from their tableau

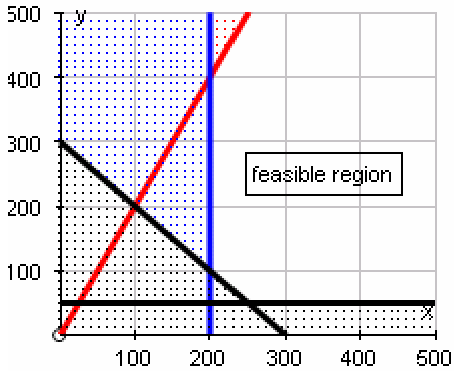
(iii) Pivot on 10 in *y* column

1	0	0	16.2	1.3	0.2	0	66.4
0	1	0	0.84	0.06	0.04	0	5.28
0	0	1	0.1	-0.1	0.1	0	3.2
0	0	0	7.64	0.76	-0.16	1	66.88

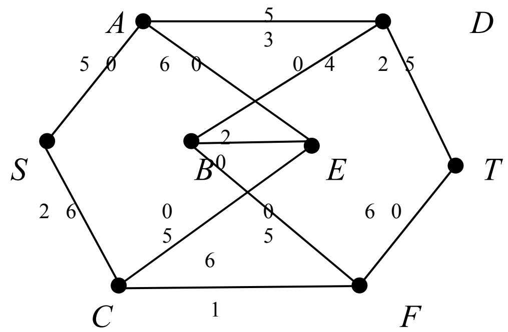
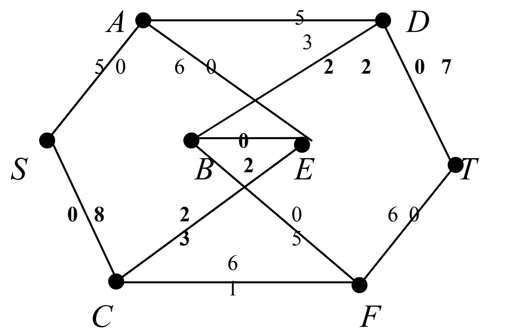
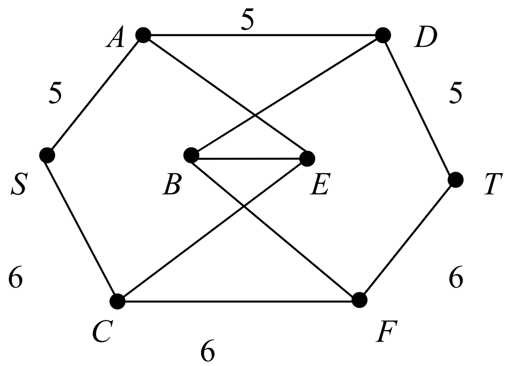
$x = 5.28, y = 3.2, z = 0$
$P = 66.4$

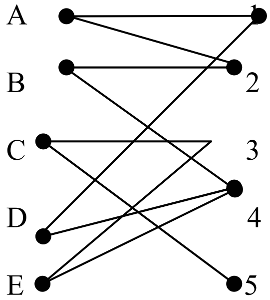
Mark	Guidance
M1	For the correct pivot choice for their tableau
A1	For dealing with the pivot row correctly
M1	For dealing with the other rows correctly
A1	For a correct tableau
B1	For the correct values of *x*, *y* and *z* at optimum
B1	For the correct value of *P* at optimum

[14]

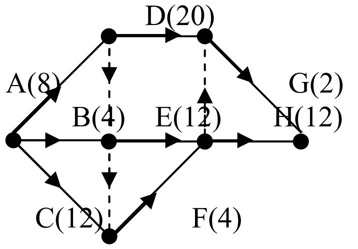
7	(i)	Minimise $70x + 80y + 50z$	B1	For 'minimise' a (non-zero) multiple of $7x+8y+5z$
		'No more than twice as many packs of type Y as packs of type X'	B1	For identifying this constraint from the list, or equivalent
		Other constraints		Ignore extra 'constraints' unless contradictions
		$x \geq 200$, $0 \leq z \leq 50$	B1	For boundary constraints on x and z
		$y \geq z$	B1	For this, or an equivalent correct answer
		$x + z \geq 220$	B1	For this, or an equivalent correct answer
		$x + y \geq 300$	B1	For this, or an equivalent correct answer Use of strict inequalities – penalise first time only
	(ii) (a)	Minimise $70x + 80y$ (+ 2500) (or scaled through) Subject to	M1	For replacing z by 50
		$y \leq 2x$ $x \geq 200$ $y \geq 50$ $x + y \geq 300$	A1	For their $y \geq 50$
			M1	For at least two appropriate lines drawn on a graph with plausibly scaled axes.
			M1	For boundary lines drawn correctly (follow through their equations provided there are at least two horizontal or vertical lines and at least two lines that 'slope')
			A1	Feasible region correctly identified (correct answer only, not follow through)
	(b)		M1	For reading off or calculating at least one of their vertices
		(200, 400), (200, 100), (250, 50)	A1	For getting these three vertices correct with no extras
		(200, 100) gives $70x + 80y = 22000$ (£245) (250, 50) gives $70x + 80y = 21500$ (£240)	M1	For calculating their cost at one of their vertices or using an appropriate line of constant cost
		Cost is minimised when $x = 250$, $y = 50$	A1	For identifying vertex (250, 50)
		Cost = £240	B1	For £240 or 24000 p (with units)
	(iii)	eg $x = 300$, $y = 0$, $z = 0$ only costs £210	M1	For finding a feasible point with $z < 50$ Or a written explanation
			A1	For finding such a feasible point with a lower cost than that in (ii)(b) <u>and</u> showing that cost is lower.
			18	

Mark Scheme 4737
June 2005

1	**(i)**	*AD*, *EB*, *CF* 8 + 2 + 7 = 17 litres per second	B1 M1 A1	For these three directed arcs and no others 8 + 7 + 0 or 8 + 7 – 2 seen ⇒ M1, A0 For 17
	(ii)		M1 A1	Accept all arrows reversed For no more than three errors For a correct labelling
	(iii)	*SCEBDT* 	B1 B1	For this path only For a correct labelling
	(iv)	Cut {*S*, *A*, *B*, *C*, *D*, *E*, *F*} {*T*} = 13 Diagram in (iii) shows a flow of 13 litres/second	B1 B1	For a this cut or the cut {*S*} {*A*, *B*, *C*, *D*, *E*, *F*, *T*}, in any form, or for 'no more can flow into *T* ',or 'no more can flow out of *S* ', or equivalent. For flow shown = 13 <u>or</u> max flow ≥ 13 ≥ min cut (but NOT just stating max flow = min cut) Value 13 given in question
	(v)	 Max flow is 11 <u>litres per second</u> Cut {*S*, *C*, *E*, *F*} {*A*, *B*, *D*, *T* } = 11	B1 B1 B1 **12**	For showing this flow, or excess capacities and potential backflows equivalent to this For 11 <u>litres per second</u> (with units) For cut <u>or</u> a <u>convincing</u> explanation in words

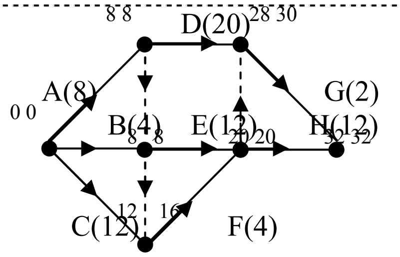
2	(i)		B1	For a correct bipartite graph
	(ii)	Denny cannot have a song that she has chosen	B1	For this reasoning
	(iii)	5 C 3 D A-1, B-2, C-5, D-3, E-4	M1 A1	Follow through their bipartite graph, if possible For this path (or in reverse), not longer path - if shown on diagram, path must be obvious For this matching, not alternative
	(iv)	A-2, B-4, C-5, D-1, E-3	B1	For a different matching from their bipartite graph
	(v)	Hungarian algorithm finds minimum cost allocation, need to subtract each score from 10 to convert maximising into minimising. Dummy row is needed to make a square matrix.	B1 B1	For a valid reference to maximising/minimising For 'make it square' or equivalent
	(vi)	F G H J K A 6 1 3 10 3 B 4 2 7 2 10 C 3 6 5 8 3 D 4 4 8 3 9 X 0 0 0 0 0	B1	For setting up initial matrix as described
		Reduce rows 5 0 2 9 2 2 0 5 0 8 0 3 2 5 0 1 1 5 0 6 0 0 0 0 0	M1 A1	For reducing rows (to give a 0 in each row) For correct reduced matrix (cao)
		Cover 0's using four lines 5 0 2 9 2 2 0 5 0 8 0 3 2 5 0 1 1 5 0 6 0 0 0 0 0	M1	For covering 0's using minimum number of lines
		Augment 4 0 1 9 1 1 0 4 0 7 0 4 2 6 0 0 1 4 0 5 0 1 0 1 0	A1	For correct augmented matrix (cao)
		Complete matching A-G, B-J, C-K, D-F	B1 13	For correct matching (listed)

3 (i)



Mark	Guidance
	Durations not necessary
M1	For a correct activity network
A1	For directions indicated correctly

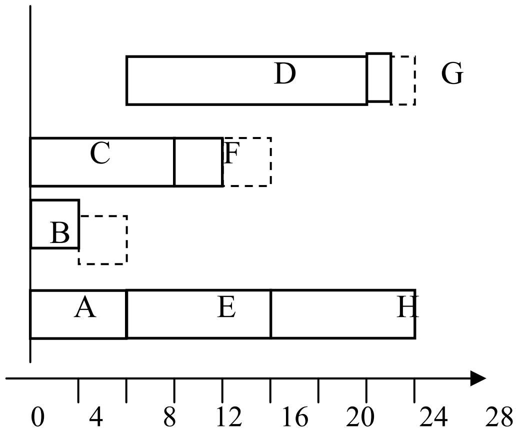
(ii)



Minimum completion time = 32 minutes
Critical activities A, E, H

Mark	Guidance
	Follow through their network if possible, provided not significantly simpler, for the passes
M1	For forward pass
A1	For forward pass correct
M1	For backwards pass
A1	For backwards pass correct
B1	For 32 stated, not just on diagram (cao)
B1	For A, E, H stated (not just on diagram (cao)

(iii)



32 time(mins)

Mark	Guidance
	Follow through their start times if possible
M1	For structure of chart correct, activities may be collected together or on individual rows
A1	For non-critical activities correct (floats optional)
A1	For critical activities correct

(iv) e.g.

Time	John	Kerry
0 – 4	A	
4 – 8	A	B
8 – 12	C	C
12 – 16	C	C
16 – 20	C	C
20 – 24	D	D
24 – 28	D	D
28 – 32	D	D
32 – 36	D	D
36 – 40	D	D
40 – 44	E	E
44 – 48	E	E
48 – 52	E	E
52 – 56	F	.
56 – 60	H	H
60 – 64	H	H
64 – 68	H	H
68 – 72		G (68 – 70)

Mark	Guidance
M1	For structure of schedule correct and all activities shown (with H appearing twice)
A1	For activities A, B, C, D, E and F correct: • A=8, B=4, C=12, D=20, E=12, F=4; • D after A; E after A, B; F after A, B, C; • C, D and E done by J and K at same time
A1	For activities G and H correct • G = 2 (may see 4), H = 12 • G, H after (D), E, F (not alongside F) • H done by each of J and K • Total time taken = 70 (minutes)
[14]	

4 (i)

stage	state	action	working	maximum
1	0	0	**4**	*
	1	0	**4**	*
2	0	0	4 + 4= 8	
		1	5 + 4= **9**	*
	1	0	6 + 4=**10**	*
	2	1	7 + 4=**11**	*
	3	0	5 + 4= 9	
		1	6 + 4=**10**	*
3	0	1	8+10=**18**	*
		3	6+10=16	
	1	0	7+ 9=16	
		2	6+11=**17**	*
	2	0	7+ 9=16	
		2	6+11=17	
		3	8+10=**18**	*
4	0	0	5+18=23	
		1	8+17=**25**	*
	1	0	7+18=**25**	*
		2	5+18=23	
5	0	0	6+25=31	
		1	8+25=**33**	*

Mark	Guidance
M1	For structure of table correct
A1	For stage and state columns correct
A1	For action values correct
M1	For all calculations correct for stages 1 and 2 (may be seen as an addition or the result and may be shown in final column)
A1	For suboptimal maxima identified correctly (may be implied from next stage)
M1	For correct calculations for stage 3 (follow through from stage 2, if possible)
A1	For suboptimal maxima correct (ft their totals) (may be implied from next stage)
M1	For correct calculations for stages 4 and 5 (follow through from stage 3)
A1	Calculations correct for entire table

Route: (0;0) – (1;0) – (2;1) – (3;0) – (4;1) – (5;0)

Giles will be able to see 33 plants

Mark	Guidance
B1	(cao) or in reverse
B1	For 33 (cao)

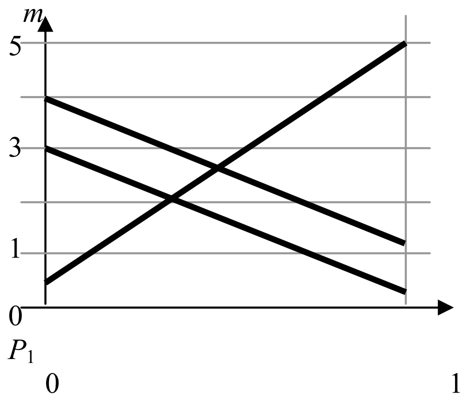
(ii) Minimax

Route: (0;0) - (1;0) – (2;3) – (3;0) – (4;0) – (5;0)

Or (0;0) - (1;1) – (2;3) – (3;0) – (4;0) – (5;0)

At stage 5 all paths have at least 6 plants

Mark	Guidance
B1	For 'minimax'
M1	For a path with at most one path > 6 plants
A1	For either correct path
B1	Or stage 3 or any equivalent argument in words
15	

5	**(i)**	What one player wins the other loses	B1	For a statement equivalent to 'total won each game is zero'
	(ii)	S and T: $3 > -2$ but $-2 < 1$ (or $-1 < 2$) S and U: $3 > 1$ (or $-1 > -2$) but $-2 < 3$ T and U: $-2 < 1$ (or $1 < 3$) but $2 > -2$	M1 A1	For considering differences, showing inequalities or considering rows where column maxima and/or minima occur For a valid explanation
		D and E: $3 > -2$ but $-2 < 1$ (or $1 < 3$) D and F: $3 > -1$ (or $1 > -2$) but $-2 < 2$ E and F: $-2 < -1$ (or $1 < 2$) but $3 > -2$	M1 A1	For considering differences, showing inequalities or considering columns where row maxima and/or minima occur For a valid explanation
	(iii)	Row minima are -2, -2, -2 $\Rightarrow$ row maximin = -2 Col maxima are 3, 3, 2 $\Rightarrow$ col minimax = 2 $2 \neq -2 \Rightarrow$ not stable	M1 M1 A1	For identifying -2 correctly <u>or</u> identifying all rows For identifying 2 correctly <u>or</u> identifying col F For a valid explanation, or equivalent in words
	(iv)	So that for $p_1, p_2, p_3 \geq 0$ we will have $m \geq 0$	B1	For explaining that this will make $m \geq 0$ (not sufficient to just say that we need to make all the entries non-negative)
	(v)	If Colin plays D, with the augmented payoffs Rhoda will expect to win $5p_1+0p_2+3p_3$, and similarly for when Colin chooses E or F m is the minimum of the augmented E(winnings)	B1 B1	For explaining any of the three expressions on the right hand side of the inequalities For explaining why $m \leq$ each expression
	(vi)	 $5p_1 = 3(1-p_1) \Rightarrow p_1 = \frac{3}{8}$ (and $p_2 = \frac{5}{8}$)	M1 A1 B1	For a graph of m against p_1 (or m against p_2) with three lines For lines (0,0)-(1,5), (0,3)-(1,0), (0,4)-(1,1) or equivalent For convincingly showing how values were obtained (ie identifying $5p_1 = 3p_2$ or equivalent Or reading off from correct point on graph) Note: $p_1 = \frac{3}{8}$ and $p_2 = \frac{5}{8}$ is given in the question
	(vii)	-0.125	B1	For $-\frac{1}{8}$, or equivalent (cao)
	(viii)	e.g. Toss the coin three times to give eight equally likely possible outcomes, allocate three outcomes to 'play S' and five to 'play T' In the long run she expects to lose $\frac{1}{8}$ per game	M1 A1 B1 **18**	For a specific example, or a description of any valid method eg HHT, HTH, THH $\rightarrow S$ all other outcomes $\rightarrow T$ For 'lose (at least) $\frac{1}{8}$ per game'

Report on the Units
June 2005

Pure Mathematics

Chief Examiner's Report

General Comments

Five units (C1, C2, C3, C4 and FP1) of the new specification were set in this session and significant numbers of candidates entered for each. Examiners reported that much good work was seen from candidates in all five units. It is pleasing that Centres have adapted well to the new specifications and that, in many cases, candidates have obviously been prepared thoroughly. There were, of course, many entries for all the legacy units and here the papers also proved accessible to the vast majority of candidates whilst including some appropriately demanding aspects.

There are certain aspects of candidates' work which recur at each examination session as being in need of further attention in many cases. One is the need to provide sufficient detail to give a convincing solution to a problem where the answer is given in the question. Another concerns the response by candidates to questions which ask for a sketch graph. What is required is a sketch showing the essential features of a graph and these features can almost always be shown effectively in a diagram in the answer booklet. What is not expected is a detailed graph plotted on graph paper; this must take a length of time out of proportion to the number of marks allocated and, often, does not succeed in showing necessary features such as asymptotes. Examiners are then further irritated in some cases when the sheet of graph paper is not attached securely to the answer booklet as the regulations require.

2631: Pure Mathematics 1

General Comments

The paper was accessible to the majority of candidates, although the final part of Q10 proved too difficult for most. Unfortunately poor algebra and arithmetic were prevalent throughout the paper, see Q9, as was a certain lack of understanding of some topics, such as factorisation, see comments below on Q7, Q9 and Q10. In addition, it should be stressed that whilst candidates could deal with routine topics such as finding stationary points, see Q10, they were unable to link their ideas from one topic to another topic, see Q1 and Q9.

Comments on Individual Questions

1) Critical points of $x = -4$ and $x = 10$ were usually correct, although too many candidates had the incorrect signs. Unfortunately candidates refused to show any method, such as a sketch of a positive quadratic function, and they appeared to just guess their answer. Most guessed incorrectly with $x \geq -4$ and $x \geq 10$ or $-4 \leq x \leq 10$ common. This illustrates the general comment above that, despite there being many methods available for solving inequalities, the linking of a sketch of a quadratic function would have indicated whether they needed to be inside or outside the critical values without any additional knowledge. Finally, most of those who had the correct critical values and knew that they should be outside of the critical values spoilt everything by expressing their answer as the combined interval of $x \leq -4$ **and** $x \geq 10$ or as a "**wrap around statement**" $10 \leq x \leq -4$.

2) (a) About 50% of candidates understood what was meant by simplify; unfortunately the remainder believed that no combination of the two numbers (or the powers of x) was required. Hence it was common to see final answers with $3/x$ being a simplification of $3x^{-1}$ and $\sqrt[3]{(2x)}$ for $2x^{2/3}$. Likewise it was common to see 2 become $2^{2/3}$.

   (b) This was usually correct but there were several cases of 4^{30} becoming 2^{2+30} and $2^{40} \times 4^{30}$ becoming 8^{70}. Furthermore, even when candidates reached $2^{40} \times 2^{60}$, this often finished as 4^{100}.

3) The coefficient 3 caused problems for many candidates with $a = 6$ being common. Likewise the fact that it was necessary to subtract $3a^2$ from 7 to acquire b resulted in numerous combinations of $3a^2$ and a^2, where $a = 2$ or $a = 6$, with 7 or 7/3. Hence $b = 5/3, 3, -29$ and -101 were frequent values for b.

4) Again many candidates started badly, from which there was no recovery, by either replacing $x^2 + y^2 = 25$ with $x + y = 5$ or by replacing $y^2 = (5 - 2x)^2$ by $25 + 4x^2$ and hence solved two trivial linear equations or $x^2 = 0$, respectively. The candidates who knew what to do, when solving for x, often omitted to consider the solution $x = 0$. In fact they just cancelled x from the equation $x^2 - 4x = 0$ and as a result forfeited the method mark for the solution of this quadratic equation. Unfortunately, many of the candidates who had worked successfully nearly to the end of the question then chose to substitute for x in the quadratic expression so finishing with (4, 3) instead of (4, –3).

5) (i) Most candidates were able to sketch a recognisable graph of $y = x^3$, although it would have been nice to see a horizontal gradient at the origin and for the sketch to have been undertaken in the answer booklet as opposed to on graph paper, since they were only asked for a sketch.

   (ii) Again, reflect was the usual answer, and flip, mirrored and invert were not deemed as acceptable. In addition, this needed combining with in/on the x-axis (or $y = 0$) or y-axis (or $x = 0$), but reflection parallel to the axis or reflection in both axes lost one of the marks.

   (iii) Only the better candidates knew how to express the translation in x in mathematical terms, although they still often finished with $y = (x + p)^3$ or $y = (x \pm p)^3$. A few candidates opted to assume the specific value of $p = 3$ or omitted the y from the right side of the expression. Unfortunately, all the other candidates translated parallel to the y-axis and produced $y = x^3 + p$.

6) This question was meant to be undertaken without using a calculator, although it was obvious in several cases that the calculator had been used to produce the correct surd forms and the rationalisation. However, on this occasion, provided that things were converted back into the appropriate form such a procedure was accepted. The rationalisation of $1/\sqrt{2}$ was often avoided by candidates opting to remember sin 45 as $\sqrt{2}/2$, with the result that the marks scored on this question were usually fairly high. Many candidates decided that it would be sensible at some stage to multiply their expression by 2, basically not understanding the difference between an expression and an equation.

7) Candidates either knew exactly what was required, although $\sqrt[3]{(-27)}$ was often believed not to exist, or never produced anything sensible other than possibly spotting $x = 1$ as a solution. Some candidates never indicated that they were replacing x^3 by some other variable and solved a quadratic for x. Obviously the better candidates understood what they were doing by continuing to take the cube root. However, it should be stressed that notation in mathematics is important and failure to follow it correctly quickly leads to confusion. A very common error, and mentioned in the general comments above, was to see $x^6 + 26x^3 - 27 = 0$ factorised as $x^3(x^3 + 26) = 27$, and hence $x^3 + 26 = 27$, leading to $x^3 = 1$, with $x = 1$ often checked as a solution in the equation.

8) (i) This part was very well done and, other than the occasional candidate who differentiated instead of integrated or who integrated x to get x^2, full marks were common.

(ii) This part proved more testing with the omission of the constant common, a quantity vital for those who realised that section (iii) needed this answer. In addition, $-1/x^2$ was often integrated to give $-1/-x$, which was not deemed a suitable form for the final answer.

(iii) Many of the candidates interpreted the question as "find the equation of the straight line through the point (1, 6) that has the gradient $(3 - 1/x^2)$". Hence, instead of using their solution from (ii) equated to y and finding c, they evaluated dy/dx at $x = 1$ and proceeded to use $y = mx + c$.

9) This question really highlighted many of the problems that some candidates were facing. Throughout they basically ignored the coefficients of x and y whenever they wanted to and interchanged the x and y coordinates at will.

(i) The gradient m was commonly given as 4 (ignoring the coefficient of y) or as 1 (ignoring both coefficients).

(ii) The attempts at the normal often saw gradients of $-m$ and $1/m$ instead of $-1/m$. However, whilst the use of (1, 2) was usually undertaken correctly, the simplification of $y - 2 = -3/4\ (x - 1)$ often had a sign error in the constant term.

(iii) At this stage a sketch of the lines l_1 and l_2 would have helped, linking their knowledge of straight lines into this situation. However, this was never seen and instead many candidates guessed incorrectly that point P where the line l_1 crosses the x-axis would produce a non-zero y coordinate, together with $x = 0$, whilst the point Q where the line l_2 crosses the y-axis would produce a non zero x coordinate, together with $y = 0$. Again, as in (i), just assuming the constant terms to be these coordinates in l_1 and l_2 was the norm. Finally, too often any final answer for the midpoint was left in the form of a fraction within a fraction, a decimal within a fraction, or the division of a fraction by 2 became the multiplication of its numerator by 2.

10) (i) This was usually well done by all but a few candidates who failed to produce x^2 for the derivative of $1/3\ x^3$.

(ii) Usually candidates successfully found x coordinates of stationary points but y values were often omitted altogether or one of them was evaluated incorrectly.

(iii) Little working was seen from some candidates when considering the sign of the second derivative; in such situations it was difficult to award marks if errors were present. Using the y co-ordinates to predict which is a maximum and which a minimum is not an acceptable method because it works only in a limited number of cases.

(iv) Full solutions of this part were rare. A few candidates equated successfully the gradients of the line and the curve to establish $x = 1$ and $x = -1$, but then substituted each x value for either the line or the curve. In fact they needed to substitute each x value into both of these equations to establish which x value had the same y coordinate in both in order to ensure that the line and curve actually met at that point. Others, probably the majority of those that had some knowledge of what was required, opted to find where the line and curve were meeting. However, this required the solution to the cubic equation $x^3 - 3x + 2 = 0$. A few spotted $x = 1$ was a solution but often forms such as $x(x^2 - 3) = -2$, leading to a factorisation similar to $x^2 - 3 = 0$ and $x = -2$ were common. Again, those who did manage a solution usually then proceeded to find the y values of the line and curve for their point of intersection and as expected these were identical or should have been. Whereas in fact they now needed to ensure for which value of x the gradients of the line and curve were identical before they could state the appropriate y coordinate

2632: Pure Mathematics 2

General Comments

This paper proved accessible to the vast majority of candidates and many excellent scripts were seen. Candidates seem to have had sufficient time to demonstrate their mathematical ability. Two topics proved unexpectedly problematic for a significant number of candidates. Many were clearly not sure of work involving radian measure, often indicating a belief that an angle measured in radians must be given in terms of π. The basic definition of modulus was apparently not known to all; many candidates seemed unaware of results such as $|-7| = 7$ and $|-6| = 6$.

Comments on Individual Questions

1) For many candidates this was a straightforward question and three marks were easily earned. For a significant number of candidates, however, the question revealed considerable uncertainty about radians. The incorrect answer 0.96π was common in part (i). Most candidates were aware of the significance of 180º in converting to degrees but the incorrect answers 172.8º and 59.7º were often noted in part (ii).

2) This question was answered well. The only error to occur with any frequency was $1 + 36x + 144x^2 + 336x^3$, the result of failure to evaluate $(4x)^2$ and $(4x)^3$ in the relevant terms.

3) Questions involving arithmetic and geometric progressions are usually answered well and this one was no exception. There were problems for some in recalling formulae accurately and some algebraic slips occurred in finding r but, generally, solutions were clear and accurate.

4) More problems were evident in solutions to this question. A large number of candidates ignored the modulus signs completely whilst others proceeded with values of $(2^x - 8)^2$ or $2^x + 8$. Some credit was available to candidates who adopted the correct procedure for the trapezium rule with incorrect y values; thus many did earn two marks for the incorrect answer 6.5. However there were also many errors in applying the trapezium rule. Some candidates took the value of h in the standard formula to be 5 whilst, in other cases, an apparent lack of vigilance in using the formula led to the calculation of 0.5(7 + 24) + 2(6 + 4 + 0 + 8).

5) Most candidates earned the mark for stating the range of f. Examiners accepted notation such as f > 4, $y > 4$ or $x > 4$ as well as the preferred $f(x) > 4$.

   Part (ii) was not answered well. A few candidates interpreted $g^{-1}(x)$ in terms of a reciprocal function or a derived function. Many others had some notion of an inverse function but expressions for $g^{-1}(x)$ such as $\frac{1}{3}e^x$ or e^{3x} were often noted. The exact value e^2 was required as the answer but equivalent exact values such as $\sqrt[3]{e^6}$ were accepted.

   Most candidates formed the correct expression for $gf(x)$ in part (iii) although absence of the necessary brackets led some astray subsequently. Realisation that differentiation required use of the chain rule was not common however and $\dfrac{3}{x^2 + 4}$ was a common incorrect answer.

6) Each part of this question required candidates to reach a given exact answer; accordingly, detailed accurate work without the involvement of decimal values was needed. In part (a), some candidates

produced an integral involving a natural logarithm or $(4x+3)^{-\frac{3}{2}}$. In many cases where candidates had integrated correctly, convincing work showing that $\frac{1}{2}\sqrt{27}-\frac{1}{2}\sqrt{3}=\sqrt{3}$ was absent.

Candidates generally fared better with part (b) and many presented solutions showing clear and accurate use of logarithm properties. The commonest error involved a step such as $\ln 225 - \ln 9 = \dfrac{\ln 225}{\ln 9}$.

7) There were many different approaches taken to confirm the value of *a* in part (i). The most efficient approach was to apply the remainder theorem; setting P(6) = 4 and solving for *a* confirms the value and many candidates did adopt this approach. Some candidates started with the assumption that *a* was –14 and then confirmed a remainder of 4; this method was accepted. Other methods involved a long division process or the consideration of a polynomial identity containing several unknown coefficients. To be successful, such methods needed care and some were betrayed by their algebraic skills.

Candidates who appreciated the relevance of part (i) to part (ii) were largely successful in part (ii) although failure to include 6 as one of the roots was common. There were some problems with solving $x^2+2x-2=0$; some candidates believed this led to $x=-2\pm\dfrac{\sqrt{12}}{2}$. A minority of candidates approached part (ii) by using a factor theorem routine, testing *x* = 1, –1, 2, …; a few eventually succeeded.

8) The general response to part (i) was disappointing. A number of candidates, perhaps misunderstanding the limit on the value of *k*, drew graphs only for positive values of *x*. Most drew an acceptable graph of $y=\mathrm{e}^{kx}$ although some showed a graph passing through the origin. The second graph was not always recognised as a parabola; when it was, it was not always shown with a maximum point on the *y*-axis. For the final mark in part (i), candidates were required to draw attention to the two points of intersection of the two graphs. Some did this convincingly but many did nothing or marked the two points of intersection of the parabola with the *x*-axis. Others also drew a graph with equation $y=\mathrm{e}^{kx}+3x^2-25k$, but no credit was available for this approach.

Most candidates recognised the need in part (ii) (a) to evaluate $\mathrm{e}^{4x}+3x^2-100$ at the two values but not all then commented on the resulting change of sign. Poor calculator work was quite common; substitution of –5.78 led to the wrong values –315 or –200 on a significant number of scripts.

Most candidates earned the mark in part (ii) (b) although those taking a first step of $4x+\ln(3x^2)-\ln 100=0$ did not. Although a few candidates embarked on an unacceptable decimal search routine in part (ii) (c), the vast majority was successful although the evidence provided was sometimes minimal.

9) Examiners were pleased to note that many candidates were sufficiently aware of the nature of exponential decay to be able to write down the correct answer to part (i) without difficulty. Others detected terms of an arithmetic progression so that 240 was a common error. Not all candidates realised that differentiation was required in part (ii); many merely substituted 100 into the expression for M_P. Where differentiation was attempted, not all candidates could do so correctly but there were many candidates for whom three marks were easily earned.

Part (iii) was challenging. Some candidates toyed with formulae for geometric progressions or equated the expression for M_P to 480. Just a few candidates showed commendable mathematical skill by establishing a correct formula for M_Q and then solving the resulting equation faultlessly.

2633: Pure Mathematics 3

General Comments

There were pronounced differences between the better candidates, who scored good marks without any apparent difficulty, and less capable candidates, who struggled and seemed poorly prepared for the examination. Use of correct notation was often poor, especially in Qs 3 and 7. Standards of layout and presentation were variable.

The paper proved to be reasonably accessible but not all candidates were able to take advantage because their general algebraic skills were not sound. It was only candidates with a firm grasp of algebraic and manipulative skills who were able to do themselves full justice.

There were very good responses in general to Qs 1(i), 4, 6(ii) and 8(i) whereas the questions causing most difficulty were Qs 6(i), 7(i) and 7(ii). There was no evidence of candidates running out of time and, in general, full attempts were made with all questions.

Comments on Individual Questions

1) There were many excellent solutions to part (i) with only a few candidates believing that the derivative of a product was equal to the product of the derivatives. Candidates fared less well with part (ii); some committed an error similar to the one mentioned in connection with a product and many were unable to simplify their correct derivative accurately.

2) Almost all candidates recorded the mark in part (i). A mark of 3 out of 5 was common in part (ii) as many candidates rarely found both roots of $\tan\theta = 0$ or of $\tan^2\theta = \frac{1}{3}$.

3) Many candidates produced excellent solutions but, in other cases, much uncertainty was revealed and integrals such as $\int (u+2)u^5\,\mathrm{d}x$, $\int xu^5\,\mathrm{d}u$, and $\int (u+2)u^5$ with no indication of du or dx were common.

4) Errors in part (i) were very rare. There were more problems in part (ii) as some candidates did not appreciate the need either to multiply their expansion from part (i) by (1 + x) or to examine the identity $1 + x \equiv (1+2x)(1-x+kx^2)$. Only about half of the solutions contained the correct set of values in part (iii); incorrect answers included $x < \left|\frac{1}{2}\right|$, $|x| < -\frac{1}{2}$, $x \neq \pm\frac{1}{2}$ and $x \neq 1$.

5) There were many excellent solutions to part (i) but there were also many attempts where there was a total inability to express $\cos^2 x$ in terms of $\cos 2x$. Attempts at part (ii) were better than at part (i); most candidates scored 4 marks with sign errors being the main source of mistakes. However, very few candidates were able to obtain the correct answer for the exact value of the definite integral.

6) Part (i) proved to be the most troublesome request in the paper and few candidates were able to eliminate θ correctly. Attempts which involved an 'identity' of the form $(a \pm b)^2 = a^2 \pm b^2$ were rife. Other solutions showed the correct elimination of $\cos^2\theta$ and $\sin^2\theta$ but still had terms involving cos θ and sin θ which caused problems. The initial step of rearranging the parametric equations to make cos θ and sin θ respectively the subjects was not adopted by many candidates. Even when equations did not represent a circle, many candidates were insistent that their equation did represent a circle. There was much greater success with part (ii) although many candidates left their answer in terms of θ.

7) Not many correct vectors were noted in part (i) and solutions were riddled with sign errors. Part (ii) prompted a better response although there were further sign errors and some candidates were

confused between an appropriate position vector and the direction of the line. In part (iii), the necessary theory was well understood but earlier errors meant that few correct solutions were seen.

8) There were almost no errors in attempts at part (i). There were two principal errors in attempts at part (ii). Perhaps half of all attempts at the integral of $(1-p)^{-1}$ featured a wrong sign and many candidates failed to include a constant of integration. Part (iii) was done well and the scheme allowed credit for the correct approach even if there had been earlier errors.

2634: Pure Mathematics 4

General Comments

The majority of candidates found the examination accessible, answering the questions in the order set. Generally, candidates appeared to be better prepared for the paper, and they were able to demonstrate their ability to deal with the concepts tested. Although there were some poor scripts, there were many more good ones. Algebraic techniques were attempted with more accuracy, and candidates proved adept at both identifying the problems and applying correct methods to them.

In questions such as 4(a), 6(i) and especially 7(iv), where answers to be proved are given to candidates, there remains a tendency to omit important steps leading to the answer. Some candidates claim too much as "obvious", whilst others appear preoccupied in getting to the answer by any means! In general, candidates should consider the number of marks available as a means of at least indicating the number of steps which may be necessary.

Candidates appeared much less likely to get involved in over-long solutions, although some time was wasted, for example in Qs 3 and 4(a). However, there was no indication that timing was a problem.

Comments on Individual Questions

1) This proved to be a reasonably straightforward question for most candidates. Only a few candidates used $Bx/(x^2 + 1)$ instead of $(Bx + C)/(x^2 + 1)$, and these candidates were still able to score 3/4 with care. A small minority attempted $B/(x^2 + 1)$.
Candidates were given credit for the method thereafter and for arriving at the correct constants. Correct values then transferred wrongly, for example $1/x - (x + 3)/(x^2 + 1)$, were not penalised.

2) i) It was not generally recognised that the answer could be written down from the formula book as $1/\sqrt{(1/a^2 - x^2)}$. Many candidates attempting to quote an answer omitted the "a" in the numerator, and there was some lack of precision or carelessness, such as $\sqrt{(1 - ax^2)}$, in the denominator. Follow-through marks in (ii) ensured that some credit could still be gained.
Candidates who started to derive the answer from scratch were often successful, although many left their answer as $a/\cos y$ (and were penalised) and others arrived at $a/\cos(\sin^{-1}ax)$ (and gained some credit).

ii) Most candidates used their results from (i) to get to f(0) and f′(0). The less successful then omitted the x in the f′(0) x term.

3) This was a mixture of partial fractions methods (divide out) and asymptotes, but candidates did not appear to be phased by the combination. Minor algebraic errors in the division (in whatever form this was attempted) were condoned as long as it led to $y = x + 3$ emerging. A few candidates obtained $y = x$ as an asymptote and were given partial credit for it. Candidates giving only $x = 1$ and $x = 2$ should perhaps have noted the number of marks available. A minority continue to give answers such as $x \to 1$ instead of the requested equation.

4) (a) This was generally well recognised and answered. There was some algebraic manipulation seen when the answer given was not appearing, relating particularly to the use of $\sum 1 = 1$. However, a number of candidates gave an elegant proof, with early factorisation the key. More ploughed their way through, often successfully. A few candidates tried to use induction, with various degrees of success, whilst the minority who used $(3n - 1)$ in the formula for $\sum r^2$ gained no marks.

(b) The relatively easy mark for (i) was used as an indicator of the difference method by most candidates. However, many missed the factor of $-\frac{1}{2}$, or used ± 2, ± 1 or $\frac{1}{2}$ instead. Others "quoted" the answer $k(\,f(1) - f(n + 1))$ for various and assorted k. This gained credit, as did closely related (but less accurate) efforts such as $k(\,f(1) - f(n))$. A minority continue to give the answer in terms of r rather than n.

5) (i) This was successfully completed by most candidates, leading to the correct conjecture in (ii).

(ii) The "odd" conjectures were treated on merit, although candidates should expect the method given in (i) to lead to a reasonable conjecture.

(iii) The induction process was better presented than normal, although candidates remain poor at bringing together clearly the two parts of the induction proof. Most correctly used the product rule, with only a minority reproducing what they have always done and using, for example, $f^{k+1}(x) = f^{k}(x) + e^x$, presumably from summation examples.

6) (i) Examiners felt that there was a substantial improvement in substitution. Many candidates made appropriate use of the chain rule in a number of different but accurate ways. However, with the answer being given, it was important that all relevant steps should be seen. Some solutions were difficult to follow because steps were omitted. The novel approach of candidates who used the substitution as in integration and wrote "$dy = -z^{-2}\,dz$" was condoned as long as the method was clearly followed through.

(ii) The integrating factor method was well recognised, with the correct I.F. of e^{-2x} being identified in most cases, although a surprising number of e^{-2z} was seen. Parts was used with mixed results, as many signs were lost. More importantly, the constant of integration was often omitted, and this led to problems when attempting to solve for y.
The final mark was given for the replacement of y for z, which a substantial number of candidates failed to do. On reflection, and considering some of the solutions which were offered, it may have been better to ask for the solution in the form $y = f(x)$. Answers such as $1/y = x/2 + \frac{1}{4} + ke^{2x}$ leading to $y = 2/x + 4 + 1/ke^{-2x}$ were surprisingly, and disappointingly, common.

7) (i) Most candidates opted for multiplication by the conjugate of z_2, although those who chose $\sqrt{3} + i$ were equally successful, with any problems arising from the -4 in the denominator. Others equated z_1/z_2 to $x + iy$, and then used $z_1 = (x + iy)\,z_2$ to find x and y by equating real and imaginary parts. Apart from minor arithmetic errors, this part was generally well done.

(ii) The lack of a sketch led many candidates to produce the incorrect arguments, and use of $\tan\theta = y/x$ should be discouraged. The problems caused by this were overcome by the follow-up marks in (iii), but caused further problems in (iv). A few candidates attempted answers from their part (i), usually unsuccessfully.

(iii) Many candidates were helped by the follow-on marks from their answers in (ii).

(iv) Few candidates scored full marks due to errors in (ii), poor diagrams and insufficient working and explanation. Again, with an answer given, a precise account was needed, using earlier results.

8) (i) Most candidates completed this part. Marks were lost when the answer was not clearly arrived at as important steps were omitted.

2635: Pure Mathematics 5

General Comments

As usual there was a wide range of ability shown by candidates. Some had little knowledge of the syllabus and it is rather surprising that they had been entered. However, there were many who had been well prepared and they showed considerable finesse in much of what they did. In general, presentation has not improved over the years and much is scrappy, indicating little pride shown by candidates in their work.

Comments on Individual Questions

1) Almost everyone coped with part (i) but it was a different matter in part (ii). Most used a tabulated form which was fine provided (a) the answer was correct and (b) it was possible to tell which was the final answer. If the answer was wrong, it was sometimes possible to see how and where a mistake had arisen – but by no means always. In these cases it was not possible to award either or both of the method marks.

2) The auxiliary equation was generally correct and only a few were unable to solve it. Even with correct roots, some candidates were not sure about which variables to use and it was not uncommon to see $t = A\mathrm{e}^{-\theta} + B\mathrm{e}^{-\frac{1}{4}\theta}$. There was some confusion about the general solution being identical to the complementary solution but most coped, either just saying that the particular integral was 0 or deriving it.

   Part (ii) started well but many did not interpret the phrase "Initially the needle is at rest…" and so were unable to produce values for *A* and *B*.

3) (i) The vast majority were able to distinguish which rectangles should be used and the result was obviously understood by most, even though it was not always explained as well as it might have been.

   (ii) Most realised that $\int_1^n x^{-4}\,\mathrm{d}x$ was involved but many were unsure just how it was connected to $\sum_{r=1}^{n} r^{-4}$. The extra '1' came from a number of sources, some right, some wrong.

4) (i)(ii) The sketches generally indicated that candidates were well aware of the geometrical significance of the Newton-Raphson process; however, a few failed to show a curve crossing the x-axis and/or a clear starting point. The case for failure was occasionally not convincing when the first iteration produced a point much closer to the root than the starting point – but it was clear that most realised that a starting point near to a stationary point held the key to a suitable possible explanation.

   (iii) There were very few failures here; almost everyone remembered to have $\mathrm{f}(x)$ as $x^3 - x - 3$ and not as $x^3 - x$ but there were one or two attempts using $x = \sqrt[3]{x+2}$ and the $x_{r+1} = \mathrm{F}(x_r)$ iteration. A few gave the result correct to 3 significant figures instead of the requested 3 decimal places, and this was treated as misreading.

5) There were some neat solutions to this from those working systematically: finding $\alpha^2 + \beta^2$, $\frac{\alpha^2+\beta^2}{\alpha\beta}$, $\frac{\alpha}{\beta}.\frac{\beta}{\alpha}$ and finally using $x^2 - \mathrm{S}x + \mathrm{P} = 0$. Those attempting the substitution method did not fare very well, tending to go round in circles with no clear idea of where they were going. There was

some confusion between the 'a', 'b' and 'c' in the given equation and those in the equation to be derived.

6) Part (i) was straight-forward book-work and little tolerance was given to mistakes. Some wrote $\tanh^{-1} x = y \Rightarrow x = \tanh y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ with consequent confusion in the x-variable. Others wrote $\tanh^{-1} x = \frac{\sinh^{-1} x}{\cosh^{-1} x}$ and moved into logarithmic form. The equation $(1-x)e^{2y} - (1+x) = 0$ was not always solved directly as $e^{2y} = \frac{1+x}{1-x}$ but treated as a quadratic in e^y with solution using the quadratic formula.

Part (ii) was generally successful apart from those candidates who used the non-identity ln a + ln b = ln(a + b). There were one or two peculiarities in the simplification but, as this had a given answer, very close inspection was made of the working.

7) The differentiation of the trig functions in part (i) was generally satisfactory but the overall simplification to $36\sin^2 t$ proved to be quite a problem. By no means did everyone manage to simplify $9\sin^2 t + 9\cos^2 t + 9\sin^2 3t + 9\cos^2 3t$ to 9 + 9 = 18. The most common way of simplifying $-18\sin t.\sin 3t - 18\cos t.\cos 3t$ was to find or quote the expansions for sin 3t and cos 3t; relatively few recognised sin t.sin 3t + cos t.cos 3t as a particular form of cos(A – B). Again, very careful inspection was made of the working.

In part (ii), most realised they needed to integrate 6 sin t between 0 and π and, apart from the occasional sign error ($\int \sin t \, dt = \cos t$), there were few errors.

8) It had been hoped that the setting of part (i) would assist candidates concerning the split of u and dv when they started part (ii). For about half the candidates, this did not prove to be the case as they decided that part (i) itself was an example of 'integration by parts' and, therefore, achieved no marks; the split in part (ii) was then u = x^n , dv = e^{-x^2} and this obtained only minimal further award.

The correct standard way of tackling part (ii) proved to have few difficulties for candidates. A number tried using u = e^{-x^2} , dv = x^n (the reverse of what might be expected); for most of these, it proved to be an easy piece of work, the only slight difficulty being that they had to replace n by $n-2$ at the end.

In part (iii), candidates were expected to use the induction formula and connect I_3 with I_1; the examiners allowed the use of any value found in part (i), right or wrong.

2636: Pure Mathematics 6

General Comments

It is pleasing to report that most candidates were able to make progress with all of the questions. Apart from Q2, the earlier questions in the paper were done better than the later ones, but even parts of Qs 6, 7 and 8 were done well by the majority. Presentation was good in most cases; few candidates handed in work which was difficult to interpret and few used unnecessarily long methods. It did not appear that candidates were pressed for time, and almost all were able to finish the paper, even if Q8(iii) proved demanding for many.

Comments on Individual Questions

1) This was a standard question which most candidates were able to tackle confidently. Almost all used the scalar product of the correct vectors, although solutions using the vector product were seen occasionally. Apart from some arithmetical errors, a very common mistake was to give the complement of the angle required, when candidates automatically used $\cos\theta$, rather than $\sin\theta$, with the scalar product.

2) The majority of candidates knew the general approach to this question. Nearly all started correctly by stating the modulus and argument of 1 + 2i, but then some errors were made in finding the cube root of $\sqrt{5}$: in several instances this became $\sqrt[3]{5}$, but in many cases it was left as $5^{\frac{1}{6}}$, rather than being evaluated as 1.31. Most candidates knew that they had to add multiples of 2π to the argument and divide by 3, or, equivalently, to divide the argument by 3 and add multiples of $\frac{2}{3}\pi$, but the calculations were not always carried out correctly. A number of candidates worked throughout in degrees and in a few cases a superfluous π was included with angles in radians (though of course these could be given correctly as 0.117π etc.).

3) The first two parts of this question were straightforward and many answers were entirely correct. There were few incorrect expressions for $\mathbf{M}^{-1}$, but several answers to part (ii) found the product as $\mathbf{M}^{-1}\mathbf{UM}$; this may have been a misreading but it is more likely to have arisen from confusion about the order in which multiplication of matrices is written. In part (iii) many obtained the correct expression for $\mathbf{D}^{-1}$, either by simply writing it down or by doing some appropriate multiplications starting from $\mathbf{D} = \mathbf{MUM}^{-1}$. Some candidates inverted **M**, **U** and $\mathbf{M}^{-1}$ without reversing the order, and some weaker candidates attempted to find the inverse of the matrix **D** as found in part (ii), which did not help.

4) (a) This was almost always entirely correct.

(b)(i) Although many candidates were aware of Lagrange's theorem, some appeared not to know it and some thought that it was about the basic properties of a group. To gain full credit for this part it was necessary to show clearly that the table given has at least one subgroup or element of order 2.

(b)(ii) It was not easy to gain full marks for this part. Some candidates simply stated that, because the table is not commutative, the set H cannot be a group; they might have justified their claim by noting that the group in part (i) is the only group of order 5, and is commutative, but, nevertheless, commutativity is not a basic property of groups. It was very common for the terms "commutativity" and "associativity" to be mixed up: thus, some answers claimed that associativity was not satisfied because $ab \neq ba$. For those who knew how to check for associativity, there were two pitfalls. First, some used the table incorrectly: when two elements, such as ab, are combined, the first written element, a, is taken from the left column and the second written element, b, is taken from the top row: thus, $ab = c$, not d. Second, the "obvious" triple to try is abc; unfortunately this triple is associative, and it was necessary to use another example, most of which do demonstrate non-associativity.

5) Many candidates obtained full marks for this question. In part (i) some answers claimed that closure is satisfied simply because addition of rational numbers is closed. To demonstrate closure of S it is necessary to take two distinct elements and to show that their sum gives an element of the same form. For associativity there was, as in Question 4, some confusion with commutativity, but many answers here correctly showed that three distinct elements were associative, or else stated that addition (of rational numbers) is associative. For those who had stated the correct inverse element, it was an easy matter to answer part (ii) correctly, as inverses require negative elements. Alternatively, there is no identity in S', but this was a less common reason given for S' not being a subgroup.

6) The form of this question was less familiar to many candidates, but fair attempts were usually made.

(i) Surprisingly, a minority of answers interchanged 0 and 3 in the matrix **A**; this was perhaps because candidates had considered the product of the row vector $(x \;\; y)$ with **A** for the transformation, rather than the product of **A** with the column vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

(ii) Most answers gained some credit for this part, but full marks were not easily obtainable. The main sources of error were to describe the stretch as an "enlargement" or the shear as a "skew", to fail to give full details of one or both of the transformations, and to give the transformations in the wrong order for the factors stated. This last point made the question quite demanding, for there are two possible answers: both a stretch, parallel to the y-axis and of factor $\frac{3}{2}$, and a shear in the x-direction are required, but the shear factor is either 2 or 3, depending whether it follows or precedes the stretch. The majority of answers gave the stretch first, but then stated a shear factor of 3. A small number of answers gave an angle for the factor and these were more likely to be correct ($\tan^{-1}$ or $\cot^{-1}$ 2 or 3).

(iii) Many answers were correct, although there were some cases of misreading which resulted in a single factor (the determinant of **A**) being given, rather than the separate factors of P and Q.

7) This question was attempted well by many candidates, which is a pleasing reflection on the teaching of this section of the specification.

(i) Many candidates used the neatest method to show that the line lies in the plane, which is to substitute the parametric coordinates of the line into the equation of the plane. Alternatives are to use two distinct points, and to use one point and to show that the direction of the line is perpendicular to the normal to the plane. A significant number of answers used only one point or checked only for perpendicularity, and these received no credit.

(ii) As is often the case with vector problems, a variety of methods is possible for this part; the mark scheme shows five methods, all of which were seen. The most concise solution, used by many candidates, is to find the vector product of the direction of l_1 and the normal to P. Most answers cancelled out the factor of 13, although arithmetical slips were quite common. The other methods require scalar and/or vector products and solution of the resulting equations. Quite often these were successful, but some candidates were unable to solve the equations or found that they had arrived at the same equation by different routes and were unable to make further progress towards the solution.

(iii) For many candidates, this followed easily from part (ii), although some started again; in cases where they had not completed part (ii) this meant that they could still get full credit for part (iii).

(iv) This was done, as intended, as an easy "write down" by most candidates, and they often scored full credit, sometimes benefiting from follow-through marks.

8) (i) This standard result was obtained correctly by nearly all candidates.

(ii) Although a considerable number of answers followed the intended method of putting $\cos 4\theta = 0$ and using its solutions, associating them with the quartic equation in x, it was not uncommon to see parts (ii) and (iii) combined in some way, with numerical solutions of the quartic equation being found in part (ii). This was marked fairly generously, even if in some cases candidates had not perhaps fully appreciated the link between the trigonometrical and the surd solutions. A very common error was to include one or more repeated trigonometrical roots among the four solutions; the most popular set of roots was $\sin\frac{1}{8}\pi, \sin\frac{3}{8}\pi, \sin\frac{5}{8}\pi, \sin\frac{7}{8}\pi$, only two of which are distinct.

(iii) Many answers started correctly by solving the quartic equation algebraically (if this had not already been done in part (ii)). However, very few then justified the choice of signs in deducing the given expression for $\sin\frac{1}{8}\pi$. Those who appreciated that the four possibilities for signs corresponded to the four distinct trigonometrical roots of part (ii) usually obtained the correct expression for $\sin\frac{11}{8}\pi$, but answers of that nature were not often seen. A fairly common error was to give a positive value for the final answer.

4721: Core Mathematics 1

General Comments

This paper proved accessible to the majority of candidates, although Q10(iv) proved challenging to most. Candidates usually worked through the paper in question order and were able to attempt every question. It was quite common for candidates to score more marks on the structured questions in the second half of the paper.

As in January, candidates demonstrated good knowledge of differentiation but poor understanding of how to solve a quadratic inequality. Weaker candidates again lost many marks because they could not solve quadratic equations.

Some candidates appeared to run out of time at the very end of the paper but this was often because they had used unnecessarily long methods to solve earlier questions.

Comments on Individual Questions

1) This question proved to be a tricky starter for many. Although nearly all candidates were able to find the roots of the quadratic equation correctly, very few showed any method (such as a sketch of a positive quadratic function or a number line) for solving the inequality. They then appeared to guess their answer, mostly incorrectly, with $x \geq -4$ and $x \geq 10$ or $-4 \leq x \leq 10$ common.

2) In part (i), the coefficient 3 caused problems for many candidates with $a = 6$ being common. Also, the fact that it was necessary to subtract $3a^2$ from 7 to acquire b resulted in numerous combinations of $3a^2$ and a^2, where a =2 or a =6, with 7 or 7/3. Hence b = 5/3, 3, –29 and –101 were frequent values for b.

   Part (ii) was often omitted completely although many candidates did gain this mark. Some gave the coordinates of the vertex, while others gave the ambiguous response $x = -2$, $y = 5$ which did not gain the mark. A minority differentiated to find the vertex and hence the line of symmetry, an approach which, while long, was usually completed successfully.

3) For part (i), as in the January paper, far too many candidates used graph paper to sketch the graph. Most candidates were able to sketch a recognisable graph of $y = x^3$, although it would have been nice to see more sketches with a horizontal gradient at the origin.

   Part (ii) was one of the best answered parts of the paper with very few candidates losing marks, although there were a few using the word ‘flip’ or ‘mirror’ instead of reflection.

   By contrast, part (iii) proved one of the most challenging questions with only the better candidates gaining both marks. Many candidates translated parallel to the wrong axis producing $y = x^3 \pm p$, while the stretch $y = px^3$ was also very common. A few candidates used the specific value of $p = 3$ or omitted the ‘$y =$’ from the expression.

4) Marks were very varied in this question. Many candidates knew exactly what was required, although $\sqrt[3]{(-27)}$ was often believed not to exist. Some candidates never indicated that they were replacing x^3 by some other variable and solved a quadratic in x. The better candidates showed that they understood what they were doing by continuing to take the cube root but others just stopped at $x = -27$ and $x = 1$, making it unclear whether they actually knew how to solve the problem set.

   It was disappointing to see so many solutions starting with a decision to cube root each term, giving $x^2 + {}^3\sqrt{26}x - 3 = 0$ or even $x^2 + 26x - 3 = 0$. By contrast, a pleasing number of candidates were able to factorise the original equation into the form $(x^3 - 1)(x^3 + 27) = 0$ and then solve.

5) This was another question where the marks varied widely, with only the very best candidates scoring all 7 marks.

Part (a) proved difficult for many candidates. They could often rewrite the two terms as $2\sqrt[3]{x^2} \times \frac{3}{x}$ but were unable to progress any further. Addition of indices was seen in a minority of scripts.

Part (b) proved similarly challenging, with very many candidates stating that $4^{30} = 2^{31}$. Another very popular incorrect answer was 8^{70}. It appeared that many candidates were not prepared for this type of problem.

Part (c) produced better solutions, with many fully correct, although some candidates did not simplify $^{104}/_{13}$. The most commonly seen errors involved multiplying numerator and denominator by $\sqrt{3}$ or $4-\sqrt{3}$.

6) Nearly all candidates scored high marks on this question. As in January, knowledge of differentiation techniques was sound.

In part (i), it was widely understood that the brackets had to be multiplied out first and this was usually done accurately, although a few candidates expanded $(x + 1)^2$ to give $x^2 + 1$.

Nearly all candidates understood the notation in part (ii) with only the occasional integration seen. Most of those who made a slip in the first part were able to gain full marks for subsequent follow-through answers.

Part (iii) was also very well handled by all but the very weakest candidates.

7) This question was well done by most and showed that candidates had a good understanding of quadratic functions.

In part (i), it was very pleasing to see so many perfect answers. A few candidates differentiated and a few took the square root but these errors were rare.

However, part (ii) responses were more varied and usually resulted in at least the loss of one mark as candidates found it difficult to provide adequate reasons for their correct choice of graphs in (a) and (b). It was interesting to see the variety of approaches, with many candidates not using their answers from part (i). Many candidates described translations in the x direction, not always correctly, others differentiated to find the minimum point, while some started to factorise and considered the signs in the brackets. A few substituted in x values, which was not always a full enough justification for their choice, and probably took up quite a lot of time.

8) Part (ii) was very poorly answered or left out altogether. Many candidates did not recognise this as the equation of a circle and some guessed that it was a parabola with intercepts of (5,0) and (0,5). Of those who knew it was a circle, many forgot to state the centre as (0,0), some did not identify the radius, a few gave radius and centre but without the word 'circle' anywhere in their answer.

In part (ii), almost all candidates understood that they were required to solve two simultaneous equations, which gained the first mark, but then many started disastrously, replacing $x^2 + y^2 = 25$ with $x + y = 5$, from which there was no recovery. Those candidates who did start off correctly made good progress, although a few failed to give $x = 0$ as a solution to the quadratic equation $x^2 - 4x = 0$, ending up with only one point of intersection. Some candidates chose to substitute for x in the quadratic expression so finishing with (4, 3) instead of (4, –3) but, in general, there were many very good solutions seen.

9) Most candidates answered part (i) correctly, the most commonly seen error being to give the gradient as 4 (ignoring the coefficient of y).

In part (ii), candidates stated the gradient of the perpendicular line correctly but made a careless sign error in the simplification and/or rearrangement of $y - 2 = -3/4\,(x - 1)$.

In part (iii), there was much confusion between the x- and y- axes and the lines $x = 0$ and $y = 0$, meaning that only a small minority of candidates found the correct coordinates for both P and Q. They also had difficulty in dealing with the fractional coordinates, as in January, leaving their final answer for the midpoint in the form of a fraction within a fraction or a decimal within a fraction. Many of those who tried to halve their fraction multiplied its numerator by 2. The best candidates handled this part confidently, often drawing a sketch.

Confusion in earlier parts of the question meant that few candidates had the correct coordinates for part (iv), and often used two points on the same axis. Most attempts involved Pythagoras' theorem so gained the method mark here.

10) There was some evidence that candidates were running short of time by this stage. Most candidates attempted all parts of the question but the number of errors in working suggested that they were pressed for time.

In part (i), candidates again showed their competence at differentiation. Most scored both marks.

Part (ii) was also generally well done although a disappointing number of candidates at this level ignored the negative square root of 9. Some forgot to find the y coordinate while others could not evaluate $⅓x^3$ correctly for one or both x values.

It was pleasing to see the variety of methods used in part (iii). The most common was the consideration of the sign of d^2y/dx^2, but many centres had taught candidates to investigate the gradient each side of the stationary point, while other candidates looked at the y values either side of $x = 3$ and $x = -3$. Sketches were clear and helpful. Many good candidates simply sketched a positive cubic, marking on the stationary points, which was also acceptable.

Full solutions of part (iv) were very rare, even from very good candidates. A few candidates successfully equated the gradients of the line and the curve to establish $x = 1$ and $x = -1$, but then substituted each x value for either the line or the curve. In fact they needed to substitute each x value into both of these equations to establish which x value had the same y coordinate in both in order to ensure that the line and curve actually met at that point. As in part (ii), too many candidates ignored the negative square root which, while leading to the correct value of q, did not constitute a fully correct solution.

Others opted to find where the line and curve met. However, this required the solution of the cubic equation $x^3 - 3x + 2 = 0$. A few candidates spotted the solution $x = 1$ but often forms such as $x(x^2 - 3) = -2$, leading to a factorisation similar to $x^2 - 3 = 0$ and $x = -2$, were common. Again those who did manage a solution usually then proceeded to find the y values of the line and curve for their point of intersection and as expected these were identical. In fact, they now needed to ensure for which value of x the gradients of the line and curve were identical before they could state the appropriate y coordinate.

In a few impressive solutions, candidates showed that the cubic had a repeated root at $x = 1$ and stated that, therefore, the line was a tangent at that point.

4722: Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates and, overall, the standard was good, though a minority of candidates struggled with even the most basic topics. There were a number of straightforward questions where candidates who had mastered routine concepts could demonstrate their knowledge, and other questions had aspects that challenged even the most able candidates. On a few questions, Q1(i), Q4(i) and Q9(aii),, a number of candidates lost marks through neglecting to answer all aspects of the question.

Only the most able candidates can manipulate logarithms accurately, and trigonometric topics also posed problems for many. There seems to be much confusion between degrees and radians, and many candidates seem unclear of the distinction between angle A, $\sin A$ and $\sin^{-1} A$. Many candidates were reluctant to use exact values, especially surds and trigonometric ratios. They must be aware that, where an exact answer has been requested, full marks will not be awarded for decimal equivalents. They must also ensure that any intermediate values in their working are accurate enough to justify the final answer to the specified degree of accuracy. Whilst some scripts contained clear and explicit methods, on others the presentation was poor making it difficult to follow methods used and decipher answers given. This is especially true if a candidate starts a second attempt at a question. On questions where the answer has been given, candidates must ensure that they provide enough detail to be convincing.

Comments on Individual Questions

1) (i) Most candidates found this to be a straightforward first question and it was generally very well answered. Virtually all candidates could list the first three terms of the sequence successfully, though a minority interpreted it as an iterative sequence. Most could identify the sequence as arithmetic, even if they couldn't accurately spell it. A few candidates thought it was a geometric sequence, or used other incorrect terminology, and a few omitted this part of the question completely.

(ii) Whilst most candidates could identify the general meaning of sigma notation, for a few this was clearly unknown to them and they searched the formula book for anything of use, usually resorting to the formula given for the sum of the square numbers. A number thought it was simply the sum of the first 100 natural numbers, and even those candidates who realised that it was the sum of the sequence in part (i) often struggled to get the values of both a and d correct.

2) (i) Whilst a few candidates stated the required equations, the vast majority simply quoted the general equations for the area of a sector and the arc length. They then gained the marks for this part by subsequent work in part (ii) when the values of 12 and 36 were substituted. Some candidates failed to gain credit by working in degrees instead of radians, and a few quoted inappropriate formulae from the List of Formulae.

(ii) Virtually all candidates could state that the value of θ was 2, though some gave the degree equivalent. Whilst some convincing proofs were seen to show that the value of r was 6, many candidates were reluctant to use simultaneous equations and relied on showing that these two values satisfied both equations.

(iii) This was generally well answered with many candidates able to quote the relevant formulae and find the final answer efficiently. A surprising number felt the need to convert θ into degrees before using the formula for the area of a triangle, possibly betraying insecurity when using radian measure. There were also some rather long-winded methods based on finding the base and height of the triangle. These methods were often successful, but not always fully accurate.

3) (i) This question was generally done very well, and most candidates showed good mastery of basic integration skills. A few candidates lost marks by omitting the constant of integration, or by still having an integral sign or d*x* in the final answer. Some candidates were unable to accurately expand the brackets, with a constant term of 4 being a surprisingly common error. Some of the weaker candidates differentiated instead of integrating, and a few simply integrated the two brackets without appreciating the need to first expand the given expression.

(ii) Some excellent solutions to this question were seen, but these were in a minority. A number of candidates could integrate to get an expression of the form $kx^{1/2}$, but ½ was the more common coefficient. Many more candidates seemed to have only limited understanding of indices and struggled to rewrite the expression as $x^{-1/2}$. Common errors included $x^{1/2}$, x^{-1} and x^{-2}. The majority of candidates understood the concept of definite integration and gained a mark by substituting the limits correctly, though a number did not explicitly show the value of 0 being used. Some of the weaker candidates seemed to think that by rewriting the expression using indices, integration had also taken place and simply substituted the limits into $x^{-1/2}$, an approach which gained no credit.

4) (i) Most candidates realised the need to use the cosine rule and attempted to do so, though a few labelled the sides incorrectly. The subsequent rearrangement of the formula was usually accurate, though a number struggled with the algebra and thought that the coefficient of cos *C* was given by $(a^2 + b^2 - 2ab)$. Whilst a few of the more able candidates used elegant methods to find an exact value for sin *BCA*, many more simply found the value of the angle itself and hence the required value. A significant number of candidates struggled to distinguish between $C = -{}^1/_3$, $\cos C = -{}^1/_3$ and $\cos^{-1} C = -{}^1/_3$. A surprising large number of candidates overlooked the request for the value of sin *BCA* entirely. Some candidates tried to do both parts of the question using basic trigonometry by assuming that right angled triangles only were involved.

(ii) A number of candidates struggled to make progress in this part as they couldn't identify the required angle property. Long-winded, and ultimately futile, methods were common. A number thought that angle *ABC* was equal to angle *ADC,* or identified parallel lines but then assumed that angle *BAC* equalled angle *ACD.* However, a pleasing number of candidates could identify the equal angles necessary and use the sine rule appropriately. Premature rounding of the angle led to inaccurate answers on a number of occasions. Some candidates thought that angle *CAD* was 70.5°, either from using supplementary angles or from an incorrect value for angle *BCA*. This also gave an answer of 18.3° but full credit was not available.

5) (i) The responses to this question were very varied, with a number of concise and accurate solutions seen, but also some very lengthy solutions that gained little or no credit. Many candidates seemed familiar with the factor and remainder theorems and could attempt the two equations, though 3^3 becoming 9 was a surprisingly common error. Of those who managed to get two correct equations, most could then solve them correctly though some candidates struggled with even these two basic simultaneous equations. However, a significant proportion of the candidates chose to use much more cumbersome methods, which also took up a lot of time. Some candidates attempted long division, and more used a method of matching coefficients. In both cases they seemed familiar with the standard methods but struggled to apply it to this situation, and were rarely successful. On the matching coefficients method, when using $(x + 1)$ as a factor, it was quite common to see the statements $C - 1 = a$ and $C = b$ with no appreciation of the need to combine them. Whilst it is important for candidates to be familiar with a variety of methods, it is also important that they appreciate the most efficient and accurate method for a given situation.

(ii) Most candidates could attempt f(2), but a number went no further upon realising that the result was not zero. However, many persevered and could gain credit for attempted factorisation even with the wrong equation. The more astute identified a combined factor of $(x + 1)(x - 2)$ immediately and then attempted the final linear factor with little further effort.

6) (i) Most candidates could attempt the binomial expansion, though a number chose to expand the three brackets and were rarely successful. When using the binomial expansion, a few made errors such as ${}^3C_2$ becoming ${}^3/_2$ or the three components of a term being added rather than multiplied, but most could make a correct statement. As in previous sessions, the more successful candidates used brackets effectively, but the majority of candidates had great difficulty in simplifying the terms as they were unable to use indices (a C1 topic) accurately. Errors included $(x^2)^3$ becoming x^5, $3(x^4)(x^{-1})$ becoming $3x^{-4}$ and, most commonly, $3(x^2)(x^{-2})$ becoming $3x$. The vast majority of candidates could gain some credit on this question, but only a few gained full marks.

(ii) The integration was generally very well done, especially the positive powers. The negative powers caused more problems, including an uncertainty of how to increase a negative number by one and how to obtain the correct coefficient.

7) (i) This was generally done well, with many candidates obtaining log 25 though this often followed ${}^{(\log 300)}/_{(\log 12)}$. Only the more able candidates realised that this could be further simplified, though a few gave 5 as the final answer. Some candidates changed the base to 10 and then used their calculator to carry out the evaluation – an approach that was successful on this occasion but would be less helpful if the question had involved algebraic expressions.

(ii) There were a variety of methods that could be used in this question and all of these were attempted by at least some candidates. Whilst many candidates gained some credit in this question, fully correct answers were few and far between. The most common mistake was $\log y = \log 3 \times \log 10^{2x}$, though other errors were also very common. It is obvious that most candidates are not fluent in the manipulation of logarithms. The most successful approach was to divide by 3 before introducing logarithms, which resulted in a concise and elegant solution.

8) (i) This question was done extremely well, with virtually all candidates gaining full marks. There was the occasional error in the power or the ratio, and some candidates attempted the sum of 4 terms rather than the fourth term.

(ii) Most candidates could gain at least some marks on this question, and for the weakest candidates it often formed a significant proportion of their total marks. Whilst a few attempted to use the formula for a sum of a geometric progression most appreciated the need to identify the first term less than 5000. Most candidates could set up a relevant equation and attempt to solve it, either by using logarithms or by trial and improvement. Those who used the latter method were often successful, despite showing little or no working, though some stopped at a value of 50,000 not 5,000. Of those who employed more formal methods, most could use logarithms effectively though some failed to rearrange the equation before introducing logarithms. On obtaining a value of 29.4, there was some uncertainty about whether to round up or down, and this was exacerbated by an error with the inequality sign introduced by dividing by log0.9. Many candidates were then uncertain about whether to add this value of n to 2000 or 2001, and a number simply stated the number of years rather than an actual date which, on this occasion, was still given full credit. Some candidates used an index of n and some used $(n - 1)$, but their intention was often not made clear until they attempted to convert the value of their index into a value for the year.

(iii) Again, this question was generally done well with most candidates able to quote the correct formula and evaluate it using a value for n based on what they thought the final year of production was. A significant number of candidates attempted the sum to infinity instead. Some of the weaker candidates actually listed all the terms and then summed them.

9) (a)(i) Very few candidates were able to give the exact values requested, and decimal answers were abundant. Whilst 0.866 and 1.732 were to be expected, there was also a plethora of other decimal solutions offered, reflecting uncertainty over how to evaluate even basic trigonometric ratios, especially in radians. The verification also caused a number of problems, with some embarking on lengthy, and incorrect, solutions involving trigonometric identities and others offering a verification involving $2\cos(^{\sqrt{3}}/_{2})$.

(ii) The graph sketching in this question was poorly done. Whilst a number of candidates could sketch one of the graphs correctly, usually $y = 2\cos x$, it was unusual to see both graphs correct. Very few candidates even attempted to find the roots of the equation, either through not noticing this part of the question or by not realising how the graphs could help them. Indeed, there were a number of candidates who drew two correct graphs which crossed at a point clearly marked $\frac{1}{2}\pi$ and yet still did not state this as a solution. Not all candidates marked scales on their graph, which did not help them in their search for solutions.

(b)(i) Most candidates seemed familiar with the trapezium rule and could attempt the question, though errors were common. There were the usual mistakes of interpreting 3 strips as meaning 3 ordinates, and using x coordinates not y coordinates but, in addition to this, many candidates struggled to evaluate $\tan x$ accurately. A significant number had their calculators in the wrong mode, but errors such as $\tan(0.1\pi)$ and $\tan^{-1}0.1$ were also common. A large proportion of the candidates failed to work to the required degree of accuracy. The question paper requests answers to be given to 3 significant figures, it was fortunate for many that full marks were awarded for 0.077, though some could not even manage this.

(ii) Underestimate was a more common answer than overestimate, and even those who got the statement correct often struggled to give a plausible reason. The most successful candidates stated that the tops of the trapezia were above the curve and provided a clear diagram to support this.

4723: Core Mathematics 3

General Comments

It was evident that many candidates had prepared well for this examination and mathematical competence was noted on many scripts. There were candidates who struggled with all aspects other than the strictly routine but an examination at this level has to assess candidates' ability to apply concepts and techniques in unfamiliar contexts or in the solution of multi-stage problems. Time did not seem to have been a problem. The early questions proved accessible to the vast majority of candidates and the later questions, particularly Qs 7 and 9, contained a few demanding requests which defeated all but the most able.

Some questions have their parts labelled (i), (ii), … whilst others have parts (a), (b),… . There is a rationale behind this labelling and there are instances when awareness of this rationale would be helpful to candidates. The labels (a) and (b) in this paper were used in Qs 4 and 6. In the former, the two parts concerned integration but were otherwise unconnected. Similarly in Q6, the topic was differentiation but the two parts considered different aspects. By contrast, the labels (i), (ii), … are used when there are much stronger links between the parts. Awareness of this would have helped many candidates in Q7; there were many candidates who treated the three parts of Q7 as three unrelated questions.

Comments on Individual Questions

1) Precision was required in answering part (i) and the answer f(x) < 10 did not earn the mark. Most candidates appreciated that the given form of the function enabled them to write down the answer and many did so correctly. A few expanded and simplified and then the answer tended to involve 1.

   Part (ii) was generally answered very well. Most candidates evaluated f(–1) and then f(6) and obtained the correct answer. A minority preferred to find and simplify an expression for ff(x) before substituting; algebraic errors meant that success was not so widespread for those adopting this approach.

2) This question was answered well. The method of squaring both sides of the equation was the more popular. There were some errors in the subsequent algebraic manipulation and, on some scripts, the root x = 0 was omitted. Those candidates considering a pair of linear equations also usually succeeded and there was less tendency with this approach for the root x = 0 to be omitted.

3) Examiners were pleased to see many correct solutions to both parts of this question. In part (i), most candidates realised that producing $e^{-0.017t} = \frac{25}{180}$ was an appropriate step to take prior to the introduction of a natural logarithm.

   In part (ii), some candidates substituted 55 immediately but most appreciated that differentiation was required and proceeded to obtain the correct answer without difficulty. For those attempting differentiation, the commonest error was a derivative of $-3.06t\,e^{-0.017t}$ though this was not widespread.

4) The vast majority of candidates was aware of the formula for a volume of revolution but not all could deal successfully with $(\frac{2}{\sqrt{x}})^2$. When $\frac{4}{x}$ or $4x^{-1}$ was obtained, an integral involving $\ln x$ usually followed although $\frac{1}{4}\ln x$ did occur a number of times. The exact answer $4\pi \ln 5$ was required but some candidates lost the final mark in part (a) by offering a decimal approximation.

   Many candidates answered part (b) without difficulty and retained sufficient accuracy during their working to be able to give the answer to the requested accuracy. A few candidates used an incorrect value for h in the formula as given in the *List of Formulae* . More common were errors apparently caused by a misunderstanding of the notation. Some, perhaps interpreting $y_0, y_1, \ldots$ as values of y

when x = 0, 1, …, effectively found an approximation for the integral between the limits 0 and 4. Others, instead of a pattern 1, 4, 2, 4, 1 for the coefficients of the y values, proceeded with a pattern of 1, 2, 4, 2, 1; possibly they had a recollection of the formula involving '4 times the odds and 2 times the evens' but incorrectly associated 4 with the odd x value 3 and 2 with the even x values 2 and 4.

5) Part (i) was generally done well although, in some cases, trigonometric uncertainties led to $\tan\alpha = \frac{3}{2}$ and therefore to a wrong value for α.

Not all candidates seemed aware of the relevance of part (i) to the solution of the equation in part (ii) and no sensible progress was made. To most candidates though, this seemed a familiar topic and they proceeded to find one value of θ. Obtaining the correct second value was less common and many merely subtracted the first value of 42.4° from 180° to produce a second value. Not all candidates worked with sufficient accuracy; some, for instance, took the value of R as 3.6 and their final answers were then not sufficiently accurate to earn the final mark of part (ii).

6) Most candidates recognised the need to use the product rule in part (a) and did so correctly. Solving the equation $\ln x + 1 = 0$ was usually carried out correctly although answers such as e^1 and $-e$ did occur.

Part (b) requires use of the quotient rule, a formula for which is given in the *List of Formulae*. Some had the terms in the numerator the wrong way round and algebraic skills were poor in many cases; brackets were omitted and simplification often led to the incorrect $\dfrac{0}{(4x-c)^2}$. Even for those with a correct simplified derivative, a convincing concluding comment was not very common.

7) This question required some skill from candidates in dealing with trigonometric identities. Those who appreciated that parts (ii) and (iii) were most efficiently tackled by using preceding parts did best. For many candidates, part (i) was not straightforward and some reached the required expression only after consideration of the formula for cos(A + B). Attempts at the proof in part (ii) were often not presented logically; some worked independently on the left and right hand sides until equality was reached. Those who substituted the expression from part (i) in the left hand side were usually able to provide a sufficiently convincing argument.

Faced with the equation in part (iii), a significant number of candidates ignored the earlier parts of the question, tried to express everything in terms of sin x and cos x and made no progress. Candidates using the result from part (ii) were usually able to form, and then solve, a quadratic equation in tan x. It was encouraging that many candidates seemed comfortable handling radian measure and were able to find the four roots.

8) Although this question contained one or two searching aspects, candidates generally answered it well. In part (i), almost all recognised that numerical substitution was required. Some substituted 5.2 and 5.3 in each of the two expressions but then struggled to provide a convincing justification for the location of P. Those who substituted in the single expression $e^{\frac{1}{5}x} - \sqrt[3]{3x+8}$ were then required to note the change of sign and most did so.

Most candidates were able to carry out the manipulation required for part (ii) although a few thought that numerical substitution, of a value such as 5.25, was needed.

The iteration process required in part (iii) was done well with most showing the necessary evidence. Surprisingly, some started with a value 0 or 1 rather than the apparently more obvious 5.2, 5.25 or 5.3. It was also surprising how many candidates, having obtained correct values for the first few iterates, concluded with a value of 5.28 rather than the correct 5.29.

Candidates knew, in general terms, how to proceed with part (iv) but often the details of the integration were incorrect. A few had the subtraction of the definite integrals the wrong way round and, in a significant number of cases, no evaluation using the limit 0 was made. A pleasing number of candidates did complete this question successfully, showing clear and precise working.

9) This question contained particularly challenging aspects though there was a number of candidates equal to the challenges. In part (i), most candidates had the idea that the transformations stretch and translation were needed. To earn all four marks, the terms 'stretch' and 'translation' were required together with correct associated detail in each case. Only partial credit was available for use of words such as 'shift', 'move' and 'squash' and in cases where the transformations were presented in the wrong order.

There was greater success with part (ii) and many found a correct expression for the inverse function. Many candidates, though, believed that f was one-one because one value of x led to one value of y; this merely confirms that f is a function.

In part (iii), most candidates recognised that each curve is a reflection of the other in the line $y = x$. But the subsequent argument in most cases seemed to be along the lines of "they reflect in $y = x$, so they cannot possibly meet". Only a few mathematically astute candidates proceeded to rearrange either $\mathrm{f}(x) = x$ or $\mathrm{f}^{-1}(x) = x$ and to recognise the need for the resulting quadratic equation to possess no real roots.

4724: Core Mathematics 4

General Comments

It is not easy predicting the response of candidates to a new paper. The specification for this unit has four sections and it was clear that many candidates had been well prepared. However, about 15% obtained less than a third of the marks and 3% produced hardly anything; this is a real cause for concern as evidence of any mathematical ability was non-existent and there appeared to be no form of preparation for the examination.

Presentation from some centres is good; from others it was abysmal – the manner of the writing of the candidate's name and the listing of the numbers of the questions answered was often an excellent indication of what will be produced.

In this paper, there were four cases of answers being given. It should be stressed to candidates that the work in such circumstances will be more closely scrutinised than usual and a higher degree of explanation will be required.

There were the usual cases of misreading; in Q1, the quartic often involved an incorrect sign as did the vectors in Qs 3 and 5. There were many instances in Q8 of both $(2+x)^2$ and $2+x^2$ appearing in the same script.

Candidates made good use of the new 1h 30m length of paper; just occasionally part (iii) in the last question was omitted but that was mainly because of the length of time spent on part (ii).

Comments on Individual Questions

1) The most common approach to this was long division and few candidates had any problems in producing either the quotient or the remainder. The same cannot be said for the identity method(s); many did not realise that they should allow for the remainder to be of the form $px+q$. Some used the identity $x^4+3x^3+5x^2+4x-1 \equiv (x^2+x+1)(ax^2+bx+c)$, a device that was suitable for finding the quotient provided the relevant coefficients were examined – but most showed that they did not realise just what was happening. A few candidates had difficulty with the word "remainder" and gave it as $-\dfrac{3}{x^2+x+1}$.

2) Almost all realised that this was a question to be solved by 'integration by parts' and most were aware of the correct split. The first problems arose at the integration of cos x and of the subsequent sin x; the use of limits also indicated that a large minority assumed that a limit of 0 will always produce 0.

3) Most were able to produce the correct equation in (i) although **r** = was often missing or replaced by L_1. In (ii), the majority knew what to do but systematic working was not always in evidence – some candidates solved equations (i) for example and (ii) for t and then substituted into (iii) to find s. However, most managed to prove that the lines were skew.

4) The majority were aware of what they should be doing in part (i) but some were not able to pursue it to the end. Most realised that $\dfrac{dx}{d\theta}=\sec^2\theta$ but then dx was sometimes converted to $\dfrac{d\theta}{\sec^2\theta}$. As is usual if the answer is given, close inspection of the working was always made and any laxity punished.

   Part (ii) involved the integration of $\cos^2\theta$ and, almost invariably, the changing of the limits from x to θ. There was a good knowledge of the integration, though signs were sometimes wrong or

$\frac{\cos^3 \vartheta}{3}$ seen; as far as the limits were concerned, 45 often appeared instead of $\frac{\pi}{4}$ and even 50 (presumably from candidates using the grads mode on their calculators).

5) Vectors often cause problems and finding the position vector of D was no exception. A basic simple diagram would have helped but these were few and far between. It was clear, either from the diagrams or from the work shown, that the labelling of the parallelogram was often being considered as $ABDC$ and, even on a few occasions, as $ACBD$. The side AB was often denoted by **a** or **ab** or ab etc. One easy way to find the position vector of D was to say: $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} = \mathbf{a} + \mathbf{c} - \mathbf{b} = 2\mathbf{j} + \mathbf{k}$. A few candidates used the simple idea that the mid-points of AC and BD were identical.

The technique of calculating an angle was generally well known; scalar product ideas and magnitudes of vectors were good. The only real weakness was that of determining which two vectors should be used; **a** and **b** were frequently seen.

6) Although a handful of candidates changed the format of the given equation to $x = \frac{3y}{y^2 - 2}$ and avoided implicit differentiation (though the derivation of the answer was then not particularly easy), the vast majority showed they fully understood the techniques of differentiating a product and y^2 (with respect to x). There were a few cases of $\frac{dy}{dx} = x.2y\frac{dy}{dx} + y^2 = 2 + 3\frac{dy}{dx}$ being seen but, fortunately, few actually brought the LHS $\frac{dy}{dx}$ into contention.

Part (ii) proved to be one of the most difficult on the paper. The statement $2 - y^2 = 0$ was very common though few knew what to do with it. Equally common was the statement that $\frac{dy}{dx} = 1$. An extremely small number used the fact that $2xy - 3 = 0$ and, in conjunction with the equation of the curve, produced either $8x^2 = -9$ or $y^2 = -2$ – thus showing that there were no real points on the curve at which the tangents would be parallel to the y – axis.

7) Part (i) was generally well done though a few worked out the cartesian equation first of all. It was not uncommon to see that, if $y = \frac{1}{t}$, then $\frac{dy}{dx} = \ln t$.
In part (ii), a good proportion of the candidates did not realise they had to find a value of t which satisfied both $t^2 = 4$ and $\frac{1}{t} = -\frac{1}{2}$; however, the derivation of the equation of a tangent was sound.

Most realised how they should start in part (iii); the derivation of the equation $t^3 - 12t - 16 = 0$ was relatively easy as was $16y^3 + 12y^2 - 1 = 0$ but few were able to put the equation $x - \frac{16}{\sqrt{x}} = 12$ into a more suitable form for solving (though it was possible to see $x = 16$ by inspection – or make use of a graphical calculator). The solutions of these equations proved more problematic though it should have been realised that $t = -2$, $y = -\frac{1}{2}$ and $x = 4$ were repeated roots of the individual equations.

8) The cover-up rule and/or the use of an identity were common methods for solving part (i). The cover-up rule worked for A and C though some thought that it was B which could be found. The repeated denominator proved an obstacle for some in the identity but the majority had a good understanding of what they were doing. Multiplication throughout by $1+x$ and $(2+x)^2$ before substituting suitable values for x was used by some centres, generally with success.

In (ii), the expansion of $(1+x)^{-1}$ rarely caused any problem, but those of $(2+x)^{-1}$ and $(2+x)^{-2}$ were less successful; $(2+x)^{-1}$ was frequently seen as $2\left(1+\frac{x}{2}\right)^{-1}$, $(2+x)^{-2}$ as $4\left(1+\frac{x}{2}\right)^{-2}$ and $\left(\frac{x}{2}\right)^2$ as $\frac{x^2}{2}$. Although the partial fractions were generally used, others worked with $(3x+4)(1+x)^{-1}(2+x)^{-2}$ (with some success) and $\frac{3x+4}{4+8x+5x^2+x^3}$ (with no success).
There were 2 common answers in (iii): $|x|<1$ and $|x|<2$ (or equivalent forms).

9) It is almost asking for trouble to seek explanations (as in the first part) but tolerance was given to poor descriptions provided there was a reasonable attempt to show why the differential equation fitted the bill. The ideas of $\frac{d\theta}{dt}$ being the rate of change of the temperature, k being the constant of proportionality, the negative sign indicating 'falling' and the $\theta-20$ representing the difference between the temperature of the object and the temperature of its surroundings were all that were required.
The setting of the question in this form at least ensured that all candidates would start with the same differential equation and that there would be a context.

In part (ii), most separated the variables or inverted the two sides of the equation; either, of course, could be successful but the latter generally involved problems with the 'k' factor – most attempting the former left the 'k' on the RHS and so it was separated from the $\theta - 20$ in the denominator of the LHS whereas the latter method generally meant integrating $\frac{-1}{k(\theta-20)}$. Some candidates attempted to use the boundary conditions before integrating; although their value of k was wrong, the process could be followed through. Others omitted to include '+c'. There were some interesting attempts to produce the required equation, relevant stages often being omitted.

Careful reading was needed in part (iii); most candidates used $\theta = 32$, a fair number used 36 and more than a handful used their own version of '68 – 32'. Relatively few candidates performed the final piece of work, subtracting 5 from their previous answer in order to show 'how much longer …'

4725: Further Pure Mathematics 1

General Comments

All the questions proved accessible and completely correct solutions to all questions were seen, with quite a number of candidates scoring full marks. Most candidates worked sequentially through the paper and few candidates appeared to be under time pressure and so did not complete an attempt at all questions.

Qs 4, 6 and 9 proved to be most demanding, and fewer completely correct solutions were produced than in the remaining questions.

It is expected that the standard of algebraic competence shown needs to be of a high standard as errors early in a solution can lead to the loss of quite a number of marks, especially when many of the answers are given in the question.

The presentation of work from the minority of candidates was of a good standard, which helps both the candidate and the examiner.

Comments on Individual Questions

1) Most candidates answered this question well, the most common error being to take $\sum_{r=1}^{n} 1$ to be 1 rather than n.

2) This was well answered by the majority of candidates, with many candidates adopting a direct approach by finding $\mathbf{A}^{-1}$, rather than using the answer to part (i) and multiplying by $\mathbf{A}^{-1}$.

3) Most candidates answered (i) and (ii) correctly, with only a few careless errors occurring. More candidates did not seem to understand how the conjugate should be used to achieve the division in (iii).

4) A significant number of candidates seemed to have no idea how to approach this question, while a small but significant number just wrote down the answers, presumably obtained from a graphical calculator. The examiners expected to see the real and imaginary parts equated, the elimination to obtain a quadratic in, for example x^2, and its solution, all in reasonable detail.

5) In (i) there were too many candidates who obtained the given answer though their numerator was, for example, clearly $r^2 + 2r + 1 - r^2 + 2r$. The method differences in (ii) was generally well understood and applied. The most common error in (iii) was obtaining 0 as the evaluation of $\frac{n+1}{n+2}$ as $n \to \infty$.

6) The locus C_1 was generally recognised as a circle, the most common error being to have the centre at (0, –2) rather than (0, 2). Fewer candidates recognised that C_2 was a straight line. In part (ii), too many gave the answers as coordinates, rather than complex numbers, and mixtures such as (2, 2i) were also seen.

7) Even though the potential for arithmetic errors was relatively high, most candidates worked with pleasing accuracy. Most candidates knew all the processes for finding the inverse of a 3×3 matrix, and most dealt with the general case in terms of a and not the particular case $a = -1$. A significant number of candidates solved the equations in (iii) by direct methods, rather than using the inverse matrix found in (ii).

8) This question was answered well by the majority of candidates. A significant number of candidates in (a) (iii) thought that $x^2 + 4x + 16$ was a quadratic <u>equation</u>.

9) The transformation geometry in matrix form was generally well understood and the first three parts often scored full marks. The "shear" in (ii) was often described using weaker synonyms and it is hoped that centres will point out that the terms given in the specification are those required by the examiners.

However, the presentation of the Induction proof in part (iv) did cause the examiners some concern. The inductive proof does require a rigorous setting out and explanation; effectively all the "answers" are given in the question, so the explanation of how the elements of $\mathbf{M}^{k+1}$ are obtained in the required form needs to be explained more fully, rather than just written down. There were many examples of candidates "fudging" the answers or just missing out vital steps or not making any reference to the final induction conclusion.

Mechanics

2637: Mechanics 1

General Comments

The level of achievement of candidates covered a wide range. There were some extremely good quality scripts seen by examiners.

Candidates should be reminded for the need to show sufficient working when asked to produce a required answer as in Qs 1(i), 3(i) and 5(iii).

Comments on Individual Questions

1) (i) This part was generally well answered; most candidates were able to use a suitable method to show the required result. However, there were a few who answered part (ii) first, using $\theta = 60$,and then using their answer for X to show that $\theta = 60$.

(ii) The majority of candidates answered this question well.

2) (i) Most candidates found the acceleration correctly. A significant number of candidates did not attempt to find the time. Those who made an attempt at the time were usually successful.

(ii) Many good answers were seen. Some attempts were seen in which the height of *A* above *B* was taken to be g, leading to $\alpha \sin^{-1}(g/3.2)$.

3) (i) Many correct answers were seen, but few candidates made clear the direction in which they were resolving, or else they showed insufficient detail to explain their solution. There were cases seen of manipulation of the values in the question to acquire the given answer.

(ii) & (iii) Examiners saw very few correct answers. The drawing of a correct force diagram for the forces acting on R_1 would have been beneficial to candidates. It was common to see the normal reaction as $0.3g$, omitting the component of tension.

4) (i) The majority of candidates answered this part correctly.

(ii) Many correct answers were seen. Common errors included:

- an initial velocity of 5 m s^{-1}
- a velocity of zero when $t = 100$
- a positive velocity for the complete motion
- the trucks shown to be accelerating between $t = 100$ and $t = 300$.

(iii) Most candidates knew a method to find the required distance but sometimes there were errors in application. Errors included finding only the area of a triangle above $v = 0.75$ rather than that of a trapezium. Others found the distance travelled for the complete journey. Those who used a constant acceleration method were generally more successful.

(iv) The answer to this part was effectively given. Many candidates found a correct distance for the second part of the journey without any indication of what this implied or what was being shown. This was especially the case where the answer to part (iii) was incorrect.

5) (i) The majority of candidates answered this part correctly. Only a minority attempted a whole system method to find the acceleration, but these candidates usually made the error of including a mass of 0.29 where (0.29 + 0.2) should have been used.

(ii) Most candidates answered this part correctly. A few candidates thought that the final velocity of *P* before hitting the ground was zero.

(iii) Many candidates found the correct speed. However it was common to see no attempt at finding the time. Those who did try made numerous errors including the use of their acceleration found in (i), or finding only half the correct time.

6) (i) Some candidates resolved forces to find the vertical and horizontal components of the resultant force, and then made reasonable attempts at finding R and θ. Others used a triangle of forces with the cosine and sine rule. However this led to an error in finding θ when candidates first tried to find the angle opposite the 500N force. Calculators gave this angles as 82.5^0 when it is in fact obtuse and equal to 97.5^0.

(ii) Many candidates did not find the mass of the child but used 400 instead. Others did not appreciate that the value of R found in (i) could be used, and used instead a resolved part of either the resultant force or of the original forces, thus finding only a component of the required acceleration.

7) This was the least successfully attempted question. Some candidates used constant acceleration throughout; most used variable acceleration techniques throughout, when in fact a combination of both was needed.

(i) Most candidates were able to differentiate accurately to find the correct value of k.

(ii) The majority of candidates attempted to solve the irrelevant quadratic $0.2t + 0.01t^2 = 25$. The correct method was very rarely seen.

(iii) Most candidates successfully integrated v, but few used the correct limits. Very few candidates appreciated that there were two parts to finding the distance.

2638: Mechanics 2

General Comments

The examination enabled the majority of candidates to show a reasonable understanding of the mechanics that they have been studying. In general, the main weaknesses were again in problems involving the equilibrium of a rigid body, Q6, or energy and work, Qs 3 and 5. Diagrams, with forces or velocities clearly marked, were often omitted and this often led to a significant loss of marks. In most cases it is a false economy to tackle mechanics questions without using a good diagram. Marks were also frequently lost through simple algebraic errors. Nevertheless there were many excellent papers and only a small percentage of candidates were totally unprepared.

Comments on the Individual Questions

1) The majority of candidates achieved the straightforward tan 32º = 6/*x*, even though many diagrams did not show that the centre of mass was on *AC*.

2) The first part was generally well done. Part (ii) was the most poorly answered question of the paper. The vast majority of candidates considered only one section of the string acting on B. As a consequence of this it was very common to score one mark out of three.

3) This was generally not well answered. In the first part some weaker candidates resorted to equations of motion or included the potential energy twice in their working. But those applying "Gain in KE = Loss of PE – Work done against resistance", or "Gain in KE = Work done by gravitational force – Work done against resistance" were usually successful. In the second part many candidates got muddled with the masses of the raindrop and the smaller droplet; others did calculations for a 20%, rather than an 80%, loss of kinetic energy. However a fair number avoided all these pitfalls to get the right answer, and there was a follow-through mark at the end for those who had gone wrong in the first part.

4) This question was straightforward and was generally well answered. However for quite a few candidates the concept of impulse was not well understood, and a mark was lost when the direction of the impulse was not shown. A small minority of candidates did not understand the meaning of "coalesce" in part (iii).

5) Candidates who attempted to find the changes in KE and PE usually did so satisfactorily, although some could not link these to find the work done. Some candidates used $1/2m(v-u)^2$ for the change in KE. Quite a number falsely assumed constant acceleration to obtain their answer, limiting the marks available in part (i) to 2, under a special ruling, instead of 4. Parts (ii) and (iii) were often answered well, most candidates knew how to calculate the power of the car's engine and how to write down an equation of motion up the hill.

6) This was a good question, testing an understanding of the forces acting on a rigid body in equilibrium; stronger candidates scored higher marks on it. However the majority had difficulty in writing down correct moments equations in both parts of the question. In the first part, too many did not know the position of the centre of mass of a uniform triangular lamina, and some did not get 40T as the moment of tension about *A*. In (ii), the perpendicular distance from B to *CD* caused difficulty; those who split the tension into horizontal and vertical components – and then took the moment of each – were usually more successful. Resolving to find the components of the force as *B* was done fairly well. Only a very few candidates used a triangle of forces and the cosine rule to solve (ii) (b).

7) The motion of projectiles was quite well understood by most candidates, but many algebraic and arithmetical mistakes caused a loss of marks. A surprising number of people expanded $-10(1+\tan\theta$ as $-10+10\tan^2\theta$ in (ii), hence causing mistakes in finding the two angles of projection; and some did not even cancel the $70^2/490$ at the start, but multiplied all through the equation by 490! Various methods were available for calculating the final distance in (iii); setting y = 0 was a sensible strategy employed by some candidates; the use of the formula for the range was seen quite often, but this was

a high risk strategy because the formula might not be quoted accurately. However there was a pleasing number of high marks for this question from the stronger candidates.

2639: Mechanics 3

General Comments

Many candidates demonstrated a good knowledge of the topics in the specification, and accordingly scored high marks on the paper. However, the questions revealed some areas of mechanics which were poorly grasped, and where such weaknesses were widespread, reference is made to them below.

In many solutions, candidates failed to indicate in any way to what their variables referred, or how numerical answers emerged from disjointed arithmetic. For the first time, there was significant evidence of graphical calculators being used to solve equations. Candidates should be aware that such working practices leave no scope for giving marks for methods or for accurate work, relating to intermediate stages, when the final answer is wrong. Writing down the expressions being evaluated, or checking that a calculator solution does satisfy the equation, provides valuable evidence for which marks can be given.

The other major problem was that some candidates did not give an answer where it was specifically required, the question having been read with insufficient care.

Comments on Individual Questions

1) (i) The use of the wrong trigonometric ratio was the only error seen regularly.

(ii) A failure to resolve the initial speed, using a wrong sign in finding the change in velocity, omission of the mass, or some combination of these, led to many wrong answers. Some candidates made the question harder by not appreciating the simplification implicit in the surface being smooth.

2) (i) Many candidates had clearly memorised the proof, sometimes omitting the minus sign. Even in scripts lacking a proof, the value of the period was frequently derived from a standard formula. Some able candidates did not give the period.

(ii) Candidates seldom started with $\theta = 0.08\sin 2t$. Use of the cosine alternative was more usual, sometimes with the incorporation of an initial quarter period time compensation. Some candidates were unwilling to use an angular formula and converted the data to a linear format, and others forgot to switch their calculators from degree to radian mode between Qs 1 and 2.

3 Most candidates understood this topic well, and excellent solutions were seen. Sign errors in the momentum and restitution equations were rare among candidates who used right to left as their positive direction. However, when the more usual alternative convention was employed, errors were common, particularly in the restitution equation.

Some scripts lacked a diagram to indicate the meaning of a candidate's symbols and equations. This caused particular problems when the horizontal component of *B* was negative. Candidates who then used a simple sketch to interpret their result as a positive value in the appropriate direction were able to complete a correct answer.

Some candidates lost one mark when the value of the angle was incorrect as a consequence of premature numerical approximation.

4 Fortunately few candidates attempted to work with lengths of the rods and angles of inclination. Correct solutions were seen frequently, but often a sign error in a moment equation prevented full marks being obtained. It was also common for solutions to be left incomplete, by not combining the components of the force on *AB* to obtain the required magnitude.

5 (i) This demonstration was achieved accurately by nearly all candidates. The only common error was the failure to obtain a force term from the power.

(ii) Many correct answers were seen, but candidates often marred their solution by leaving out the minus from the coefficient of $\ln(65000 - v^3)$,on integration, or having the 1050 or 2100 as its numerical value. Some candidates used $v = 0$ when $x = 0$.

6 (i) Very many candidates found Q 6 the most difficult of the paper. This might have been partly due to the lack of printed answers, and the need to deduce the behaviour of the particle prior to the string going slack. However, even at this first stage of the problem errors arose from incorrect resolving of the weight in a radial direction.

(ii) Few candidates appreciated the need to use an energy equation at this stage, to find the speed of the particle at *A*. The use of the inappropriate formula $v^2 = mgr(1 - \cos\theta)$ was common, and giving the transverse component of acceleration was another frequent error. Even when a numerical value for v^2 was found, the acceleration formula used was often mv^2/r instead of just v^2/r.

(iii) Many scripts had an energy equation used here for the first time.

Errors in the equation obtained by applying Newton's Second Law were an incorrect weight component and incorrect signs.

7 (i) Nearly all candidates gained full marks, the only error being a failure to give an explicit expression for *a*. A few candidates did not appreciate that *x* was the only letter which could appear in their formula for *a*.

(ii) Candidates were well able to give a convincing derivation of the given formula.

(iii) This part was generally correct and the most common error, finding *v* where $x = 0$ (at which stage the rope begins to impede the descent) was understandable.

(iv) A quadratic equation was usually correctly obtained and then solved.

(v) Again the answer was usually correct. The use of $x = 0$ was once more the only frequent mistake.

2640: Mechanics 4

General Comments

The candidates' work in this unit was generally of a very high standard, with about a third of candidates scoring more than 50 marks (out of 60), and only 15% scoring fewer than 30 marks. About 10% of the candidates scored full marks.

Comments on Individual Questions

1) This question, on Constant angular acceleration, was very well answered, with 90 % of candidates scoring full marks.

2) This question, on finding the centre of mass of a lamina, was well understood, and the integration as usually carried out accurately; about 80% of candidates scored full marks. Some did have difficulty with the *y*-coordinate; those integrating with respect to *y* often considered $\int xy dy$ instead of $\int (9-x) y dy$, and some assumed that $\hat{y}=\sqrt{\square}$

3) About half the candidates scored full marks on this question. On part (i), most candidates found the moment of inertia (using the perpendicular axes rule) and applied the formula for the period f a compound pendulum correctly. In part (ii), most candidates considered the work done by the couple, although the signs of the energy terms were often incorrect. There were several attempts to use constant acceleration formulae.

4) This question, on relative motion, was answered correctly by about a third of the candidates. Many very quick and efficient solutions were offered. However, about a third of the candidates scored no marks at all (for every other question the proportion scoring no marks was lower than 10%). It was often assumed that for the closest approach the velocities of the two boats should be perpendicular.

5) Most candidates used an appropriate integral to find the moment of inertia of the solid of revolution, although some used the method for finding the moment of inertia of a lamina. Minor errors, often involving powers of *a*, or failure to find the volume correctly, frequently prevented the valid derivation of the printed answer, but nearly half the candidates did obtain full marks.

6) This question, on finding the force acting at the axis of rotation, was quite well understood, although only about a third of the candidates obtained full marks. Parts (i) and (ii) were usually answered correctly, but in part (iii) there were many sign errors in the equations o motion, and the radius was very often taken to be *a* instead of ½ *a*

7) The energy approach to equilibrium was well understood, and parts (i) and (ii) were very often answered correctly. Errors occurred mainly in the gravitational potential energy term, which often had the wrong sign, and sometimes *AB* was taken to be a uniform rod instead of a light rod with a point mass at *B*. Part (iii) was often omitted, but there was also a lot of good work. Many had an incorrect expression for the kinetic energy, leading to a wrong answer for the period of small oscillation, but about a quarter of the candidates obtained full marks on this question.

4728: Mechanics 1

General Comments

There was a very wide range of ability displayed by the candidates for this paper. Many candidates displayed an excellent understanding of the techniques involved and organised their solutions in a clear and logical manner. There were, however, a number of weak candidates who had little knowledge of the forces acting in a given situation, nor of their direction and therefore had difficulty in producing the force diagrams which are usually an essential part of the solution to a question.

Some marks were lost unnecessarily by giving answers correct to two significant figures rather than three, as required by the rubric. Other marks were lost by prematurely approximating a calculated value which was then used in a further calculation.

Comments on Individual Questions

1) (i) Only a very small minority of candidates stated that the tension is the same in both parts of the string because the ring is smooth. Many gave as the reason either that the ring is in equilibrium or that the string was taut.

(ii) Horizontal resolution of the forces was usually attempted but the tension in *BR* was frequently ignored, even though it had been referred to in part (i).

(iii) Much better attempts were made to resolve vertically and many candidates were able to obtain the correct answer for *m* using the given value of the tension.

2) (i) Most candidates applied Newton's second law to the particles *A* and *B* to obtain two equations in *T* and *a*, although either the weights of the particles or the air resistances were quite often absent and the directions of the tensions were not always correct. Only a minority of candidates applied Newton's second law to the complete system to obtain one equation in *a* only.

(ii) There were some good attempts at eliminating either *T* or *a* from the two equations obtained, although poor algebraic techniques hindered a few candidates.

3) (i) The principle of conservation of momentum was widely known, but its application was sometimes marred by candidates who ignored the directions in which the particles are travelling.

(ii) Most candidates handled the constant acceleration formula $v^2 = u^2 + 2as$ extremely well in order to find the individual distances travelled by spheres *P* and *Q*. Occasionally, these distances were then either not added together or subtracted.

4) (i) This was a challenging question for many candidates and a variety of methods were used.
Those who used the equation $s = ut + {}^{1}/_{2}at^{2}$ for the two stages often made the mistake of using the same initial speed in each equation.
An alternative method which involved using the speeds at the mid-interval times did not often progress beyond finding the average speeds during the two stages, as many candidates did not attempt to find the mid-interval times of 0.4s and 1.4s. Such candidates used a time of 2s between these points instead, with the result that $a = (5 – 2.5)/2 = 1.25$ ms^{-2} was a commonly seen error.

(ii) This part of the question was often successfully attempted.

5) (i) There was widespread confusion over the directions of the frictional force and normal reaction at *A* and many candidates were unable to draw a correct force diagram from which to work. The equations produced by resolving the forces on *A* vertically and horizontally were frequently incorrect, common errors including F and N being interchanged, the tension acting in the wrong direction or the weight taken to be 2g N.

(ii) Many candidates were able to obtain the result Tcos□ = 7cos□ but did not convincingly show that this was 7×0.28 and therefore equal to 1.96. When answers are given in the question, it becomes more important to show clearly all the intermediate working.

(iii) A large proportion of candidates were able to answer this part successfully.

6) (i)(a) There was an encouraging number of correct solutions. However, some candidates found the configuration in this question difficult to visualise and attempted instead to answer a question about a block on an inclined plane, with an accompanying normal reaction. A common error was then to resolve in the direction of *P* to obtain the incorrect $P = 0.04g\cos20^{o}$, assuming, in this part of the question, that the forces acting on the particle were maintaining equilibrium. Other incorrect solutions showed *P* acting at an angle α to the horizontal rather than to the upward vertical.

(b) Many candidates used a correct method for finding *R*, either resolving horizontally or applying Pythagoras' Theorem (or occasionally cosine rule).

(c) In the majority of solutions, the resultant force *R* found in (b) was correctly used to find the acceleration but there were some scripts in which the force producing the acceleration was wrongly taken to be either *P* only or a combination of *P* and *R*.

(ii) There were many completely correct solutions to this part of the question.
Errors which occurred included misreading 0.08 as 0.8, 0.04 as 0.4, using 0.04 rather than 0.04g as the weight and taking the 0.08 N force to be acting vertically rather than horizontally. Most candidates used the correct method to obtain the magnitude of the resultant force but there were more problems associated with finding its direction, which stemmed mainly from the resultant being incorrectly shown on their diagrams.

7) (i) The vast majority of candidates obtained the correct answer to this part. Other candidates usually made only minor arithmetical errors.

(ii) Again, this was well attempted and although some candidates calculated the deceleration as 100/25, most obtained the correct magnitude. The main error was to state the acceleration as a positive quantity.

(iii) It was encouraging to see that almost all candidates attempted to differentiate in order to find an expression for the acceleration, although some also tried to 'differentiate' the factor 10^{-6}. There were only a few scripts in which $a = v/t$ was used.

(iv) The correct method was generally used, and most marks were lost here as a result of prematurely approximating *Q*'s deceleration as 0.248, leading to the answer 0.002 rather than the more accurate answer 0.0025 which was required.

(v) Many candidates realised that *Q* has a maximum velocity when its acceleration is zero. However there was a significant minority who thought it was sufficient to find its velocity (32 $ms^{-1)}$ when t = 400 and to show that it was less than this at both t = 399 and t = 401. Some candidates were content with finding the speed of 32 ms^{-1} without making any attempt whatsoever to show that it was a maximum.

(vi) The distance travelled by *P* when $t = 400$ was often incorrectly found, a common error being the use of a final speed of 25 ms^{-1} rather than the correct 22 ms^{-1} . There were a few scripts in which this distance was taken as 9000m, the answer found in (i). A high proportion of candidates attempted to integrate to find *Q*'s displacement and although the factor 10^{-6} was again sometimes a problem, the integration was usually executed well and there were many correct answers obtained for this value. Some candidates, however, wrongly assumed that *Q* was moving with constant acceleration.

4729: Mechanics 2

General Comments

The majority of candidates showed reasonable understanding of all topics, except for resolution of forces and/or taking moments. These are essential to the answering of Qs 3 (ii), 5 and 7. However, marks were frequently lost through arithmetic and algebraic errors. This may have been a consequence of rushing as a significant number of candidates appeared to be short of time. There were many excellent scripts and only a small percentage of the candidature was totally unprepared. Candidates should pay attention to the precise requirements of questions and to the use of clear diagrams with components of forces shown where appropriate.

Comments on individual questions

1) Most candidates demonstrated an appreciation of the fact that, on the point of toppling, the centre of mass lies vertically above the lowest point of the base. Mistakes occurred with the use of $1/3h$ rather than $1/4h$ and because of confusion between r and $r/2$. A follow through mark was awarded for "yes it will topple" if the candidate's r was more than 2.5, provided a valid reason was given.

2) This question was generally well answered. A few solutions were unnecessarily complicated when a general angle of projection was used. The majority of candidates simply considered vertical and horizontal components of velocity and gained full marks.

3) Part (i) was generally well answered. Part (ii) was the most poorly answered question of the paper. The vast majority of candidates considered only one section of the string acting on *B*. As a consequence of this, it was very common to score one mark out of three. Part (iii) was answered more satisfactorily and, because a follow-through mark was available, many candidates scored both marks. However, a significant number of candidates lost this mark as they failed to square the velocity despite quoting $\frac{1}{2}mv^2$ for the kinetic energy.

4) This question was generally well answered. Marks were lost for not giving the magnitude of the impulse in (ii) and for not stating the direction of this impulse. A small minority of candidates did not understand the meaning of coalesce. This is a requirement of the M1 specification (4728).

5) There were many examples of fiddling numbers to obtain the answer given in (i). In part (ii) many candidates assumed the direction of the force acting on the rod at *A* is either vertical or along *AB*. It is advisable that horizontal and vertical components of forces are shown on a clear diagram in order to answer questions such as this. As stated in the general comments, the resolution of forces and the taking of moments are currently the techniques which require the greatest improvement. There were a few successful solutions gained through using a triangle of forces and the cosine rule.

6) Candidates who attempted to find the change in K.E. and in P.E. usually did so satisfactorily, although some could not link these to find the work done. Some candidates used $\frac{1}{2}m(v-u)^2$ for the change in K.E. Many candidates implicitly assumed constant acceleration, calculated it, but then used it incorrectly. Part (ii) was generally well answered although there was a small number of fiddled attempts to obtain the given answer. The driving force was normally correctly found in (iii), but mistakes were often made in not using Newton's second law correctly or in confusing 'retardation' with 'retarding force'.

7) The first part was generally well done, although several candidates assumed the whole barrier to be of uniform density. Some candidates treated the masses as weights and omitted g. In both parts mistakes commonly occurred with sign and distance errors. In dealing with the second part a significant number of candidates 'broke' the barrier by rotating the arm but not the counter-weight. The question asked for the angle between *AB* and the horizontal. There were many small angles given, less than 20°, which if correct would mean that the barrier is impractical.

8) This question was generally well attempted, the main difficulty being to get through the problem without making an arithmetic or algebraic error. The most common mistake of candidates was to expand $-10(1+\tan^2\theta)$ as $-10 + 10\tan^2\theta$. In (iii) most candidates realised the need to set $y = 0$, and thus obtain a quadratic equation in $\tan\theta$. Some candidates found the time to mid-range but failed to double this time. However, there was a pleasing number of totally successful solutions.

Probability and Statistics

Chief Examiner's Report

The standard of work on the Probability and Statistics part of the examination remains generally high, although certain weaknesses continue to be apparent.

Centres are particularly asked to note the following points which have been agreed by the Examiners responsible for these papers in order to encourage good practice.

- Answers given to an excessive number of significant figures (such as "probability = 0.11853315"), which have not in the past lost marks, may in future be penalised.
- Hypothesis tests are likely no longer to include the explicit instruction "stating your hypotheses clearly"; any answer to a hypothesis test question should include a statement of hypotheses unless they are already given in the question..
- Likewise, questions that ask for critical regions may not ask explicitly for associated relevant probabilities, but these should always be given.
- Conclusions to hypothesis tests should be stated in terms that acknowledge the uncertainty involved. Thus "the mean height is not 1.8" is too assertive and may not gain full credit; a statement such as "there is significant evidence that the mean height is not 1.8" is much to be preferred.

2641 Probability and Statistics 1

General Comments

The overall standard of the candidates was quite good. There were few very strong candidates and hardly any very weak candidates. The questions on correlation and regression were answered very well. There was some evidence of candidates being short of time.

The attention of centres is particularly drawn to the points noted in the Chief Examiner's report above.

Comments on Individual Questions

1) (i) Almost all candidates scored the first mark for the mean of y, although a few added 11 here instead of in (ii).There was considerable misuse of the divisor 80 in the formula for variance.

(ii) Most candidates knew that they needed to add 11 to their answer from (i) to find the mean of x, but some decided to make a fresh start. Some obtained the correct answer, but many worked out (35.2+11)/80. Better candidates knew that the variances of y and x were the same. Some added 11 to the variance of y and many tried to make a fresh start, usually with no success.

Many candidates wasted a considerable amount of time in an attempt to score just 1 mark and usually failed to obtain it.

2) (i) Most candidates used the correct 9!/(2!3!4!) and almost all of these obtained the correct answer.

(ii) Most candidates did not know how to answer this part. Some candidates had some idea but were confused about how to deal with the 4 Gs. Some treated them as one item, and then worked out 6!/(2!3!).Others multiplied the correct 5!/(2!3!) by 4!. Both these errors were allowed a method mark.

(iii) Almost all candidates knew that they needed to work out answer(ii)/answer(i).

3) (i) Most candidates scored well on this part, but a significant number made one of the following two errors.
1. The key was misread to give answers of 27.5,35 and 45.5. It was usually unclear whether the candidate thought that these values were £ or pence.A special rule mark was available to these candidates,

2. The lower quartile was given as £2.50, obtained by counting from the left, rather than from the stem. This error produced an incorrect answer for the median but an acceptable value for the upper quartile.

(ii) Almost all candidates drew a correct box plot for their values.

(iii) Most candidates pointed out that store *A* was more variable. Most candidates did not realise that a comment about skewness or symmetry was required. Most of those who did scored the mark, because various comments about skewness could be correct, depending on which method was used. The mark was awarded, provided nothing contradictory was seen.

4) (i) Most candidates realised that they needed to evaluate (5/6)^3 to find *a*. Many then used sum of probabilities =1 to find *b*, but a considerable number worked out *b* independently. Weaker candidates used the given *b* to find *a*, which was not acceptable.

(ii) Most candidates knew how to do this part.

(iii) Better candidates used the binomial distribution correctly, many failed to use ${}^5C_3$.

5) (i) Most candidates drew a correct scatter diagram, but some lost marks for failing to label the points. A considerable number chose to plot the values for the first born on the vertical axis. Although unconventional, this was acceptable.

(ii) Most candidates confused Spearman's coefficient with the pmcc, and incorrectly concluded that the points should lie on a straight line. Better candidates pointed out that e.g. point *B* was too high

(iii) Most candidates knew how to do this part. As usual, a few did not rank the data.

(iv) Most candidates made a sensible comment.

(v) Most candidates knew that no answers would change.

6) (i) Most candidates answered this part correctly. A few lost a mark for giving a 3dp answer, when 4 were required.

(ii) Roughly half the candidates scored this mark. Of those who did not, many did not give a reason.

(iii) Most chose to work out the *y* on *x* line, and did so correctly. Many wasted time by working out both lines.

(iv) Many candidates answered incorrectly by using $x = 56$, rather than the correct $y = 56$.

(v) Most candidates scored one of the two marks available by referring to either the high value of *r*, or 100 being outside the data range. Few mentioned both.

7) (i) More candidates than usual made at least one correct comment.

(ii) Few candidates answered this part correctly, although many scored some credit for using Bin(20,0.03).The use of Bin(20,0.6)was common.

(iii) Few realised that a Geometric distribution was required.

(iv) Only the very best candidates made progress with this part. Many tried to use the formula for the Geometric distribution with non-integral powers.

2642: Probability and Statistics 2

General Comments

Most candidates were adequately prepared for the examination, though a significant number could not cope with enough of the statistical concepts. Many were able to display a thorough knowledge of most of the calculation techniques and generally found the paper accessible. However, there were some testing parts of questions.

Most candidates had sufficient time to complete the paper, but some possibly spent too long on Q6 to be able to do full justice to Q7.
In general work was well presented, with methods and working clearly shown.

It is essential that candidates identify the appropriate distribution that is to be used in a question by checking the sizes of parameters. Too many candidates automatically use a normal approximation regardless of whether it is valid. Thus Q1 needs Poisson (1.2); Q6 needs several Binomial distributions; and Q7 needs several Poisson distributions. It would assist the understanding and answering of the questions if candidates clearly stated which distribution they have chosen, and what are the values of the relevant parameters. Thus

Q6	(i)	B(25, 0.8)	Q7	(ii)	Po(5.4)
	(ii)	B(20, 0.0468)		(iii)	Po(3)
	(iii)	B(25, 0.6)		(iv)	Po(4.8)

Some confusion remains over reading from the cumulative probability tables and interpreting the ranges so obtained.

The attention of Centres is particularly drawn to the points noted in the Chief Examiner's report above.

Comments on Individul Questions

1) Most recognised that a Poisson distribution was appropriate. Tables could be used or $P(X \leq 2)$ calculated. A few mistakenly used a normal distribution.

2) Many attempted this question well, but simple sign and bracket errors were quite frequent. Some of these produced $d = -8.56$, which was not questioned. Some candidates inserted a spurious continuity correction. It was essential to use $z = 1.96$.

3) Only basic integration was required, but many did not follow correct procedures.
   - (i) The integral had to be equated to 1.
   - (ii) Most tried $\int t\mathrm{f}(t)\mathrm{d}t$, but far too many made mistakes with t/t^2 and k. The correct limits, 1 and 4, were necessary, not, for example, 0 and 4, A few looked for the median.
   - (iii) It was necessary to match the integral limits with the appropriate probability: (t_0, 4) with 0.1, or (1, t_0) with 0.9.

4) Responses to this routine question were disappointing.

   - (i) This question was an explicit request for part of the solution to a standard question that has not in the past been subdivided. It seemed to confuse many, with 0.0967/50 and other answers being common.

   - (ii) There is a worryingly increased tendency, noted in last year's examination, to confuse the sample and population means, so hypothesis statements such as "H_0: $\overline{X}$ = 1.72" (or "μ = 1.72") were widespread. The core idea of hypothesis testing is to assume a value for the population mean and find the likelihood of obtaining a result as extreme as the sample mean – *not* the other way round. More candidates than usual omitted the $\sqrt{50}$ factor. Weaker candidates compared a z-value with a probability, rather than comparing one z-value with another, or one probability with another. Some thought the test was one-tailed, and many failed to give their final conclusion in the context of heights of plants, merely saying something like "the mean is not 1.8".

5) (i) The probability of $W = 10$ was required, not $W \leq 10$. Those using tables needed to subtract two correct values.

(ii) Many knew the condition $np > 5$ for the normal approximation to be valid, but surprisingly few used $nq > 5$ as well. Some used trial and improvement rather than solving the inequality, but this was usually taken no further than $0.2 < p < 0.8$, which is insufficient. Those who used $npq > 5$ (which is not the rule given in the specification) had more complicated algebra to negotiate, but if they obtained the correct answer by this method they could score full marks.

(iii) There were many correct answers, but as usual many also chose the wrong, or no, continuity correction or used 7.2 instead of $\sqrt{7.2}$.

6) This was found to be much the hardest question, even though the final question of 2642 June 2003 was effectively the same question.

(i) Those who used B(25, 0.8) usually got the right answer, although some failed to state the relevant probabilities, and some looked for an upper tail around 23 instead of the lower tail. The wording of the question indicates that the answer is "$k = 16$", not "$k \leq 16$".

(ii) There was much woolly thinking; answers using the wrong distribution, such as 20×0.8, were often seen. The answer does not need to be an integer.

(iii) Better candidates realised that the critical value found in part (i) was to be used again here, rather than looking for a new k value with probability near to 0.05; this was the main lesson to be learnt from the 2003 question.

(iv) This was found to be the hardest part. Two approaches were available. The simpler involved first finding the probability that one test resulted in rejection of the null hypothesis, and then using $2p'(1 - p')$. The longer method required eight probabilities to be found, such as $0.9532 \times 0.7265 \times 0.5^2$. Some candidates found some of these but rarely all, and omission of 0.5^2 was very usual in this method.

7) This question required the Poisson distribution throughout.

In part (i) the conditions were poorly stated. In this sort of question it is not a good idea to use the concept of "randomness"; it needs further explanation if it is to mean anything other than "R is a random variable". For marking purposes, examiners take statements such as "the coins are scattered randomly" to be equivalent to saying that their positions are independent. The other important condition is that the coins are scattered at a constant *average* rate. Strictly, "constant rate" implies that there is no randomness involved; those who said that the number of coins per square metre was constant were not awarded the mark. However, this particular context brings up a further issue – are the coins scattered singly? If they are likely to be found in hoards this would invalidate the model, although candidates who used the word "singly" rarely gave any indication that they were engaging with the context of the question instead of regurgitating a sentence in a textbook.
Part (ii) was almost always done correctly, apart form those who stayed with 0.75 instead of 5.4.
In parts (iii) and (iv), some again confused the given values of λ with the parameters needed for the Poisson distribution. In part (iii) many obtained the correct probabilities but then gave the final answer as 6 instead of 7. In part (iv) a pleasing proportion could use the definition of a Type II error in the context of the question.

2643: Probabilty & Statistics 3

General Comments

This paper contained a number of straightforward questions which gave the average candidate the opportunity to display their basic understanding of the topics in the Specification and to accumulate a good number of easy marks without encountering any special difficulties.

These included fairly routine questions on a paired sample *t*-test, a chi-squared test applied to a 2×2 contingency table and a test on the difference of proportions, and these were generally well done.

There were questions on topics that students always find difficult, for example Q5 on continuous distributions with a two-part probability density function. This applies also to questions which asked for interpretations, the interpretation of a confidence interval in Q2 and the interpretation of the paired sample *t*-test result in Q4. Although these are always found to be difficult, the correct interpretation of results is a very important skill and in that respect the number of correct answers to the second part of Q4 was very encouraging.

A number of other questions, or parts of questions, were generally found to be more difficult, notably the last part of Q5 and much of Q6.

Overall the paper produced a good range of marks but with rather fewer marks in the high fifties than in previous years.

There were fewer candidates this year, approximately 750 compared with 900 in June 2004.

Even taking into account the relative difficulty of the paper, although there were still a lot of very good candidates, there were significantly fewer of the really outstanding ones.

There was no real evidence of many candidates having insufficient time to attempt all the questions.

The attention of centres is particularly drawn to the points noted in the Chief Examiner's report above.

Comments on Individual Questions

1) (i) This was a nice straightforward start to the paper and most candidates gained full marks.

(ii) Many candidates were confused by the way in which this part was phrased and failed to translate it correctly into a useable form such as "number of skimmed milk and of semi-skimmed bottles are both zero and the number of full cream milk bottles is two". Consequently only about half the candidates produced correct answers.

2) (i) Many students find the concept of a confidence interval confusing and a relatively small proportion were able to give a clear and correct interpretation of this confidence interval. There were many answers that referred to the sample mean instead of the population mean. The incorrect statement given in the question seemed to lead many in the wrong direction, trying to change parts of the statement rather than writing a completely new statement.

(ii) This part was generally answered well with the majority of students getting full marks.

3) Although this was a straightforward chi-square question, candidates were required to construct their own contingency table from the data given. As it was a 2×2 table they also had to use Yates' correction. The most common mistake was to use $(O - E - 0.5)^2$ rather than $(|O - E| - 0.5)^2$. However this question was generally well done and more candidates gained full marks on this question than on any other.

4) (i) This was a paired sample *t*-test. The data was clearly displayed in a manner that should have pointed in that direction. Many good, correct solutions were seen. However a surprisingly large number carried out a two-sample test which is incorrect – these are not independent samples – and also much more difficult.

There has been a steady improvement in the number of candidates who state their hypotheses clearly and correctly .There was still a significant minority who used the form H_o: $d = 0$. The hypotheses must refer to the population mean. The conventional and acceptable form is H_o: $\mu_d = 0$; H_1: $\mu_d > 0$.

(ii) This question raised an important issue, requiring students to point out that whilst the test showed that the mean biomass had reduced, the test could not show that the reduction was *caused* by the building of lagoons. There were an encouraging number of good answers, some even giving possible alternative reasons such as the change of season in the six month gap.

5) (i) The majority of candidates completed this part correctly.

(ii) There were a considerable number of correct answers, though many candidates first found $P(X \leq 5)$ which is more difficult than finding $P(X \geq 5)$ directly. A surprisingly large number of candidates started this part by attempting to find a general expression for the cumulative distribution function which again made this part more difficult.

(iii) An encouraging number of candidates first identified that the median lay in the range $1 \leq m \leq 3$ and then had little difficulty finding the median. Many others gave confused answers involving both parts of the pdf. Many lost a mark by giving the answer as 2.75, rather than 2.75 hundred litres or better still 275 litres.

(iv) The question says "When the weekly sales are x hundred litres, the profit *per litre* is £0.4x", so the profit on x hundred litres is £0.4x multiplied by x and by 100. So we need to find E(£40X^2). Most candidates calculated $E(X)$ or at best $E(40X)$ instead of $E(40X^2)$

Very few candidates got this final part correct.

6) This question involves the difference of two normally distributed random variables, the mass of a metal plate and the mass of a circular disc which is drilled out of it.

(i) Candidates have to recognise that for a "component [to] have a mass of 5 grams, correct to the nearest gram", the mass of the component has to be between 4.5 and 5.5 grams. This type of question is fairly common but many candidates seemed not to be familiar with it. As a result many candidates did not complete this part successfully.

(ii) This required candidates to derive a given inequality. It was a testing question. Some attempts at this part were clearly wrong but many others contained insufficient explanation and justification for the steps in their argument. There were a fairly small number of answers that were awarded full marks.

(iii) The best method was to evaluate the function $f(n) = 5.16n + 0.4838\sqrt{n} - 125$ for n = 23 and then n = 24, showing that f(23) < 0 and f(24) > 0. This is enough to demonstrate that 23 is the highest possible value satisfying the inequality. There were a pleasing number of candidates who did exactly this.

Many candidates chose to solve the quadratic in $\sqrt{n}$ but then a good deal of further justification is required.

7) (i) This was a relatively straightforward question on the difference of proportions. There were a number of good solutions although quite a few candidates used the less satisfactory test involving $\sqrt{\left(\frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}\right)}$.

(ii) "Show that the maximum value of $p(1 - p)$ is ¼" probably seemed to be a strange request on a statistics paper. Few thought to do a simple differentiation and many gave unconvincing explanations.

(iii) Those that completed part(ii) satisfactorily, and many who did not, often produced a good answer to the last part, finding the greatest possible width of the confidence interval.

2644: Probability and Statistics 4

General Comments

From an entry similar to that of June 2004, both in numbers and ability, the results followed a similar pattern. There were many excellent candidates who displayed deep understanding of principles matched by an ability to present their work clearly and accurately. Only a small minority was not able to cope with the demands of the paper.

The work on the hypothesis tests (Qs 3 and 4) was usually good but some believed that non-parametric tests do not require any distributional assumptions. It was pleasing to find that a majority expressed the hypotheses in terms of relevant medians and the test conclusion in context, as are expected.

The attention of centres is particularly drawn to the points noted in the Chief Examiner's report above.

Comments on Individual Questions

1) This was found to be a straightforward start to the paper and very many obtained full marks. Most knew that M′(0) gave the mean but a few forgot that M(0)=1. Some used the expansion of M(t) to obtain ab=⅛ which involves equal work.

2) Many were able to complete the first two parts without difficulty, but some confused conditions for independence in part (iii).

3) A majority was aware of the symmetry requirement but some used *even distribution* which is not an acceptable description. Most candidates were familiar with the required Wilcoxon test and could apply it accurately but some used the sign test, which gained little credit. As stated above, it is expected that hypotheses be given in terms of medians rather than in the general terms of the question.

4) This was the most searching of the questions and required some mathematical sophistication. The hints of parts (i) and (ii) did help and there were some very good solutions. Some, however, merely stated the direction of the bias without giving a reason.

5) Part (i) was found to be difficult, but the remaining parts were tackled with confidence so the question proved to be generally high scoring.

   The final part was rather unstable to small variations in the individual values and even 4 SF in parts (ii) and (iii) will not give an answer correct to 3SF. In this type of question it is better to give answers in fraction form.

6) (i) This is a standard piece of bookwork and requires careful presentation. An infinite series with the correct starting value is necessary and it was expected that the terms of the series for $e^{\lambda t}$ should be of the form $(\lambda t)^n/n!$. Sometimes the final expression was left as $e^{\lambda(t-1)}$ which was not the given form.

   (ii) The method of multiplying the pgfs was usually known but some did not convert their result to the form given in part (i).

   (iii) There are several possible reasons for X-Y not having a Poisson distribution and these were often known. Some good candidates tried to use an argument based on $G_{X-Y}(t) = G_X(t) / G_{1Y}(t)$, which is incorrect. In fact $G_{X-Y}(t) = G_X(t)G_Y(1/t)$ and the result could be argued from this.

(iv) This was found correctly by a majority of candidates either from the formula, from tables or from an expansion of the relevant pgf.

(v) Although there were some good solutions many forgot to divide by the answer to part(iv).

7) (i) The Wilcoxon two-sample test requires identical continuous distributions, apart from location. Only a few candidates had met this.

(ii) The required test was usually known and only a small minority tried a paired-sample test. The actual procedures for both Wilcoxon tests are given fully in the formula booklet and so they are marked quite strictly.

(iii) Very many were able to negotiate this end to the paper but some did not use the value of W calculated in part (ii). Others omitted a continuity correction and some confused the finding of the required probability with carrying out a significance test.

In applying the continuity correction some used both w+0.5 and w-0.5 and so obtained two probabilities. This was accepted if the correct answer was then chosen, but not otherwise.

4732: Probability and Statistics 1

General Comments

Almost all candidates showed a good understanding of at least some of the mathematics tested in this paper. There were many very good scripts.

Many candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby possibly losing marks. In some cases, many marks were lost for this reason alone. Some candidates confused three significant figures with three decimal places.

In some calculations a calculator can give the answer immediately, without the need to show working. However, candidates need to be aware that they risk losing all the available marks if the answer is incorrect but no working is shown. It is wise to show some working or at least to double-check the calculation. For example, in question 4(i) an answer of 0.868 or –0.867, without working, risks the loss of all three marks.

Only a very few candidates seemed to run out of time.

Most candidates failed to fill in the question numbers on the front page of their answer booklet.

The attention of centres is particularly drawn to the points noted in the Chief Examiner's report above.

Use of statistical formulae

A disappointing number of candidates appeared to make no use of the formula booklet, but used formulae from memory. More often than not, these formulae were incorrect. Others tried to use the given formulae, but clearly did not understand how to use them properly. Some candidates used the less convenient versions of formulae from the formula booklet, eg $\Sigma(x - \mu)^2 p$ and $\Sigma (x - \bar{x})(y - \bar{y})$. The volume of arithmetic involved in these versions led to errors in most cases.

Text books give statistical formulae in a huge variety of versions, but centres should note the particular versions which are given in the formula booklet. Much confusion could be avoided if candidates were taught to use these versions exclusively. In fact candidates would benefit from direct teaching on the proper use of the formulae booklet. They need to understand which formulae are the simplest to use and also how to use them. Centres might wish to emphasise to students that the best way to calculate b is to use

$S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$ and $S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$ and then $b = S_{xy} / S_{xx}$. This is far more likely to yield correct answers than using $\frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ because of the heavy arithmetical content of the latter. Centres might also wish to emphasise the proper use of Σxp and $\Sigma x^2 p - \mu^2$.

Some candidates were unable to use the binomial tables properly. Others appeared to be unaware of their existence.

Comments on Individual Questions

1) (i) Most candidates answered this part correctly. A few placed the "1" in the numerator of the formula. A few others omitted the '1 –'. A small minority of candidates calculated the product-moment correlation coefficient. This is a correct method here, but is very much longer than necessary and gave great scope for arithmetical errors.

(ii) Some candidates understood that reverse ranking were required, and usually answered correctly. However, a large number of candidates did not appreciate this and instead found the required value of Σd^2, (40), and then attempted Trial and Improvement to find the appropriate ranks. Almost none of these candidates succeeded.

2) (i) A small minority of candidates did not recognise the geometric distribution, but introduced binomial coefficients or read values from binomial tables. Those who used the correct distribution frequently made errors. In part (a) some candidates gave 0.14^4 while others gave the wrong power of 0.86. In part (b) many candidates included 0.14. Others found $(1 - 0.86)^7$. A large number used the long method, adding up all the probabilities from $r = 1$ to 7, often making an arithmetical error. A few included an extra term, either for $r = 8$ or even $r = 0$.

(ii) This part was well answered. A few candidates used np with some value of n. Some found q/p^2.

3) (i) Most candidates recognised the binomial distribution and took values from the correct table. In part (a) many candidates failed to subtract from 1. Others found $1 - P(r \leq 8)$ instead of $1 - P(r \leq 7)$. In part (b) a common error was to find $P(r \leq 9) - P(r \leq 4)$. In both parts some candidates used the values in the table as if they were individual, rather than cumulative, probabilities. Others attempted to use the formula rather than tables, but these generally got lost in the arithmetic.

(ii) This part was generally better answered than part (i). Some candidates used $n = 6$ instead of 16.

4) (i) Most candidates substituted correctly into the most efficient formulae from the formula sheet, although some then made arithmetical errors. A few used the formulae incorrectly, e.g. $\Sigma x^2 - \left(\frac{\Sigma x}{5}\right)^2$. Those who used other versions of the formulae usually showed a misunderstanding of the formulae. In particular, Σx^2 was sometimes interpreted as $(\Sigma x)^2$, and $\Sigma(x - \bar{x})^2$ was often interpreted as $(\Sigma x - \bar{x})^2$.

(ii) Many candidates understood the point and stated that there would be no change or only a minor change due to rounding errors. Many weaker candidates thought that the value would change. Some stated that the value would be, for example, "more negative" or "inaccurate". A few seemed unaware that the value of the pmcc lies between –1 and 1.

(iii) Almost all candidates attempted the correct regression line. Most used the correct method for the gradient, *b*. Some started again, using $S_{xy} = \Sigma(x - \bar{x})(y - \bar{y})$ etc, rather than using their values of S_{xy} and S_{xx} from (i). These candidates often made arithmetical errors. A good number used the correct method for the equation of the regression line (although a few used *r* instead of *b*). Here, however, premature rounding and arithmetical errors were common. A few omitted the equation of the regression line, and proceeded directly to the estimate for *y*. A small minority chose the *x* on *y* line, found *b*' correctly, but then used $a' = \bar{y} - b'\bar{x}$. Few candidates used their calculator to find the equation directly.

5) This was a straightforward question, but many candidates lost marks through inaccuracy or failure to read the question carefully.

(i) Inaccurate reading of the two quartiles was common. 900 – 300 was also seen.

(ii) An answer of 56 (the 40th percentile instead of the 60th) was as common as the correct answer. A few gave 100 – 56 = 44.

(iii) Most candidates gave a correct answer although a few read the graph inaccurately or omitted to subtract from 1200.

(iv) Many candidates multiplied by 5 instead of raising to the power 5. Others ignored the phrase "with replacement" and either used combinations or multiplied fractions with decreasing numerators and denominators.

(v) Very few candidates understood the significance of the even spread of marks in the range 35 to 55. Many concluded that the new estimate of the IQR would be less than the original one, for all sorts of spurious reasons. Most gave irrelevant responses mentioning "skew" or the fact that the IQR only includes the middle 50% of the marks. Some stated that the IQR was 55 – 35 = 20. Others stated that the even spread confirms that the estimate of the IQR is accurate, or that it does the opposite. Many answers suggested that candidates did not understand the significance of the IQR.

6) High scores were common on this question.

(i) Many candidates found all six values correctly. Some found only *a* and *b* correctly. A few gave values of 1 and 0 for *e* and *f*. If incorrect values were correctly used in the rest of the question, method marks could still be obtained.

(ii) Most candidates gave correct solutions. Some felt the need to resort to decimals rather than multiplying and adding fractions. A few "fiddled" the solution with, for example, $^3/_5 \times {}^3/_4$. Some assumed the value for P(R = 2), used it to find k and then used their value of k to find P(R = 2).

(iii) This part was answered well on the whole. A few candidates used the long method, working from first principles, often correctly.

(iv) This part was well answered by some, but yielded all the usual types of confusion such as $\frac{\Sigma x}{4}$, $\frac{\Sigma xp}{4}$, $\frac{\Sigma p}{4}$, $\Sigma(xp)^2$, Σxp^2 and the omission of "$- \mu^2$". As in Q4, misunderstanding of the given formulae, or failure to use them, caused problems. A few candidates used $\frac{\Sigma x}{4}$ for the mean but then used $\Sigma x^2p - (\Sigma xp)^2$ for the variance.

7) Some able candidates failed to score many marks on this question. The use of combinations to find probabilities is a topic that seems to require greater emphasis in teaching.

(i) This was answered correctly by most candidates. A few found $^{18}P_7$.

In parts (ii) to (iv), many candidates tried to use a direct probability method, without combinations. Usually these methods showed a complete misunderstanding of the situation, for example, $^2/_5 \times ^2/_6 \times ^3/_7$ in part (ii). The direct probability methods for these parts are rather difficult. They involve the product of 7 fractions (or 6 in part (iv)), with denominators 18, 17, 16 etc., and then require multiplication and division by various factorials. Candidates who attempted these methods rarely succeeded.

In all three parts a few candidates found the correct numbers of combinations but lost many marks by not going on to find the probabilities.

(ii) The three correct combinations were often seen, but many candidates added them. A product of three fractions, instead of combinations, was common.

(iii) $^7C_5$ was frequently seen, but $^{11}C_2$ was less common. Candidates who tried to list all the possibilities, without combining Gloucester and Hereford into one category, rarely succeeded. Many candidates added combinations instead of multiplying. Some multiplied two or three fractions.

(iv) The most common error was to select only two from each town. Some candidates then proceeded to multiply by (or add) $^{12}C_1$ to try to fill up the 7^{th} space. Those who understood the need for (2, 2, 3) etc often achieve the correct answer, although some added instead of multiplying. A few candidates tried to use a complement method (1 – P(at least one town has 0 or 1 person)), but since this involves 19 different products of three combinations each, success was exceedingly rare!

4733: Probability and Statistics 2

General Comments

Although this was certainly not an easy paper, the quality of answers was very high and it was a pleasure to read so many good scripts. Presumably many of the candidates were first-year Further Mathematicians. There were only a few candidates who were clearly insufficiently prepared for the examination, although there was sometimes a sense that more practice on past papers could have produced improved results. At the same time, it is easy to distinguish those Centres that have noted the comments made in previous Reports from those that have not.

The attention of Centres is particularly drawn to the points noted in the Chief Examiner's report above.

Comments on Particular Questions

1) In the first part, examiners were looking for both the statement that the method was biased, and a corresponding reason. Many candidates wrote that the method was not random, and on this occasion they were given credit for this answer, although there is no *a priori* reason why a non-random method (such as systematic sampling) should be biased. The overriding reason for the unsuitability of the method is of course that many members of the population have no chance of being included.

   In part (ii), centres are again reminded that methods involving selecting names or numbers from a hat are not acceptable in this specification. Some candidates thought that members of the population should be given random numbers, rather than sequential ones.

2) This question was very well answered; most candidates scored full marks, and only a few had difficulty coping with the negative signs.

3) This question also produced a very large proportion of correct answers. Few candidates used a normal approximation in part (i), and the proportion of correct continuity corrections in part (ii) was also pleasingly high.

4) By contrast, responses to this routine question were disappointing.

   (i) This question was an explicit request for part of the solution to a standard question that has not in the past been subdivided. It seemed to confuse many, with 0.0967/50 and other answers being common.

   (ii) There is a worryingly increased tendency, noted in last year's examination, to confuse the sample and population means, so hypothesis statements such as "H_0: $\overline{X}$ = 1.72" (or "μ = 1.72") were widespread. The core idea of hypothesis testing is to assume a value for the population mean and find the likelihood of obtaining a result as extreme as the sample mean – *not* the other way round. More candidates than usual omitted the $\sqrt{50}$ factor. Some thought the test was one-tailed, and many failed to give their final conclusion in the context of heights of plants, merely saying something like "the mean is not 1.8".

5) (i) Most answered this synoptic request correctly.

   (ii) Many knew the condition $np > 5$ for the normal approximation to be valid, but surprisingly few used $nq > 5$ as well. Some used trial and improvement rather than solving the inequality, but this was usually taken no further than $0.2 < p < 0.8$, which is insufficient. Those who used $npq > 5$ (which is not the rule given in the specification) had more complicated algebra to negotiate, but if they obtained the correct answer by this method they could score full marks.

   (iii) Most correctly negotiated the appropriate continuity corrections here and obtained the correct answer; a few weaker candidates calculated P(< 10) only.

6) This was found to be much the hardest question, even though the final question of 2642 June 2003 was effectively the same question. Most could get part (i) right, but (ii) produced some woolly thinking; answers using the wrong distribution, such as 20×0.8, were often seen. The answer does not need to be an integer. In part (iii), better candidates realised that the critical value found in part (i) was to be used again here; this was the main lesson to be learnt from the 2003 question. About 6% of the candidates got the last part completely right; those who divided it up into many different cases usually omitted the $(0.5)^2$ factor, as well as about half of the possibilities. Two approaches were available. The simpler involved first finding the probability that one test resulted in rejection of the null hypothesis, and then using $2p'(1 - p')$. The longer method required eight probabilities to be found, such as $0.9532 \times 0.7265 \times 0.5^2$. Some candidates found some of these but rarely all, and omission of 0.5^2 was very usual in this method.

7) Many dealt very confidently with this question, and better candidates scored full marks on it with apparent ease. Others trapped themselves by using the wrong formula for the probability density function; kx was the usual wrong version, although $k(x - 3)$ was also seen several times. Such candidates made the question seem much harder than it was; for instance, failure to use symmetry, or the formula, to calculate μ wasted much time (and lost marks); some calculated $\frac{1}{2}(11 - 3) = 4$. In part (iii) it was disappointing that many tried to find $P(X < 9)$ by integration, or even using a normal distribution. Most could see what was happening in the last part, the most common mistake being to use $\frac{n}{n-1} \times \sigma^2$ for the variance.

8) In part (i) the conditions were poorly stated. In this sort of question it is not a good idea to use the concept of "randomness"; it needs further explanation if it is to mean anything other than "R is a random variable". For marking purposes, Examiners take statements such as "the coins are scattered randomly" to be equivalent to saying that their positions are independent. The other important condition is that the coins are scattered at a constant *average* rate. Strictly, "constant rate" implies that there is no randomness involved; those who said that the number of coins per square metre was constant were not awarded the mark. However, this particular context brings up a further issue – are the coins scattered singly? If they are likely to be found in hoards this would invalidate the model, although candidates who used the word "singly" rarely gave any indication that they were engaging with the context of the question instead of regurgitating a sentence in a textbook.

Part (ii) was almost always done correctly. In parts (iii) and (iv), some confused the given values of λ with the parameters needed for the Poisson distribution. In part (iii) many obtained the correct probabilities but then gave the final answer as 6 instead of 7. In part (iv) a pleasing proportion could use the definition of a Type II error in the context of the question.

2645: Discrete Mathematics 1

General Comments

Candidates who had learnt the standard algorithms were usually able to score reasonably well on the paper. Most candidates were able to score some marks on every question but very few candidates scored more than 50 out of a maximum mark of 60. There was some evidence that candidates who had not learnt the standard algorithms correctly and attempted questions more than once did struggle to complete the paper in the permitted time, but this was not a general problem.

Comments on Individual Questions

1) Most candidates were able to answer the first part of this question well if they knew the bubble sort algorithm.

   (i) Most candidates were able to complete the bubble sort correctly. Candidates who did make mistakes often used the shuttle sort instead. A significant number of candidates failed to show that the bubble sort used 10 comparisons and 9 swaps in sufficient detail and merely tallied without any supporting evidence to justify their tallies. The answer had been given to them in the question.

   (ii) A significant number of candidates answered this question incorrectly giving 0.5 seconds as their answer. They failed to recognise the quadratic order of the algorithm.

2) The first part of this question was answered well by most candidates. However, the rest of the question was answered poorly mainly due to a lack of clarity and accuracy in responses.

   (i) Answered well by nearly all candidates.

   (ii) This question was not answered well. Many candidates identified that an arc has 2 ends, but failed to conclude that the sum of all the orders was therefore even, thus meaning an even number of odd nodes. Other candidates indicated how additional arcs affected the order of nodes, but failed to derive their conclusions from a null graph. Some candidates attempted to use Euler's relationship to answer the question.

   (iii) This question was not answered well. Few candidates correctly identified the sum of the orders of nodes for 5 arcs as 10 and thus gained no marks. Candidates who did manage to identify this correctly, then often failed to give clear explanations as to why a simple graph with nodes ordered 1,3,3,3 is not possible, or as to why there is only one graph with nodes ordered 2,2,3,3 possible.

3) Most candidates were able to gain some marks on the first two parts of this question if they had learnt the appropriate algorithm.

   (i) Many candidates answered this question well gaining full marks. However, some candidates who drew the correct minimum connector for the network, failed to give a valid order or to give the total weight.

   (ii) Many candidates answered this question well gaining full marks. Where errors did occur candidates knew how to use the nearest neighbour algorithm but failed to return back to *A*. Some candidates drew a correct diagram for the cycle but failed to indicate any direction.

(iii) This question was not answered well by many candidates. Where candidates did understand the question and gained marks, very few gained full marks.
In part (a) many candidates failed to list the vertices on each face correctly using letters. In some cases there were insufficient letters to represent a face, or they stated the numerical values on the faces they felt met at *AG*.
Parts (b) and (c) were answered better by the candidates who did understand the question but errors were still commonplace.

4) There were many good answers to this question. Most candidates were able to set up and apply the Simplex algorithm, but a few candidates could not read of their solution from their final tableau. Numerical errors were also common in their final tableau.

(i) Generally answered well although a few candidates failed to introduce sufficient slack variables for the question.

(ii) Many candidates were able to successfully complete one iteration of the Simplex algorithm, but numerical errors were common in their final tableau. Candidates who set their initial tableau up as (8 -1 -4 0 0 0) failed to pivot correctly and as a result their value of P decreased. Some candidates were unable to correctly state the values of x, y, z and P from their final solution.

5) Most candidates were able to gain several marks on this question but few managed to get the whole question correct. Candidates who used graph paper were more successful than those who drew their graph in the answer booklet.

(i) Most candidates gained some but not all marks in attempting to represent the inequalities on a graph. Candidates tended to draw the lines $5x + 2y = 20$ and $4x + 3y = 24$ correctly, but a number of candidates then failed to draw the line $y = 2x$ correctly, drawing $2y = x$ instead. Candidates who did not use graph paper in many cases lost accuracy when drawing $y = 2x$ on their graph.

A significant number of candidates failed to shade the regions not satisfied by the inequalities correctly. A common error was candidates who failed to shade appropriately for the trivial constraint $y \geq 0$.

(ii) Most candidates gained the first mark correctly but many made errors when introducing a slack variable to $y \geq 2x$.

(iii) This question was generally answered well by all candidates who drew a graph.

(iv) Having successfully identified any vertex on their graph, most candidates were able to calculate the corresponding value of the objective function at their point or points correctly to gain a method mark. However, accuracy was often lost from reading off from poor diagrams or from not identifying the correct vertex required to maximise P. Where the appropriate constraints were solved simultaneously to find values for x and y accuracy was improved greatly.

6) On the whole this question was not answered well. The candidates who did make a serious attempt at it did gain marks, but many of them lost the accuracy marks associated with each section.

(i) Generally answered well by most candidates. Weaker candidates stated the objective to be minimise $x+y+z$ or tried to explain in words what to do.

(ii) Most candidates gained some marks on this question but few gained full marks. Many candidates simplified the constraint to $30x+27.5y+22.5z \geq 250$ but went no further with the simplification.

(iii) Most candidates again gained some marks on this question but very few gained full marks. Many correctly identified the trivial constraints and the constraint for the number of edging tiles needed. Having found this constraint some candidates then failed to simplify it correctly, whilst some left it as an equality. Relatively few candidates successfully identified an appropriate constraint for at least half as many small tiles as large tiles. If this was done correctly then fewer still were able to simplify the constraint fully.

7) The first two parts of this question was generally answered well, although very few candidates successfully completed the last part.

(i) Most candidates appeared to know how to use Dijkstra's algorithm correctly. Some candidates did fail to update their temporary labels correctly and had missing values at nodes *F* and *H*. Some candidates failed to list the shortest route the algorithm gave and merely stated the shortest length.

(ii) This question was generally answered quite well. Most candidates correctly identified the odd nodes and attempted to pair them, although some candidates did not attempt all three pairings or give sufficient evidence to show that this was the case. Having successfully identified 30, or the pairing of *AB* and *CH*, some candidates clearly did not know what to do with this value.

(iii) Most candidates failed to gain any marks on this question. The most common error when an attempt was made was taking the length of arc *BD* (=15) off 155 to give 140km.

2646: Discrete Mathematics 2

General Comments

A small but significant minority of candidates did not seem to be ready to sit this paper. Some candidates did not know the standard methods and were using approaches that were more appropriate for an AS paper than for an A2 paper. Candidates should take care when reading the questions and over the presentation of their answers. The best candidates explained their reasoning and gave full, accurate and efficient solutions.

Comments on Individual Questions

1) A slight twist on the usual bipartite graphs question, but most of the candidates dealt with it well. Candidates should list their alternating paths clearly, and alternating paths should be efficient, aiming to make as few changes as possible. In part (iv), some candidates augmented their solution from part (iii) instead of going back to the original incomplete matching.

2) Several candidates minimised instead of maximising, although some of these were able to find the correct matching by inspection. Candidates were not penalised for writing 'quaterback' instead of 'quarterback'.

3) Candidates always find the labelling procedure difficult and this year was no exception. The use of an insert meant that candidates were able to achieve tidier answers than usual. In part (i), the best candidates were able to understand the notation and identify the required arcs as being *AD*, *CF* and *EB* (in that direction). Many candidates included extra arcs, or arcs that did not exist, and consequently got the wrong value for the cut.

   Most candidates were able to attempt the labelling procedure in part (ii), although often they only labelled the arcs where there was flow and left the middle five arcs blank. Candidates who had set up the initial labelling correctly were usually able to augment the flow, although some omitted to record the flow augmenting path *SCEBDT*.

   In part (iv), some candidates summed the flows through their cut arcs in error, and so all their cuts came out as 13. Even with a correct cut identified, it was not enough to just find a cut of 13 and then state the theorem, what was required was a cut of 13 and a statement that we have already constructed a flow of 13.

4) Several candidates had problems with the precedences, in particular for activity *H*. Even so, they were able to carry out forward and backwards passes to find the completion time and the critical activities. Some candidates did not write the completion time down and just left it to be deduced from the diagram, this incurred a one mark penalty. Most candidates realised that the changes given in part (ii) had no effect on the completion time, and some gave explanations of why this was the case.

   In part (iii), several candidates did not appear to have read the question carefully enough and so went straight for a solution using only three people. Also, several candidates do not understand the difference between a cascade chart, a resource histogram and a schedule. In a cascade chart each activity starts at its earliest possible start time either with one activity to a row or with a row for the critical activities and rows for the non-critical runs, the horizontal axis shows time and the vertical axis is unlabelled. In a resource histogram the horizontal axis shows time and the vertical axis shows number of people (or whatever is appropriate to the question), activities are plotted at their earliest start times to form a solid block of histogram with no holes in it and no activities hanging out over gaps. In a schedule the time axis may run horizontally or vertically and the activities are assigned to people to show which activity each person is doing at each time, people may have rest periods in a scheduling diagram.

5) Some candidates wasted time drawing out a network of the situation. Most were able to construct an appropriate format for the tabulation, although some omitted the 'action' column and a few worked the stages the wrong way round. The action values should be the state label for the state being moved into, not the (stage, state) label and not just a consecutive list of values starting from 0 or from 1.

Apart from arithmetic slips, there were many good solutions. Some candidates omitted to write down the route or to state the number of plants that would be seen using this route.

6) Another topic that candidates find difficult. Most candidates were able to say what 'zero-sum' means, although others appeared to think that zero-sum means 'two-person' or that it means 'stable'. In part (ii), candidates needed to show that no row was dominant over any other row and that no column was dominant over any other column, some just gave the definitions of dominance in an abstract context. Specific instances of dominance are required, together with some sort of an explanation. For example, when Rhoda plays row *S*: $DS = 3$ and $ES = -2$, so if Colin plays *D* he loses more than if he plays *E*, and so *D* does not dominate *E*. This can, of course, be condensed into something like $-2 < 3$ so *D* does not dominate *E*. Some candidates seemed to have forgotten that Colin's gains were the negatives of the entries in the table.

In showing that the game is not stable, most candidates chose to find the play-safe strategies for each player. However, many candidates erroneously identified -1 as the row minimum for row *S*, and so their argument was not acceptable.

For part (iv), most candidates said that 2 had been added to 'get rid of the negatives', or sometimes they claimed that it 'made everything positive', but then if they said any more it was to say, rather vaguely, that the Simplex method will not work with negatives. In reality, the matrix entries need to be non-negative so that for all values of p_1, p_2, and p_3 the value of *m* is non-negative.

Many candidates were able to identify where the expressions $5p_1 + 3p_3$, and so on, had come from, but only the very best candidates could explain that the minimum expected pay-off (*m*) is the smallest of the three values $5p_1 + 3p_3$, and so on, for each combination of probabilities, and hence $m \leq$ each of the expressions. This is a difficult concept and those who referred to the graph and finding the highest position on the lower boundary were usually the most successful.

Several candidates used the given values of the probabilities to substitute into the inequalities to find a value for *m*, and some remembered to subtract 2 to convert to a value for *M*, however, some then chose the largest value (or the first value), rather than the least value. Again, reference to the graph sometimes helped.

Finally, for the many candidates who reached the end of the paper, part (vii) required a mechanism for choosing strategy *S* with probability $\frac{3}{8}$ and strategy *T* with probability $\frac{5}{8}$, to achieve a long-run minimum expected gain of at least $-\frac{1}{8}$ per game. Most candidates just let her choose *S* if the coin landed heads and *T* if it landed tails, but some were able to devise appropriate methods, usually by using the results from three coin tosses.

4736: Decision Mathematics 1

General Comments

Most candidates gained some marks on every question they attempted, although some candidates ran short of time. Candidates were usually able to deal with standard situations and straightforward applications of the algorithms, but the weaker candidates were unable to deal with the parts that required a deeper understanding. Many candidates did not take the time to read the questions carefully and rushed into answers that were not appropriate.

Although the use of coloured pens may be helpful in diagrams, candidates should be advised that text should be in blue or black ink and that red ink should not be used. Examiners often had difficulty in reading candidates' answers due to poor handwriting or overwriting of numerical answers. Some candidates did not use graph paper which resulted in messy graphs.

Candidates need to balance the time spent on a question against the number of marks it is worth.

Comments on Individual Questions

1) This straightforward opening question was generally well done, although in part (a)(i) several candidates applied first-fit rather than first-fit decreasing. When applying first-fit and first-fit decreasing candidates need to remember to always go back and check the bins (bags) that have already been partially filled, in the order that they were used. A few candidates wasted time by formally applying a sorting algorithm to put the list into decreasing order, this is not necessary in packing problems, unless the question specifically asks for it. Most candidates were able to find a better packing for part (a)(ii), although some had already used an ad hoc approach in the first part.

Several candidates assumed a linear order algorithm in part (b), some seeming to think that this referred to packing algorithms still, and quite a few worked with a quadratic order algorithm, even though the question had stated cubic order. If an algorithm is of cubic order then for large problems the run time is approximately proportional to the cube of the size of the problem, scaling the problem size by a factor of 5 will scale the run time by a factor of 5^3 approximately, so the run time for the larger problem is approximately $5^3 \times 4$ seconds = 500 seconds.

2) Part (i) was usually answered correctly but candidates, as always, found it difficult to explain the results in parts (ii) and (iii), although there did seem to be some appreciation of the concepts involved. Some candidates happily interchanged the terms 'vertex' (or 'node') and 'arc' (or 'edge'), resulting in some very peculiar answers. The majority of attempts to parts (ii) and (iii) were too vague to earn much credit.

In part (ii), several candidates gave answers that referred specifically to the graph from part (i) rather than 'any graph', as stated in the question. Many candidates stated, incorrectly, that 'an odd node must connect to another odd node' whilst others appreciated that there could be a path of even nodes connecting the two odd nodes. The most successful answers were from candidates who either explained that since each arc has two ends the sum of the orders of the vertices must be even, and then went on to explain why this means that, whatever the number of even vertices, there must be an even number of odd vertices. Another approach that worked well provided it was done carefully was to start from a null graph and consider what happens to the orders of the vertices when arcs are added (including the case of a loop from a vertex to itself, since the question did not say that the graph needed to be simple).

In part (iii), candidates were advised to consider the orders of the vertices. Since the graph has five arcs, the sum of the orders must be ten. Since the graph connects four vertices, each of the orders must be at least one, and since the graph is simple, no order can be more than three. Also, the number of odd vertices must be even. This gives two possibilities, namely 1, 3, 3, 3 and 2, 2, 3, 3. It is easy to explain why the first of these is not simple and an exhaustive argument can be used to show that for the second possibility the two odd vertices cannot be adjacent in a simple graph.

3) Generally answered well, including the unfamiliar application in part (iii). Most candidates were able to construct an appropriate minimum connector (spanning tree) in part (i), although a minority wasted a lot of time by using the matrix formulation of Prim's algorithm, and some of these did not draw a diagram to show their minimum connector. A few candidates made errors in listing the order in which the tree had been built or in adding up the total weight of the tree.

In part (ii), most candidates were able to apply the nearest neighbour method correctly, a few stopped their route at *E*, not appreciating that the method says to choose the 'least weight arc to a vertex that has not already been visited' and some candidates did not complete the cycle by connecting the final vertex back to *A*. Candidates who draw their solutions need to remember to indicate the direction of travel.

Some candidates could not sort out what was happening in part (iii), but most had a good try at it. In part (a), candidates were asked to list the vertices of each of the two faces, this required two sets of four vertices not just a list of the six vertices concerned. Some candidates listed the edges and rather more gave the face numbers. Parts (b) and (c) were generally well answered by the candidates who attempted them.

4) For many candidates this was their most successful answer. Having the template on the insert seems to help candidates to keep their working tidier. In part (i), many candidates still give unnecessary values at the temporary labels (only values that are an improvement on any current value need to be recorded), the route taken to reach the temporary values does not need to be recorded on the diagram. On this occasion we condoned the inclusion of 'extra' temporary labels, but for a strict application of Dijkstra's algorithm these would be wrong.

The candidates who understood that parts (ii) and (iii) were about the route inspection (Chinese Postman) algorithm generally answered these parts correctly. Some candidates just tried to find routes that satisfied the requirements using ad hoc methods, but usually they made errors, either in the route or in adding up the weights. In part (iii), many candidates did not appreciate that *G* needed to become odd and so the only extra arc needed was to double up on *FG*. Some candidates added arcs to their solution to part (ii), rather than going back to the stem. An alarming number of candidates gave answers that were smaller than 120 km.

Candidates need to expect to have to extend the applications of the standard algorithms and not just concentrate on straightforward problems.

5) A few candidates were unable to follow the instructions correctly, and many others made small arithmetic errors, but most candidates were able to achieve full marks for part (i). Some candidates did not accumulate the sums, and just listed the individual values of X and X^2, the fact that some of them then needed to find the square root of a negative number did not prompt them to consider that they had made a mistake. In part (ii) candidates needed to record both the number of additions and the number of multiplications carried out in Step 3 before finding the total number of arithmetic operations. Several candidates left out the subtraction, or counted the additions twice. Candidates who had correctly found the total 25 in part (ii) sometimes then assumed a quadratic order algorithm and consequently got the time wrong in part (iv). Many candidates did appreciate that the algorithm was of linear order and there were several correct answers to part (iv).

Some candidates spent far too much time on this question, with answers sometimes running to several pages of working.

6) A small number of candidates were not able to even set up the initial tableau correctly. Some candidates omitted the slack variables and some made copying errors, or even left out a complete row of the tableau. The candidates who did not rearrange the objective first often went on to choose the *y*- or *z*-column for their pivot and usually got themselves into a mess. Candidates should be encouraged

to rearrange the profit equation so that it equals zero and them set up their tableau using these coefficients. The candidates who correctly chose to pivot on the x-column should have chosen to pivot on the 10 from the first constraint, but some chose the –6 as giving the smaller ratio, having overlooked the requirement that it needs to be the smallest non-negative ratio.

Giving evidence of the pivot operations, such as writing $R_3 - (-6 \times \text{npr})$, helped candidates to score method marks even when they had made arithmetic errors.

Some candidates did not record the values from the right-hand side of the equations (40, 72, 48) and so could not interpret the results of their iterations, and some candidates ignored the request in part (ii) for the values of x, y, z and P resulting from the first iteration.

The markscheme generally allowed candidates who had some idea of how to apply the Simplex algorithm to obtain good credit for the things that they were able to do.

7) Most candidates made an attempt at this question, but there were few really good answers. Candidates find it difficult to extract algebraic constraints from text, even when the information is set out clearly. A common error in part (i) was to omit the non-negativity constraints ($x \geq 0$, $y \geq 0$, $z \geq 0$) or to fail to give the objective in an algebraic form.

In part (ii)(a), most candidates were able to substitute $z = 50$ into their constraints, but some just chose to ignore any constraint involving z. There were very few correct graphs drawn, even from candidates who had the correct constraints. The line $y = 2x$ seemed to cause some problems, and some candidates seemed confused about which axis was their x-axis and which was their y-axis. Many candidates who attempted the shading seemed to have assumed that the feasible region would be an enclosed (bounded) region.

Most of the candidates who obtained reasonable, even if incorrect, graphs in part (ii)(a) were able to follow through to obtain at least the method marks in part (ii)(b). The candidates who answered part (iii) either tried to give a general argument or gave a specific example to show that the cost could be reduced. The candidates who gave a specific example tended to have more convincing answers, although a statement such as 'z can be 0' without reference to x and y was not sufficient.

4737: Decision Mathematics 2

General Comments

Several candidates seemed to have difficulty in completing this paper in the time allowed. In some cases this was due to excessive time having been spent on drawing unnecessary diagrams or reworking questions several times. Some candidates did not seem to be ready to sit this paper, using approaches that were more appropriate for an AS paper than for an A2 paper. Candidates should take care when reading the questions and over the presentation of their answers. The best candidates explained their reasoning and gave full, accurate and efficient solutions.

Comments on Individual Questions

1) Candidates always find the labelling procedure difficult so it was perhaps an unexpected first question. The use of an insert meant that candidates were able to achieve tidier answers than usual. In part (i), the best candidates were able to understand the notation and identify the required arcs as being *AD*, *CF* and *EB* (in that direction). Many candidates included extra arcs, or arcs that did not exist, and consequently got the wrong value for the cut.

   Most candidates were able to attempt the labelling procedure in part (ii), although often they only labelled the arcs where there was flow and left the middle five arcs blank. Candidates who had set up the initial labelling correctly were usually able to augment the flow, although some omitted to record the flow augmenting path *SCEBDT*.

   In part (iv), some candidates summed the flows through their cut arcs in error, and so all their cuts came out as 13. Even with a correct cut identified, it was not enough to just find a cut of 13 and then state the theorem, what was required was a cut of 13 and a statement that we have already constructed a flow of 13.

   A few candidates gave very good answers to part (v), but although most could state that the flow is now 11 (although often without units) and show the flow on a diagram, they could rarely explain why this was the maximum. Ideally they would give the cut $\{S, C, E, F\}$, $\{A, B, D, T\}$. Explanations in words rarely covered both the reasons why *BE* was redundant and why no more than 5 could flow around the 'top' route and no more than 6 could flow around the 'bottom' route.

2) There were many good answers to parts (i) to (iv) of this question. The bipartite graph was usually drawn correctly and the matchings were found. Some candidates did not show their alternating path clearly, ideally the alternating path should be listed. The alternating path needs to be efficient, aiming to make as few changes as possible.

   The Hungarian algorithm was usually done well, apart from candidates who did not set up the initial matrix as described and then solved a minimisation problem. Some candidates did not augment efficiently, making two augmentations of size 1 instead of a single augmentation of size 2.

3) Most candidates were able to construct an appropriate activity network. Candidates who used activity of node were penalised in part (i) only. Several candidates used vast numbers of inefficient dummy activities to ensure that the precedences were correct, this was accepted this time, but is not to be encouraged.

   Most candidates were able to carry out a forwards and backwards pass through their network, some omitted to write down the critical activities and the minimum project completion time. The cascade charts were usually drawn well, although candidates who did not use graph paper had some problems.

   Most candidates had a good attempt at providing a schedule, they seemed to understand what was required, but lost track of the precedences so that often *F* ended up alongside either *G* or *H*.

4) In part (i), some candidates wasted time drawing out a network of the situation. Most were able to construct an appropriate format for the tabulation, although some omitted the 'action' column and a few worked the stages the wrong way round. The action values should be the state label for the state being moved into, not the (stage, state) label and not just a consecutive list of values starting from 0 or from 1.

Apart from arithmetic slips, there were many good solutions. Some candidates omitted to write down the route or to state the number of plants that would be seen using this route.

Many candidates identified the problem in part (ii) as being a minimax problem, and several were able to give an appropriate route and to explain why every route has at least one path (at stage 5 or at stage 3) where there are at least 6 plants.

5) Another topic that candidates find difficult. Most candidates were able to say what 'zero-sum' means, although others appeared to think that zero-sum means 'two-person' or that it means 'stable'. In part (ii), candidates needed to show that no row was dominant over any other row and that no column was dominant over any other column, some just gave the definitions of dominance in an abstract context. Specific instances of dominance are required, together with some sort of an explanation. For example, when Rhoda plays row S: $DS = 3$ and $ES = -2$, so if Colin plays D he loses more than if he plays E, and so D does not dominate E. This can, of course, be condensed into something like $-2 < 3$ so D does not dominate E. Some candidates seemed to have forgotten that Colin's gains were the negatives of the entries in the table.

In showing that the game is not stable, most candidates chose to find the play-safe strategies for each player. However, many candidates erroneously identified -1 as the row minimum for row S, and so their argument was not acceptable.

For part (iv), most candidates said that 2 had been added to 'get rid of the negatives', or sometimes they claimed that it 'made everything positive', but then if they said any more it was to rather vaguely say that the Simplex method will not work with negatives. In reality, the matrix entries need to be non-negative so that for all values of p_1, p_2, and p_3 the value of m is non-negative.

Many candidates were able to identify where the expressions $5p_1 + 3p_3$, and so on, had come from, but only the very best candidates could explain that the minimum expected pay-off (m) is the smallest of the three values $5p_1 + 3p_3$, and so on, for each combination of probabilities, and hence $m \leq$ each of the expressions. This is a difficult concept and those who referred to the graph and finding the highest position on the lower boundary were usually the most successful.

Part (vi) was often left unanswered, this is an aspect of this topic that candidates usually find easy. In part (vii), several candidates used the given values of the probabilities to substitute into the inequalities to find a value for m, and some remembered to subtract 2 to convert to a value for M, however, some then chose the largest value (or the first value), rather than the least value. For those who had drawn an appropriate graph, reference to the graph usually helped.

Finally, for the many candidates who reached the end of the paper, part (viii) required a mechanism for choosing strategy S with probability $\frac{3}{8}$ and strategy T with probability $\frac{5}{8}$, to achieve a long-run minimum expected gain of at least $-\frac{1}{8}$ per game. Most candidates just let her choose S if the coin landed heads and T if it landed tails, but some were able to devise appropriate methods, usually by using the results from three coin tosses.

7840, 7842, 7844, 3840, 3841, 3842, 3843, 3844 AS and A2 Mathematics
June 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
2631	Raw	60	49	43	37	31	25	0
2632	Raw	60	50	43	37	31	25	0
2633	Raw	60	48	41	35	29	23	0
2634	Raw	60	49	43	38	33	28	0
2635	Raw	60	45	39	34	29	24	0
2636	Raw	60	46	40	35	30	25	0
2637	Raw	60	46	40	34	29	24	0
2638	Raw	60	45	38	32	26	20	0
2639	Raw	60	48	42	36	30	24	0
2640	Raw	60	47	40	34	28	22	0
2641	Raw	60	44	38	33	28	23	0
2642	Raw	60	47	41	35	29	24	0
2643	Raw	60	39	35	31	27	23	0
2644	Raw	60	45	39	34	29	24	0
2645	Raw	60	42	36	31	26	21	0
2646	Raw	60	42	36	31	26	21	0
2647	Raw	60	48	42	36	30	24	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7840/7842/7844	600	480	420	360	300	240	0
3840/3841/3842/ 3843/3844	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
2631	30.0	50.3	69.5	81.1	89.6	100.0	1559
2632	30.4	51.8	65.8	76.3	84.7	100.0	3383
2633	19.3	35.9	49.9	61.7	73.4	100.0	6595
2634	38.4	57.4	69.5	78.8	85.0	100.0	1220
2635	37.9	54.0	65.6	78.8	88.4	100.0	1057
2636	36.6	54.1	65.0	76.3	83.7	100.0	1061
2637	29.9	46.8	64.2	75.9	85.5	100.0	1736
2638	26.1	43.2	57.0	70.0	80.8	100.0	2929
2639	45.2	59.7	72.5	82.3	88.2	100.0	865
2640	44.7	62.3	79.5	87.9	93.5	100.0	215
2641	18.9	36.7	53.5	70.5	83.0	100.0	1384
2642	26.2	42.9	58.5	71.3	81.1	100.0	3800
2643	35.4	48.3	63.0	74.6	84.0	100.0	743
2644	46.8	65.3	80.7	87.9	91.9	100.0	124
2645	27.9	51.7	66.3	78.4	86.8	100.0	924
2646	22.1	43.5	61.6	75.4	85.1	100.0	814
2647	63.2	87.7	93.0	96.5	96.5	100.0	57

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7840	43.6	64.2	78.5	89.4	95.9	100.0	8080
7842	50.0	50.0	50.0	50.0	50.0	100.0	4
7844	60.2	76.9	86.6	92.9	97.1	100.0	1272
3840	58.4	73.2	82.3	89.1	96.1	100.0	889
3841	24.3	44.4	61.9	79.0	93.5	100.0	943
3842	0	0	50.0	75.0	100.0	100.0	4
3843	50.0	61.1	61.1	66.7	83.3	100.0	18
3844	60.6	75.0	84.5	91.6	95.5	100.0	820

7890, 3890, 3891, 3892 AS and A2 Mathematics
June 2005 Assessment Session

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4721	Raw	72	56	49	42	35	28	0
4722	Raw	72	54	47	40	33	26	0
4723	Raw	72	57	49	41	34	27	0
4724	Raw	72	54	46	38	31	24	0
4725	Raw	72	61	53	45	38	31	0
4728	Raw	72	49	41	34	27	20	0
4729	Raw	72	53	45	38	31	24	0
4732	Raw	72	58	50	43	36	29	0
4733	Raw	72	57	50	43	36	29	0
4736	Raw	72	49	43	37	31	25	0
4737	Raw	72	51	44	38	32	26	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7890	600	480	420	360	300	240	0
3890/3891/3892	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
4721	31.5	46.2	59.8	71.3	81.2	100.0	12500
4722	33.9	46.7	58.5	69.0	78.2	100.0	15490
4723	39.2	58.4	71.7	79.7	86.4	100.0	2899
4724	25.9	42.2	57.8	68.1	78.0	100.0	2521
4725	52.4	69.3	79.8	86.2	91.7	100.0	1101
4728	35.0	47.2	57.5	67.5	78.1	100.0	5058
4729	45.3	61.8	74.2	81.8	87.8	100.0	468
4732	34.4	50.7	63.6	74.5	82.6	100.0	8677
4733	40.6	57.8	71.0	81.2	87.5	100.0	303
4736	25.5	41.2	55.9	70.0	80.7	100.0	2444
4737	29.7	49.2	67.4	80.1	87.7	100.0	236

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7890	35.2	58.1	74.2	86.3	94.4	100.0	1790
3890	30.7	45.6	59.1	71.7	82.0	100.0	10927
3891	0	0	0	0	0	100.0	1
3892	54.6	71.8	87.7	95.5	98.6	100.0	220

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